FALLACY No. 25

Contributed by Mr. D. Howell, S.W. Herts. Further Education Centre, Watford.

Let $a^x = -1$ (1) Squaring, $1^{2x} = +1 = a^0$

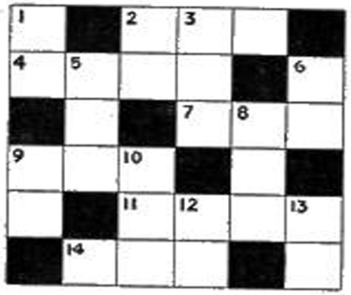
$\therefore$ $2x = 0$

$\therefore$ $x = 0$

Hence, from (1) $a^0 = -1$

But $a^0 = +1$, $\therefore 1 = -1$.

JUNIOR CROSS-FIGURE No. 24



ACROSS

2. $12(\frac{2}{3} \div \frac{8}{11} \div \frac{1}{27})$.
4. $\frac{7}{12}$ of 11 ac. $-$ 5 dn. $-$ $\frac{1}{2}$ of 14 ac. $+1$.
7. Square yards in $\frac{3}{20}$ of an acre.
9. y^2 (see 1 dn.)
11. L.C.M. of 420, 115, 322.
14. $\dfrac{444}{\sqrt{4}} - \sqrt{4}$.

DOWN

1. Value of x given by $x - y = 3$, $5x - 6y = 3$.
2. Pence in $\frac{3}{4}$ of half-a-crown $\div$ $\frac{3}{8}$ of a shilling.
3. Average speed in m.p.h. of a Comet which flew 2,950 miles in $6\frac{1}{4}$ hours.
5. Reverse of 10 dn.
6. Number of sides of a regular polygon whose interior angle is 150°.
8. Twice 2 ac.
9. Area in square yards of a border 3 ft. wide surrounding a carpet 3 yards square.
10. See 5 dn.
12. $\frac{1}{3}xy$ (see 1 dn.)
13. 3s 3d. as a decimal of £3 5s.

I.L.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 25

PIED PIPER.
15 at 2s. 1d. and 33 at 4s. 1d.

POSTMAN'S KNOCK.
There are only 10 different digits, *viz.*, 0, 1, 2, – – – 9.

SENIOR CROSS-FIGURE No. 25.
ACROSS: (2) 488; (5) .7782; (7) 49°; (8) 448; (10) 135°; (12) 7, 6, 5; (13) 441; (15) 80; (16) 6162; (19) 189.
DOWN: (1) 27; (2) 48; (3) 824; (4) 29; (6) 76340; (7) 48636; (9) 47; (11) 54°; (14) 168; (15) 84; (17) 19; (18) 24.

SEVENTH HEAVEN.
There are two possible ways of dividing the casks of wine.

	A	B	C		A	B	C
Full	3	3	1	OR	3	2	2
Half-full	1	1	5		1	3	3
Empty	3	3	1		3	2	2

ATENSION.
The tension in the first coupling must be equal and opposite to the frictional resistance to six trucks. The tension in the second coupling must be equal and opposite to the frictional resistance to five trucks; and so on.

Since the tension in the first coupling is 100 lb. and the trucks are equally loaded; the remaining tensions are: $\frac{5}{6}$ of 100 lb. $= 83\frac{1}{3}$ lb.; $\frac{4}{6}$ of 100 lb. $= 66\frac{2}{3}$ lb.; $\frac{3}{6}$ of 100 $=$ 50 lb.; $\frac{2}{6}$ of 100 lb. $= 33\frac{1}{3}$ lb. and $\frac{1}{6}$ of 100 lb. $= 16\frac{2}{3}$ lb.

£. s. d. $\dfrac{240}{253}$

JUNIOR CROSS-FIGURE No. 23.
ACROSS: (2) 144; (3) 263; (5) 37.5%; (7) 191.
DOWN: (1) 3456 ins.; (3) 28.3 sq. ins.; (4) 35.5 lb.; (6) 789; (7) 12; (8) 18d.

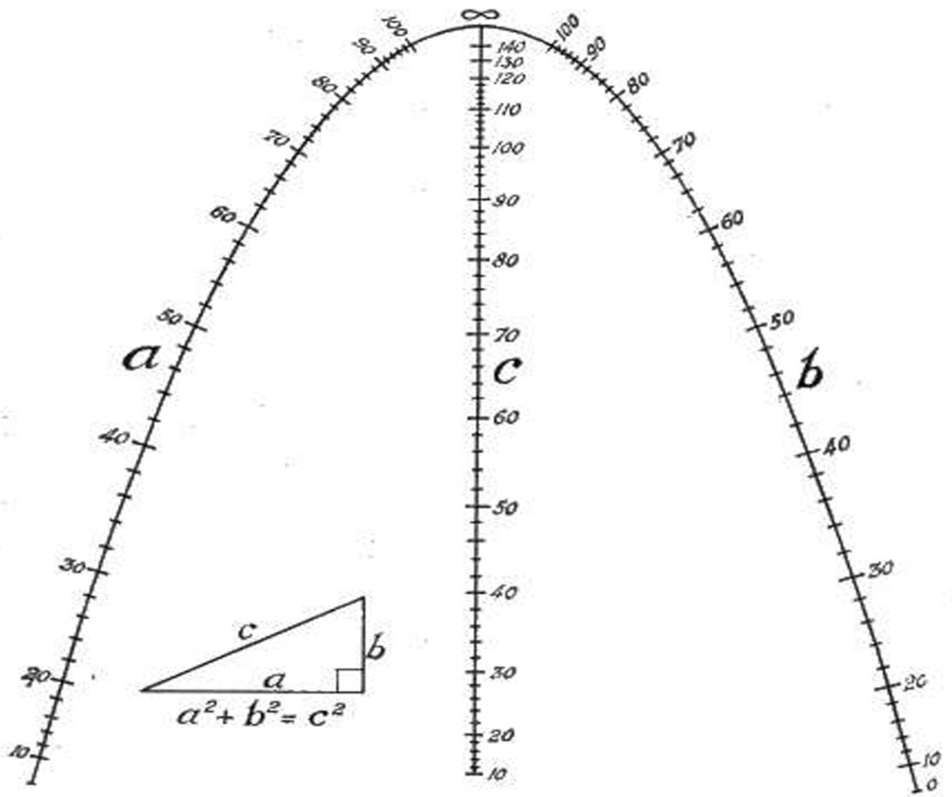
I.L.C.

33214 45713 86875 19435 06430 21845 31910 48481

MATHEMATICAL PIE

No. 26 Editorial Offices: 97 Chequer Road, Doncaster FEBRUARY, 1959

NOMOGRAM FOR PYTHAGORAS



If any two sides of a right-angled triangle are known, the nomogram can be used to find the third side. To find the hypotenuse of a right-angled triangle whose perpendicular sides are 33 in. and 56 in., lay a straight-edge from 33 on the *a*-scale to 56 on the *b*-scale. It cuts the *c*-scale at 65. The hypotenuse is therefore 65 in. If the measurements do not lie conveniently on the scales a scale factor can be used.

C.V.G.

60101 50330 86179 28680 92087 47609 17824 93858

MATHEMATICAL INSTRUMENTS No. 6.

The Sacred Calabash

The people of Hawaii regularly made the long voyage of 2,300 miles to Tahiti in the Society Islands. The canoe crews were selected with great care and undertook a period of strenuous training. Great double canoes, ninety feet long with a covered deck-house on the platform between the two hulls, were loaded with stores and when all was ready the Sacred Calabash was carried on board.

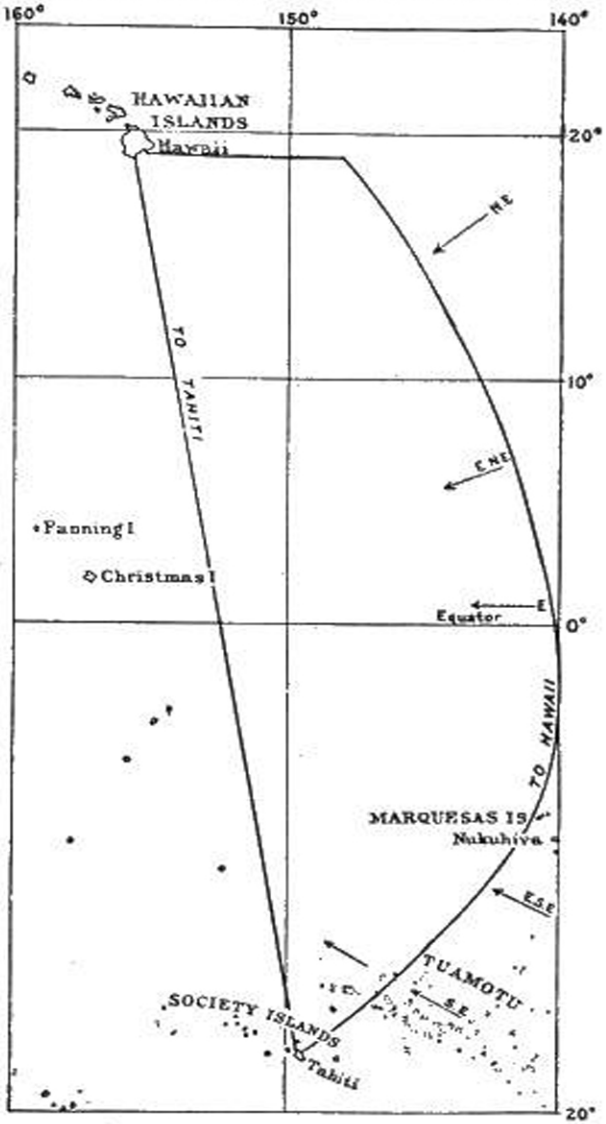


Diagram showing approximate courses from Hawaii to Tahiti and return.
Arrows indicate direction of prevailing winds

For the first half of the journey the canoe was out of sight of land. A course was steered a little to the east of south. The Sun was the guide by day and the Pole Star by night until it dropped below the horizon as the equator was crossed. South of the equator the travellers entered an area with many groups of islands and atolls and these were their guide to Tahiti.

On the return journey the course was laid to the east of north through the Marquesas, because the prevailing winds and currents would tend to carry the canoe to the westward. The islanders had a thorough knowledge of winds and currents. There still survive many " charts " made of strips of wood bound together, which are believed to represent ocean currents, but no one now knows how they were used. When the canoes had left behind the last of the

90097 14909 67598 52613 85549 78189 31297 84821

Do they ever stop? The fact is that π is not a nice tidy number like 8, for example: if we represent 8 by a lump of sugar, π is more like a spoonful of treacle with a thin streamer dangling from it. Since you cannot wait all day for the streamer to stop running you bring the process to a halt by giving the spoon a twist. This is just what we do with π. The rough and ready twist that most people know gives the value $\frac{22}{7}$ to π; a much better and nearer value is 3·1416—more useful because it has the factors $3 \times 7 \times 8 \times 11 \times 0{\cdot}0017$.

Since ancient times, mathematicians have tried to get nearer and nearer to the true value of π and, in the sixteenth century, Van Ceulen spent most of his life calculating π to 35 decimal places. Those who hoped to find an exact figure were doomed to disappointment because π is not only *irrational*, it is also *transcendental* (sixth form words) and never terminates when expressed as a decimal.

When electronic computing machines came into use about ten years ago, it was not long before a machine was set the task of extending knowledge in respect of an approximation to π. The ENIAC machine determined π to 2,035 places in 1949, and the Editor of MATHEMATICAL PIE started the publication of this determination in Issue No. 18, May 1956, by printing the numbers along the bottom of the pages. Since 320 figures are printed in an average issue, the ENIAC determination would have been used up in seven issues. A new determination has been made, however, by Mr. G. E. Felton using the Ferranti Pegasus Computer which carried the evaluation to 10,000 places in 33 hours machine time. On an ordinary desk calculator, this work would have taken about a hundred years! New machines now being designed should do it in an hour!

Mr. Felton has kindly given permission for his determination to be used in MATHEMATICAL PIE and gives some interesting details of how it was done. He writes:

" The calculation was done on the Pegasus Computer at the Ferranti London Computer Centre on various weekends between August 1956 and April 1957. Pegasus is a medium-sized machine which does an addition or subtraction in about 300 micro-seconds; a multiplication in about 2 milliseconds and a division in about $5\frac{1}{4}$ milli-seconds. It has a store with a capacity of about 4000 numbers, each of 11 decimal digits and a sign. Internal operations are carried out in the binary scale, but all the conversions into and out of this scale are done by the computer itself. The formula used for the calculation was the following, due, I believe, to Klingenstierna:

$$\pi = 32 \text{ arc cot } 10 - 4 \text{ arc cot } 239 - 16 \text{ arc cot } 515.$$

The three arc cotangents were evaluated with the Gregory series:

$$\text{arc cot } x = \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + - - - - -$$

" For the value of 32 arc cot 10 about 5000 terms had to be evaluated, each to 10021 decimals; this took about 12 hours on the computer.

" I am now trying to check the value by using the formula of Gauss:

$$\pi = 48 \text{ arc cot } 18 + 32 \text{ arc cot } 57 - 20 \text{ arc cot } 239.$$

" This check is much more lengthy than the original calculation and has so far only reached 7733 digits. There is disagreement beyond 7480 digits and I am at present investigating this when I get the opportunity.

" I have also used a similar process to evaluate a number of logarithms to just over 1000 digits ".

We feel sure all our readers are grateful to Mr. Felton for giving us an insight into this fascinating work and wish him all success in his researches.

NOTE.—Those readers not familiar with the " arc " notation should read arc cot x as " the angle whose cotangent is x; it is preferable to the alternative " $\cot^{-1} x$ " for more than one reason.

J.F.H.

03742 00731 05785 39062 19838 74478 08478 48968

original, his presentation of the subject was successful in improving the teaching of the theory of numbers and in demonstrating geometry. The early Christian scholars lived too intense a religious life and their persecutions were too close for them to devote time to such a speculative subject as mathematics, so it is not surprising that the subject lay almost dormant until Christianity had become powerful.

One of the greatest of the Church scholars was Baeda, the Venerable Bede (c. 673–735), who has been called " the father of English learning ". His mathematical interests were in the ancient number theory, the calendar and the finger symbolism in number.

The next great European scholar in mathematics was Alcuin (735–804), who studied in Italy, taught in York and later became an abbot. He wrote on arithmetic, geometry and astronomy and his name is connected with a collection of puzzle problems.

After the death of Alcuin, the invasion of Britain by the Danes destroyed the security necessary for intellectual development and there followed a marked decay in British learning. I.L.C.

FUN WITH NUMBERS No. 5

From " Le Facteur X ".

(a) Put brackets in the following statement so that it is now true :

$$3 + 4 - 5 - 3 - 1 - 5 = 12.$$

(b) Can you justify the expression $a + b \times a + b + a \times b = 122$ by inserting suitable brackets and choosing a and b from amongst the three numbers 3, 4 and 5?

STAMP COLLECTORS' CORNER No. 6

Belgium 1942,
1·75 fr. + 50c *dark blue.*

GERARD KREMER (1512–1594), who latinised his name to GERADUS MERCATOR, studied at Louvain and became a surveyor and instrument maker. He made and published surveys of Flanders, and also published maps of the world. His first world maps were based on Ptolemy's but his later maps were corrected to agree with the observations made by navigators. After charges of heresy he left Flanders and settled in Germany. In 1568 he brought out his navigational map in which the globe is represented in such a way that the compass bearing from one point to another can be measured directly from the map. This projection is still used for nearly all navigational maps. C.V.G.

PIE AND π

Little Jack Horner sat in the corner
Finding the value of π ;
The masses of digits
Gave his fingers the fidgets,
So he copied the figures from *Pie.*

New readers of *Mathematical Pie* are sometimes puzzled by the rows of figures at the bottom of each page and, from time to time, the Editor receives an enquiry as to what they represent. The answer is that these figures are part of an approximation to the value of π, a number of such importance in mathematics that its connection with the circumference and diameter of a circle is almost a sideline. Why are there so many figures?

23144 29524 84937 18711 01457 65403 59027 99344

South Pacific islands 1,500 miles of empty sea separated them from home. They continued on their course by keeping the wind on the starboard beam until the equator was crossed. Then the Sacred Calabash became their guide.

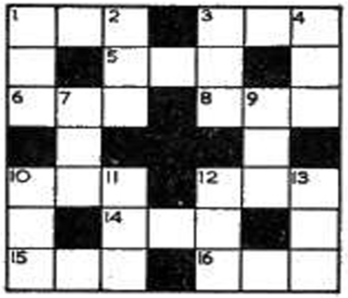
This instrument was made from a gourd over three feet in length, rather like an enormous coconut. The top had been removed, the inside cleaned out, and four holes, equally spaced below the rim, bored through the shell. When the Calabash was filled with water and held upright so that the water was level with the four holes the rim was horizontal. The four holes were so placed that in Hawaii an observer looking through any one of the holes could just see the Pole Star over the opposite rim. During the long journey northward observations were made of the Pole Star. When, at last, it could be seen on the rim of the Calabash course was altered to the west, and the canoe continued on the latitude of Hawaii, frequently checking position, until the peaks of Mauna Loa and Mauna Kea rose above the horizon to guide the voyagers back to harbour. C.V.G.

CHARLIE COOK

(From the Mathematics Student Journal)

$$\frac{17^3 + 7^3}{17^3 + 10^3} = \frac{17 + 7}{17 + 10}$$

SENIOR CROSS-FIGURE No. 26



ACROSS

1. Median AX of triangle ABC when AB = 10, BC = 12, CA = 8.
3. 5% Compound Interest on £2 for 2 years (in £).
5. Smaller of 2 consecutive odd numbers whose squares differ by 480.
6. Interior angle of regular polygon with 15 sides.
8. 3 figure number such that the middle digit is the average of the first and last digits.
10. 20y (see 11 dn.)
12. Volume in cubic inches of right circular cone 1 ft. high and base-radius 3.5 ins.
14. $\dfrac{125^{\frac{1}{3}} \times 32^{\frac{4}{5}}}{4\sqrt{25} \times 27^{-\frac{1}{3}}}$
15. H.C.F. of $6x^2 - 11x - 2$ and $x^2 - 4$ when $x = 8.73$.
16. A number divisible by 29, the product of whose digits is 70 and sum 14.

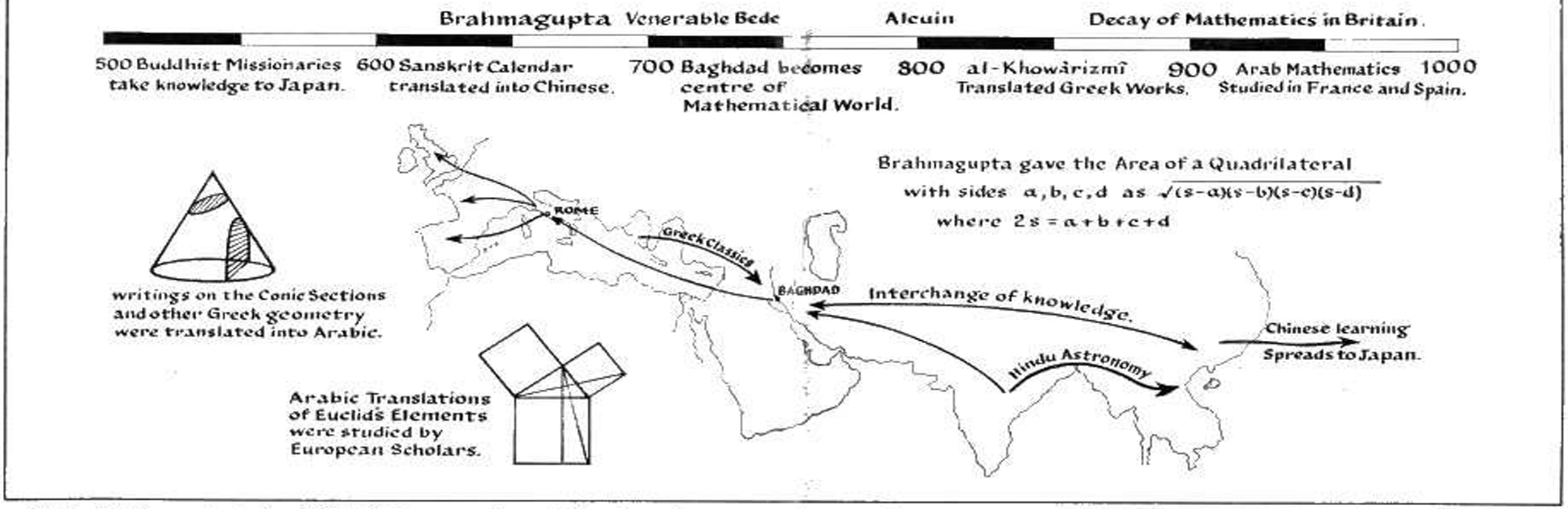
DOWN

1. Reverse of total surface area, in sq. ins., of cone (see 12 ac.)
2. $0.48\sqrt{\{(0.8)^2 + 31.25 \times 0.072\}}$.
3. See 7 dn.
4. $y^2 + 10$ (see 11 dn.)
7. Twice 3 dn.
9. $\dfrac{B}{C}$ as a decimal, if A is 20% more than B and 25% less than C.
10. 8 ac. + 12 dn.
11. 0.1% of prime root of $y^2 - 25y + 46 = 0$.
12. 36th term of A.P. whose 3rd term is 8 and whose 7th term is 4 times the second term.
13. [illegible] of 16 ac.

Check clue : Sum of all the digits inserted is 155. I.L.C.

68299 89487 22658 80485 75640 14270 47755 51323

The Spread of Mathematics 500-1,000 A.D.



During the five centuries from 500 A.D. the general trend of mathematics was westward rather than towards the East, though there was quite considerable progress in the Orient.

The increasing amount of trade between the Middle East and the Orient strongly influenced and developed the art of calculation, while the eastward travelling pilgrims and the movement of armies gave rise to an exchange of knowledge relating to abstract mathematics and astronomy.

Much of the mathematical work in China at this time was connected with the improvement of the calendar, and it was during the 7th century that a Sanskrit calendar was translated into Chinese. Following a visit to India between 629 and 645, the Chinese mathematician Hüan-tsang spent the rest of his life translating Hindu mathematical scripts.

During the 8th century, the interchange of Arab and Chinese ambassadors resulted in a further and wider exchange of knowledge. The friendship of these two countries was particularly strong during the reign of Harun-al-Rashid (well known from the Tales of the Arabian Nights), when Baghdad was rapidly becoming the centre of the mathematical world.

Japanese mathematical development was strongly influenced after the introduction of Buddhism in 552, and it was about two years after this that Japan adopted the Chinese system of chronology. From then on Japanese intellectual life was almost completely the result of Chinese influence. Chinese measures were adopted, an observatory was founded and, in 701, a

79641 45152 37462 34364 54285 84447 95265 86782

university system was established in which nine Chinese works were specified for students of mathematics.

The mathematics produced in India during this period was a mixture of brilliant and very ordinary works, but it is worth noting the contributions of Aryabhatas the Elder, Varahamihira the astronomer, Brahmagupta and Mahaviracarya. The first of these produced a collection of astronomical tables, several works on arithmetic and showed a knowledge of quadratic equations and indeterminate linear equations. The most prominent among them was Brahmagupta, who lived during the 7th century. At the age of 30 he wrote an astronomical work of 21 chapters entitled Brahmasiddhanta. The rest of his work included areas in arithmetic, the application of algebra to astronomy and indeterminate equations.

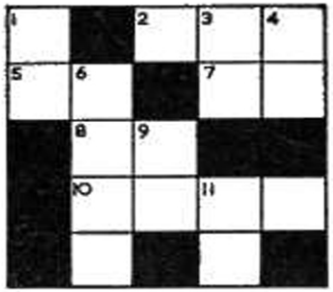
The greatest encouragement in the study of mathematics was to be found in Baghdad, where Hindu astronomy and mathematics were studied and developed by many scholars. The works of Brahmagupta were translated and the classics of Greek mathematics were introduced into the court of the Caliphs.

It was through the work in Baghdad that mathematics spread to the west, where newly civilized countries were slowly assimilating Roman culture.

The mathematical works of Boethius, a Roman citizen, were held in high regard in the monastic schools of the west. Though his work was not

10511 41354 73573 95231 13427 16610 21359 69536

JUNIOR CROSS-FIGURE No. 25



ACROSS

2. $2x+3=25$. Find x^2.
5. Simple interest on £200 at $3\frac{1}{2}\%$ for 3 years.
7. One more than a perfect square.
8. $(n+1)(n-1)$ when $n=8$.
10. Number of square yards in $1\frac{1}{4}$ acres.

DOWN

1. Number of sides of a regular polygon with interior angle 150°.
3. $x+2y=37$. Find $(x+y)$.
   $2x+y=32$.
4. A prime number.
6. L.C.M. of 16, 28, and 60.
9. One angle of an isosceles triangle when another is 73°.
11. Three times 4 down plus 3 down.

B.A.

APOLOGIES

The editor apologises for an error in the figures of π. Page 198, group 5 should read 65549 instead of 85549.

SOLUTIONS TO PROBLEMS IN ISSUE No. 26

SENIOR CROSS-FIGURE No. 26

ACROSS: (1) 678; (3) 105; (5) 119; (6) 136; (8) 369; (10) 460; (12) 154; (14) 240; (15) 673; (16) 725.
DOWN: (1) 671; (2) 816; (3) 193; (4) 539; (7) 386; (9) 625; (10) 476; (11) 023; (12) 107; (13) 435.

FUN WITH NUMBERS No. 5

(a) $3+4-5-(3-1)(-5)=12$.
$3+4-(5-3-1)(-5)=12$.
$3+(4-5)(-3-1-5)=12$.
(b) $a+b\times(a+b+a)\times b=122$.
when $a=5$, $b=3$.

CHARLIE COOK

$$\frac{17^3+7^3}{17^3+10^3}=\frac{(17+7)(17^2-17\times7+7^2)}{(17+10)(17^2-17\times10+10^2)}=\frac{(17+7)\times219}{(17+10)\times219}=\frac{17+7}{17+10}$$

So in this case, Charlie was correct.

FALLACY No. 25

When extracting square roots, the positive and negative sign must be considered separately.

JUNIOR CROSS-FIGURE No. 24

We apologise for the error in the 4 across clue.
ACROSS: (2) 243; (4) 5232; (7) 242; (9) 144; (11) 9660; (14) 220.
DOWN: (1) 15; (2) 27; (3) 472; (5) 294; (6) 12; (8) 486 (9) 16; (10) 492 (12) 60 (13) 05.

B.A.

JUNIOR CROSS-FIGURE No. 24

The clue to 4 across should have read

$\frac{7}{12}$ of 11 across—5 down—$\frac{1}{2}$ of 14 across$+46$.

The editor has received letters pointing out this error from Joan Shaw of Combe Down, Bath; Form 2A, Ruxley County Secondary School, Ewell, Surrey; and Form IIK, County High School, Chelmsford; and the following verse from Form Upper IVA, Manchester High School for Girls.

Your Junior Crossword's wrong.
It doesn't take so very long
To find that No. 4 across
Has had a quite substantial loss.
It should end sevens without a doubt,
It ends 32 but you'll soon find out.

34088 19071 04863 31734 64965 14539 05796 26856

MATHEMATICAL PIE

No. 27 — Editorial Offices: 97 Chequer Road, Doncaster — MAY 1959

WRITING NUMBERS.

No. 1—Concrete Numbers

Have you ever wondered where the numerals which we use every day (Arabic numerals) come from? Sometimes we use a different set of numerals, *e.g.*, on the face of a clock, or the chapters in a Bible, these are Roman numerals. If you collect stamps you will know that a stamp bought in Rome will have Arabic numerals on it, not Roman, and that a stamp from an Arab country will have numerals quite different from our "Arabic" numerals (see Pie No. 18).

If you think back there must have been a time when no one in the world knew how to write either words or figures. Writing was only invented 6,000 years ago, but there have been men in the world for 50,000 years.

The factors which made men desire to find a way of writing numbers were trade and the need for a calendar. The earliest known calendars date back to between 5,000 and 4,000 B.C. How can we guess what happened before then?

Fortunately there still exist, in remote parts of the world, races of men who are backward and primitive. Their number words and methods of counting have been studied and found to be much alike in general pattern, although these people live at great distances from one another and are therefore unlikely to have influenced one another. It is fair to assume then, that our ancestors and those other civilised peoples began Arithmetic in a similar way.

In the beginning numbers were inseparable from objects—*e.g.*, the first three number words of the Malays mean literally "one stone, two stones, three stones." These are called concrete numbers and the earliest attempts at writing took the form of a series of pictures which would convey a message, rather like the comic strip of today.

NORTH AMERICAN INDIAN. CRETAN.

The illustrations above are taken from picture writing and show concrete numerals—4 families, a 3-day journey and a 2-month voyage.

How would you represent a three-day cricket match in this manner?

R.H.C.

00537 06146 80674 91927 81911 97939 95206 14196

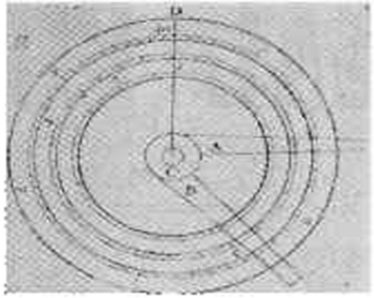


Fig. 1

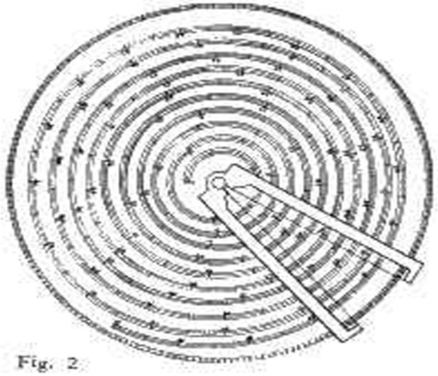


Fig. 2

MATHEMATICAL INSTRUMENTS

No. 7—The circular scales

The first circular calculator was described by William Oughtred in 1632. It consisted of a brass disc on which was engraved a number of concentric scales (Fig. 1). One was a scale of logarithms from 1 to 10, divided by ·01 from 1 to 5, and by ·02 from 5 to 10. This scale formed a complete circle and the graduations for 1 and for 10 coincided. The scale could be read as 10 to 100 or as 100 to 1000. The other scales represented sines and tangents. At the centre was pivoted a pair of pointers, looking rather like a pair of compasses. These were made so that they could be set at any angle to one another and would then turn together keeping the angle constant. To multiply 5·6 by 3·7 one pointer is moved to 1 and held there while the other pointer is moved to 3·7. The pointers are then moved together and the first pointer moved to 5·6. The second pointer now shows 2·07. We can see that the product is between 10 and 100. Therefore the answer is 20·7.

Instruments with spiral scales were used in the seventeen hundreds. They are said to have been invented by Mr. Brown. Samuel Pepys bought a " sliding rule " from Mr. Brown in 1664. This may have been the same man. A spiral scale of five turns can have the same number of divisions as a circle of five times the radius, and can be used for very accurate calculations, provided the user does not read the answer off the wrong part of the spiral. The illustration (Fig. 2) shows a Nicholson scale of 1797.

In the modern instruments the pointers are replaced by lines on two glass discs in front of the scales and the spiral usually has six turns to facilitate the calculation of square roots and cube roots. C.V.G.

THE FIRST SHALL BE LAST

1. Write down any number with 4 digits. (*e.g.* 8493).
2. Rewrite it in the same order, but put the first digit last.
3. Repeat stage 2 (9384); and again (3849).
4. Add the four numbers you have obtained. Divide the total by the sum of the digits (8+4+9+3).

The result is always the same, no matter what digits are used. Why?
J.G.

THE BINARY SYSTEM

Some readers may have heard that the new electronic computors work in the binary system, and perhaps this information puzzles them. According to the dictionary, a *binary* system is a " system involving two ". The decimal system of numbers that we normally use is based on *ten*, probably because we have a total of ten digits on our two hands. On this basis, it would have made our arithmetic easier if we had twelve digits, because 12 is better off than 10 in respect of factors.

When we look at some well-known number—say 365—we see what is really a short-hand way of writing $(3\times10^2)+(6\times10)+(5\times1)$. In the scale of 12, it would be $(2\times12^2)+(6\times12)+(5\times1)$, or shortly, 265. In the scale of 8, 365 is $(5\times8^2)+(5\times8)+(5\times1)$ or 555. What, then, is 365 in the binary system or scale of 2? The number is easily split up into $256+64+32+8+4+1$, *i.e.*, $2^8+2^6+2^5+2^3+2^2+1$ or $(1\times2^8)+(0\times2^7)+(1\times2^6)+(1\times2^5)+(0\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2)+1$, which is written shortly 101101101. This number, then, is the binary way of expressing the number of days in a normal year.

Why use the binary system in computors? Since 2 itself is written as 10 in the binary system, it can be seen that there are only two numerals, namely 1 and 0. This is very convenient in designing an electronic calculator because a simple two-position switch can be made to represent 0 in one position and 1 in the other position. Since electronic tubes or valves can be made to work as such switches, a binary number can be sent through a computing machine at very high speed in a sort of morse code which is " processed " by the machine according to the instructions given to it and is finally decoded and printed as a table of ordinary numbers ready to be used in research or industrial applications.

You can now try your own skill in turning other well-known numbers from the decimal scale into binary numbers: (*a*) 112, (*b*) 1760, (*c*) 2240, (*d*) 4840. (Hint: write out a list of powers of 2). J.F.H.

CHARLIE COOK

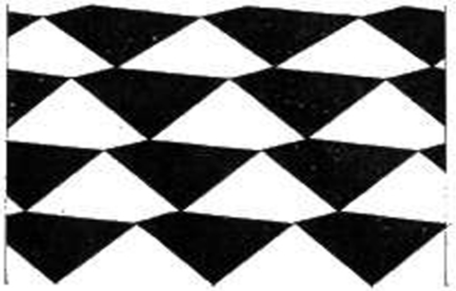
(*From the Mathematics Student Journal*)

$$\sqrt{5\tfrac{5}{24}} = 5\sqrt{\tfrac{5}{24}}\ !\,!$$

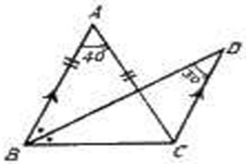
Find other similar Charlie Cook roots.

A Patchwork Pattern

Did you know that a set of identical quadrilaterals of any shape can be fitted together to cover a plane surface? C.V.G.



FALLACY No. 26



ABC is an isosceles triangle having AB = AC. The bisector of angle ABC meets the line through C parallel to BA in D. Given that angle A = 40° and that angle D = 30°, calculate angle BCD.

Method 1.

$\angle ABC = 70°$ (base angle of the isosceles $\triangle$ ABC).
$\angle BCD = 180° - \angle ABC$ (allied angles are supplementary),
$\therefore \angle BCD = 180° - 70°$,
$\therefore \angle BCD = 110°$.

Method 2.

$\angle DBC = \frac{1}{2}$ of $\angle ABC$ (BD bisects $\angle ABC$),
$\therefore \angle DBC = 35°$.
$\angle BCD = 180° - \angle DBC - \angle BDC$ (angle sum of $\triangle$ BCD).
$\therefore \angle BCD = 180° - 35° - 30°$,
$\therefore \angle BCD = 115°$.

Which is correct? I.H.

A PUZZLE FOR SQUARES

Think of a chess-board with 8 × 8 1-inch squares. It is obviously possible to cover it with 32 dominos, 2″ × 1″. Now if you cut out the opposite diagonal corner squares, can you or can you not cover the remaining 62 squares with 31 such dominoes?

Dad's not just square, he's octagonal.
Reprinted by permission of the "Daily Mirror".

6.5 SPECIAL

A man travels home by train every day. The train gets to the local station at 6 p.m., and his chauffeur fetches him punctually by car from the station and drives him home. One day the man comes by an earlier train, and is at the station at 5 p.m. Rather than wait an hour, he starts walking towards home, meeting the chauffeur on the way on his way to the station, and returning the rest of the way in the car. If he gets home ½ hour before the usual time, for how long did he walk?

DUCKS AND DRAKES

There does not seem to be enough information in the following problem to make a solution possible; but there is one answer, and one only.

A man bought an odd number of ducks at 10 shillings each, and one drake for less than 10 shillings. The total number of shillings he paid was a perfect square. What did the drake cost? J.G.

21825 62599 66150 14215 03068 03844 77345 49202

DEFINITION PIE

The following definitions have been gathered from various sources:

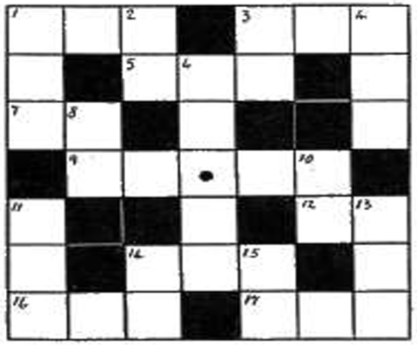
PIE: Magpie.
PIE: A kind of Woodpecker.
PIE: (sea-pie) Oyster catcher.
PIE: Pica (ecclesiastical rules; a standard of type size).
PIE: Meat, fruit, etc., baked in a dish with a cover or with a complete envelope of pastry.
PIE: (printer's) Indiscriminately mixed, disarranged type.
PIE: A small copper coin, one third of a pice.
PIE: (Mathematical) A lively journal published three times a year and circulating all over the world. It contains items of interest for children of all ages. J.F.H.

THE PATH OF TRUE LOVE BECOMES COMPLICATED

(*Adapted from* "*Magazine of Knowledge and Pleasure Vol. II, March* 1748).

"I have solicited an old gentleman to let me pay addresses to his daughter but on no terms will he hear of it without I first tell him his daughter's age which, he says, is found by this rule: To twice her age, add four more than the square root of double her age. This sum, added to its square will be equal to 3660. Query, her age?" J.F.H.

SENIOR CROSS-FIGURE No. 27



VABCD is a right rectangular based pyramid. X, Y and N are mid points of AB, BC and AC respectively.

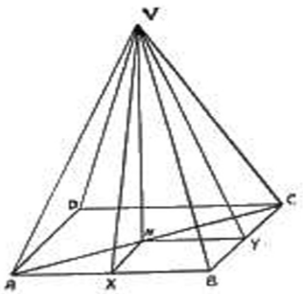
CLUES ACROSS

1. 6 ∠VXN to the nearest degree.
3. ∠VAB to one decimal place.
5. ∠VYN to one decimal place.
7. Area of XNYB in square units.
9. VY.
12. Perimeter of ABCD.
14. VB².
16. 2 ∠AVC to one decimal place.
17. 2 ∠VXN to the nearest degree.

CLUES DOWN

1. 3 ∠NVX to one decimal place.
2. ∠VAN to nearest degree.
3. 5 (AC + XY).
4. Volume of VABCD in cubic units.
6. VX.
8. VC + CD.
10. 4 VA.
11. ∠ACB in degrees and minutes.
13. 6 ∠XVN to one decimal place.
14. ∠NVX to the nearest degree.
15. Reverse the area of VBY to the nearest square unit.

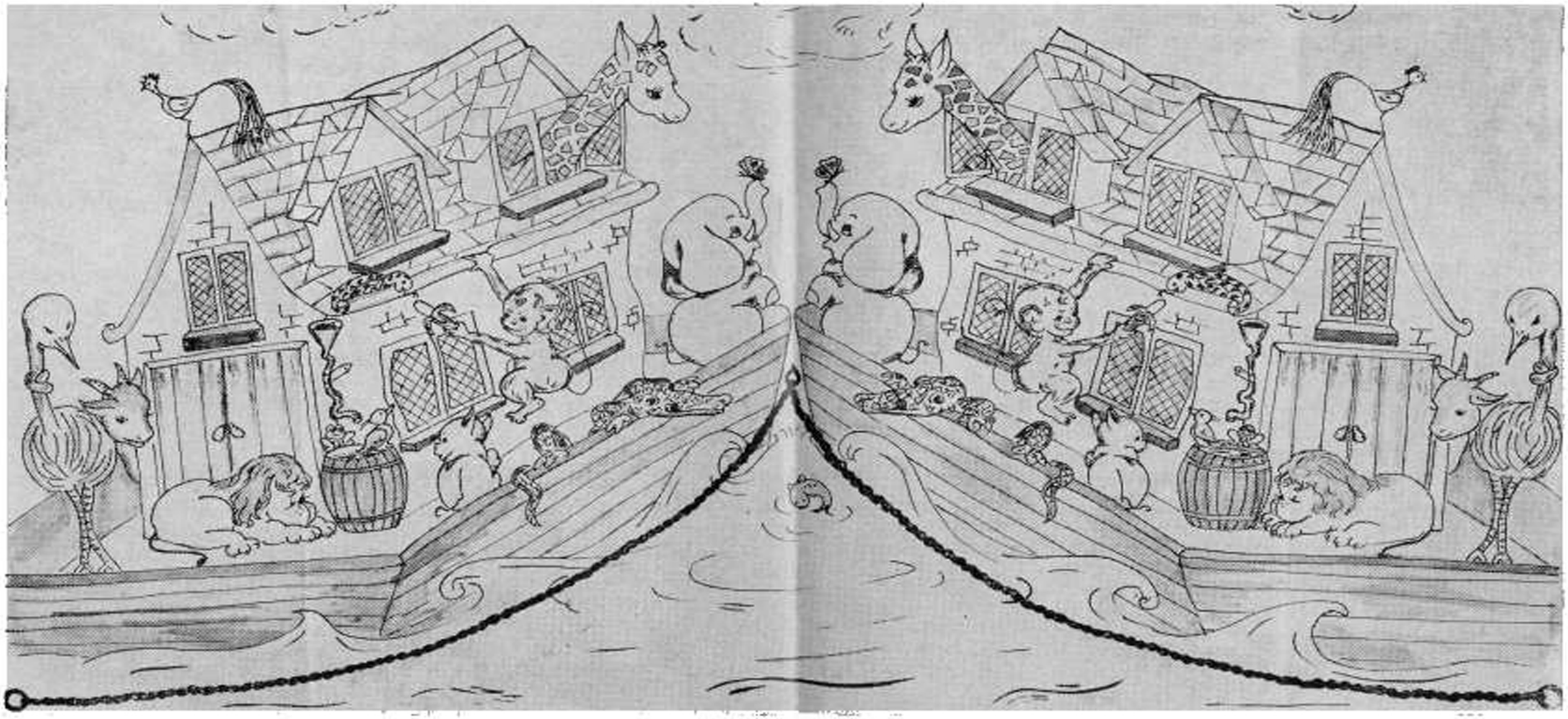
B.A.



AB = 8, BC = 6, VN = 12 units.

95618 14675 14269 12397 48940 90718 64942 31961

Equal Chords Stand on Equal Arcs



The above illustration was submitted by Jennifer Tresise, 6th Form, Bishop Blackall School, Exeter, to whom we have forwarded a book token for such a splendid effort. We invite our readers to send in further mathematical cartoons.

The caption of Jennifer's drawing can be more fully stated as "*If in equal circles, or in the same circle, two chords are equal, the arcs which they cut off are equal.*" This theorem is usually proved by using two other theorems which, in their turn, depend on yet other theorems.

A theorem may be likened to a brick in a building; it is supported by other theorems just as a brick is supported by other bricks on which it rests. We cannot carry this likeness too far because our system of geometry is more satisfactory when the basic principles, or axioms, are as few and as simple as possible. These foundations, being few in number, must be massive in form and of unquestioned reliability.

It is worth while turning back to the early pages of your textbook when you have been working at a subject for a year or so. You will be pleasantly surprised to find that some of the early statements that once seemed trivial make a lot more sense in the light of experience than they did when you first read them.

56794 52080 95146 55022 52316 03881 93014 20937

If we go beyond our text books, we find that a complete system of geometry was established nearly 2,300 years ago by Euclid of Alexandria and was most carefully expounded in thirteen books referred to as the "Elements." In these some 465 propositions are set out, in other branches of mathematics as well as geometry, the standard reached finally being well beyond that achieved by the average student even today.

Naturally there are some blemishes apparent in Euclid's work, but it is hardly a brave thing to criticise it after 2,000 years of subsequent thought. Some folks may refer to a horse-drawn vehicle as slow but the fact remains that such a vehicle can traverse the City of London at rush-hour just as speedily as the most expensive car.

Some modern geometry books do not refer to Euclid at all and in one respect, at least, are in consequence deficient because it will be found that a certain theorem has different numbers given to it in geometry books written by Mr. X and Mr. Y—and may even have different numbers in two different books written by the same author. Such a state of affairs is confusing to both students and teachers. Failing the issue of a British Standard that would systemise things once and for all, we can go back to Euclid's system. Thus the caption of the picture above has the reference Euclid III 28, *i.e.*, the 28th proposition in the third book—nothing could be simpler! J.F.H.

62137 85595 66389 37787 08303 90697 92077 34672

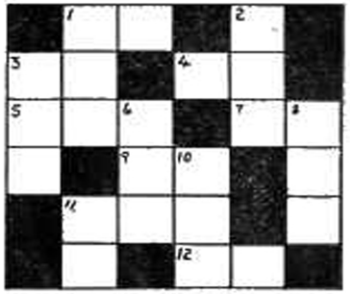
QUOTATIONS

The advance and perfecting of mathematics are closely joined to the prosperity of the Nation.—NAPOLEON.

Hold nothing as certain save what can be demonstrated.—NEWTON.

If the Greeks had not cultivated conic sections, Kepler could not have superseded Ptolemy.—WHEWELL.

JUNIOR CROSS FIGURE No. 26



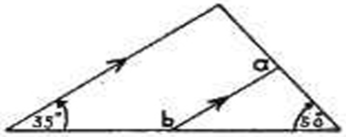
CLUES ACROSS

1. Number of sides of a polygon with interior angle 150°.
3. A third of 2 down plus five, minus ten.
4. Simple interest in pounds on £170 invested for 4 years at 5%.
5. Number of 6 inch square blocks to cover a hall floor $15\frac{1}{2}$ ft. by 11 ft.
7. A prime number.
9. $6.7^2 - 3.3^2$.
11. Product of four consecutive prime numbers.
12. a.

CLUES DOWN

1. Interior angle of a regular pentagon.
2. b.
3. Add 1 across to 9 across and multiply by $x+y$. $\begin{cases} 2x+y=13 \\ x+2y=17 \end{cases}$
6. 11xy.
8. $(p-q)(p+q)$ when $p=20$ and $q=5$.
10. Area, in sq. in., of a rectangle 1 ft. 5 in. by 2 ft.
11. A prime number.

Check clue. One digit is not used. The sum of the digits is 77. B.A.



SOLUTIONS TO PROBLEMS IN ISSUE No. 27

THE FIRST SHALL BE LAST

Write the number abcd = 1000 a + 100 b + 10 c + d
The other numbers will be dabc = 1000 d + 100 a + 10 b + c
cdab = 1000 c + 100 d + 10 a + b
bcda = 1000 b + 100 c + 10 d + a
Adding we have 1000 (a + b + c + d) + 100 (a + b + c + d) + 10 (a + b + c + d) + (a + b + c + d).
Divide this number by (a + b + c + d) and the result is 1,111.

SENIOR CROSS-FIGURE No. 27

CLUES ACROSS : (1) 456 ; (3) 721 ; (5) 715 ; (7) 12 ; (9) 12.65 ; (12) 28 ; (14) 169 ; (16) 864 ; (17) 152.
CLUES DOWN : (1) 421 ; (2) 67 ; (3) 75 ; (4) 192 ; (6) 12.36 ; (8) 21 ; (10) 52 ; (11) 538 ; (13) 842 ; (14) 14 ; (15) 91.

FALLACY No. 26

The fallacy arises because ∠BDC is given as 30°. It must be 35° if the other information is true.

A PUZZLE FOR SQUARES

The chess-board cannot be covered in this way as the two squares which were removed are of the same colour.

6.5 SPECIAL

The man meets the car when it is at a distance which takes $\frac{1}{4}$ hour to travel to the station. Hence he walks for 45 minutes.

DUCKS AND DRAKES

He bought 3 ducks and the drake cost 6 shillings. The total cost was then 36 shillings.

JUNIOR CROSS-FIGURE No. 25

CLUES ACROSS : (2) 121 ; (5) 21 ; (7) 37 ; (8) 63 ; (10) 8470.
CLUES DOWN : (1) 12 ; (3) 23 ; (4) 17 ; (6) 1680 ; (9) 34 ; (11) 74. B.A.

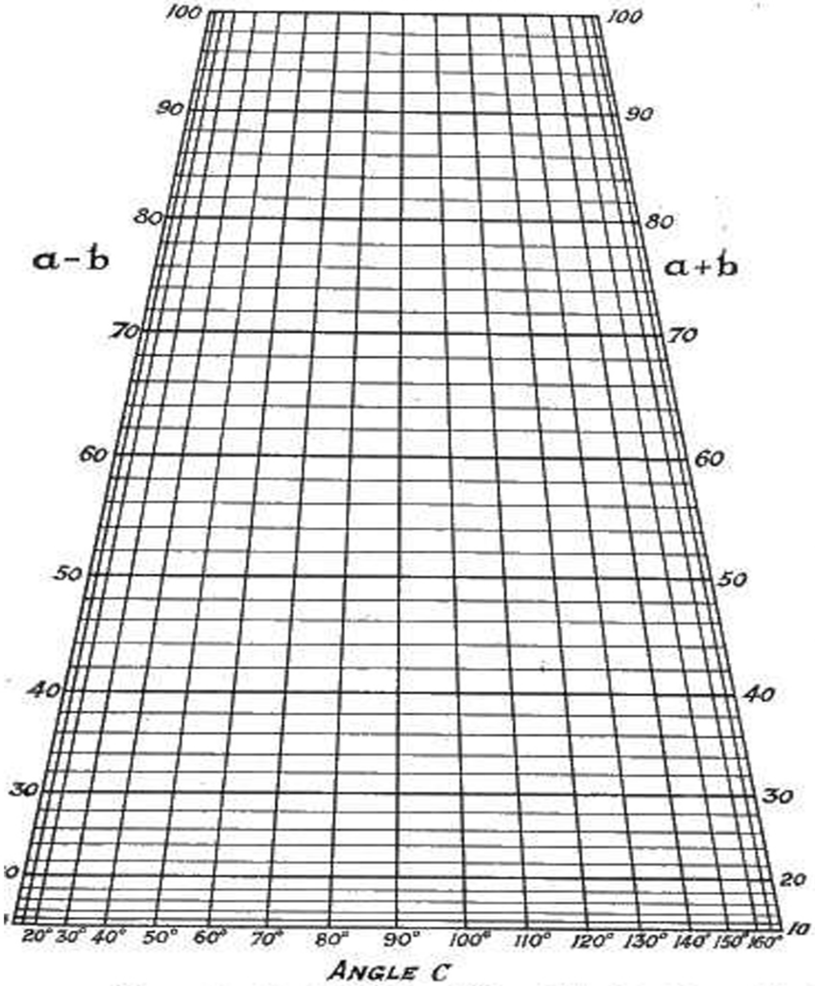
00675 10334 67110 31412 67111 36990 86585 16398

MATHEMATICAL PIE

No. 28 Editorial Offices : 97 Chequer Road, Doncaster OCTOBER 1959

A NOMOGRAM FOR THE COSINE FORMULA

$$c^2 = a^2 + b^2 - 2ab \cos C$$



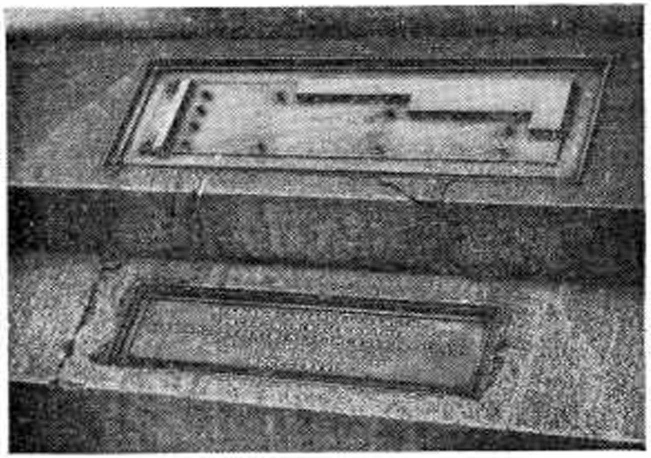
This nomogram can be used to find the angles of a triangle when the three sides are given, and to find the third side when two sides and the included angle are given. The horizontal lines represent values of c, the vertical lines values of C, and the two side scales represent a + b and a − b.

Example : In △ABC A=60°, a=5 in., b=3 in. Find c.

Lay a straight-edge from the point a+b=8 to the point a − b=2. The straight-edge cuts the line C=60° at c=4.35. C.V.G.

10055 08106 65879 69981 63574 73638 40525 71459

SMALLER FEET



One of the many public semi-standards for one foot, two feet, and one yard.

American feet are to be a little smaller. As greater accuracy of measurement has been achieved the old standards of length have become unsatisfactory. The English standard yard has varied by about one-twenty-fifth of an inch since the first legal standard was made over 700 years ago. In 1922 it was discovered that the current standard (made in 1845) was slowly shrinking. The inch was therefore defined as 25.39996 millimetres. In 1951 it was redefined as 25.4 millimetres exactly.

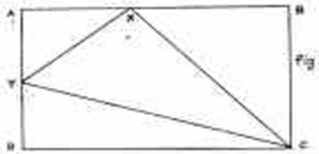
In the United States each state maintained its own standards until 1832 when a Senate Committee found considerable disagreement between these standards and ordered the distribution of new standards based on the British standard yard. In 1893 the Senate redefined the standard yard as $\frac{3600}{3937}$ metres. This makes the inch equal to 25.4000508 millimetres. Since 1951, however, the American Bureau of Standards has used the British inch and this is now to be made the legal United States standard.

The metre used for the definition is not the French standard. It is the length of 1,553,164.13 wave lengths of Red Cadmium light. This figure was obtained as the average of a number of determinations which differed only in the last figure, and has been adopted as the international standard. It will probably soon be superseded by a definition in terms of Mercury light. C.V.G.

Continued from page 215

CLUES DOWN

1. Value of $(a+b^2)$ when $2a+b=31$ and $2b+a=32$.
2. Difference between $n(n+h)^2$ and $n^2(n+b)$ when $n=13$ and $b=1$.
3. ABCD are four successive points on a line. AD=24 in., AC=18.84 in., BD=18.61 in. What is BC?
4. Radius of a circle drawn through the vertices of a rectangle 30 in. by 16 in.
5. An even number of three figures which may be described as x to the power x.
9. Number of cu. in. in two equal cubes of 1 ft. side.



10. Four consecutive integers.
12. How many m.p.h. is 105.6 ft p.s.?
13. Hypotenuse of a right-angled triangle; base 60in., area 750 sq. in.
15. Fig. 3. Area of triangle XYC. AB=20 ft., BC=12 ft., AX=8 ft., and AY=6 ft. J.G.

CHARLIE COOK'S DINNER GOES UNCOOKED!

Mrs. Cook was busy cooking the dinner when the heat supply to the stove suddenly failed. She found that the slot-meter required a shilling to be inserted to restore the supply. Unfortunately, she had no change so, handing Charlie a one-pound note, she said: "Run round to the grocer's and get me twenty shillings' worth of silver". On his return, Charlie handed his mother the change and ran out to play before she counted it. She found that there were twenty coins and that these certainly added up to twenty shillings; while there were half-crowns, florins and sixpences, however, there was not a single shilling! What coins *did* Charlie bring back? J.F.H.

STAMP COLLECTORS' CORNER No. 7

France 1937,
90c *Copper Red.*

RENE DESCARTES (1596–1650), was the man who developed the work on graphs. After education in a Jesuit college he became dissatisfied with accepted philosophy and turned to mathematics, as Pythagoras had done, as a means of understanding the universe. After a few years divided between mathematics and gambling he became a soldier. During this time he invented the idea of using the methods of algebra in the solution of geometrical problems. In 1621 he gave up soldiering and began a series of investigations into problems of physics and astronomy. He wrote a treatise which was ready for publication when Galileo was brought before the Inquisition. Publication was suspended, but at last, in 1637, with the encouragement of Cardinal Richelieu, "A Discourse on Method" was printed. C.V.G.

FALLACY No. 27

The following problem with its solution was found by the Editor whilst looking through an old magazine, "The Universal Magazine of Knowledge and Pleasure" Vol. II, April 1748—

"A gentleman hath an oblong garden, whose length is 620 feet and breadth 400 feet, round which he would make a trench 15 feet deep, so as the earth flung up should raise the garden one foot higher. Qu. The breadth of the trench?"

In the May edition, the following solution is given—

620 feet long	620 feet long
400 feet broad	400 feet broad
248000 sq. feet in the G.	1020 half the compass
	2
	2040 feet in the compass
30600)248000 solid feet of earth	15 feet in depth
	30600 square in compass and depth.
	306) 2480
	220 The trench must be 8 feet 1 wide.
Feet 8, 1 wide	14

And the answer is wrong. Can you follow the method, and show why it is wrong?

It is, therefore, all the more surprising that the Babylonians and Egyptians could calculate areas and volumes and make astronomical calculations with considerable accuracy.

For many centuries numbers were used solely for the purposes of calculation. It was not until the Golden Age of the Greeks that the properties of numbers were considered and arithmetic was divided into calculation on the one hand and the theory of numbers on the other. I.L.C.

KNOWLEDGE INCREASES

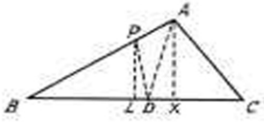


Fig. 1.

From a piece of fairly stiff paper—drawing paper will do very well—cut out a number of fairly large triangles such as ABC (*Fig.* 1). Fold it so that AC lies on AB, and then crease it plainly at the fold. The crease, AD, will show the bisector of the angle A. Flatten out the triangle again and form, in the same way, the crease bisectors of the angles B and C. Examine your handiwork and you will see that the three bisecting creases appear to meet at one point (the incentre).

Now take another triangle; fold and crease it so that C lies on B. You will obtain the perpendicular bisector LP of BC. If you crease similarly to show the perpendicular bisectors of the other two sides, you will see again that the three creases appear to meet at one point (the circumcentre).

Using another triangle, fold and crease so that C lies at any point on BC. The crease will be at right angles to BC. If before creasing you adjust the fold so that the crease passes through A, you will obtain AX, the perpendicular height (or altitude). By refolding, you may obtain the perpendiculars from B and C to the opposite sides. Again the three creases appear to meet at a point (the orthocentre).

Finally, with a fourth triangle, make three creases from the corners to the middle points of the opposite sides. You will thus obtain the centroid, or centre of gravity, of the triangle.

The four points you obtain will all be different unless the triangle is isosceles or equilateral. A large triangle folded to show all the points may suggest to you that three of them lie in one straight line. Which of the four is the stranger? J.G.

A three-day cricket match by Sandra Norcliffe, Holme Valley Grammar School, Honley.

72291 09816 90915 28017 35067 12748 58322 28718

PENNY PLAIN, TWOPENCE COLOURED!

This is a game of "just suppose". Imagine that there are two kinds of numbers, very much like ordinary ones, one kind being red (**R**) and the other kind deep blue or indigo (**I**). The colour letters **R** or **I** can be put in front of figures so that **R**10 means "red ten" and **I**7 means "indigo seven". For the purpose of our game suppose that the numbers are in pairs, *e.g.*, **R**10+**I**7, where the two sorts are kept separate just as $10x+7y$ would be written in algebra. In the rules of the game we can have addition and subtraction, just as in algebra, so that (**R**10+**I**7) — (**R**5 — **I**2) equals **R**5+**I**9.

The real fun of the game starts when we multiply or divide, because the colours, kept separate in addition and subtraction, now get mixed; suppose they sort themselves out, however, according to these simple rules: **R**×**R**=**R**; **R**×**I**=**I**; **I**×**I**= — **R**; (take note of that minus sign !).

Let us try using these rules to multiply the two numbers mentioned above, that is, to find the meaning of (**R**10+**I**7)×(**R**5 — **I**2). Taking the terms in order we get **R**10.**R**5 — **R**10.**I**2+**I**7.**R**5 — **I**7.**I**2 which makes, by the rules, **R**50 — **I**20+**I**35+**R**14 or **R**64+**I**15.

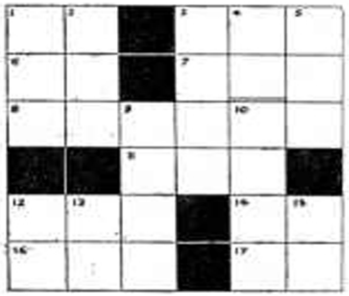
You can now try your hand at the game with the following:

a. (**R**4+**I**3)×(**R**3+**I**2)
b. (**R**3+**I**4)×(**R**2+**I**3)
c. (**R**4+**I**3)×(**R**4 — **I**3)

d. For division, first practice by dividing the products in a, b and c by their factors, then try (**R**9+**I**38)÷(**R**4+**I**3).

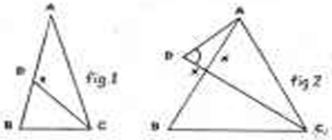
The principle of this simple game is of great service in many mathematical problems ranging from the solution of difficult equations to the practical one of designing a motor car silencer ! J.F.H.

SENIOR CROSS FIGURE No. 28



CLUES ACROSS

1. Value of $\frac{x^3-x}{x^2-x}$ when x is 40.
3. Fig. 1. The angle x when B is 78°, AB=AC and AD=DC.
6. The next number in the sequence 3, 8, 15, 24, 35, ---.
7. If 0.055 of a leap year is past, how many complete days remain?
8. Value of $\frac{11ab+312}{5}$ when a=123 and b=456.
11. Volume of rectangular block. Base is 19″×6″. Total external surface area is 428 sq. in.
12. Value of $(4x+1)$ $(x+6)$ when the first bracket is 6 less than three times the second.
14. Fig. 2. The angle x. B is 63° AB=AC =CD and AD=AX.
16. Value of x^2 when $(x - 10)$ $(x+4)=$ $(x - 6)$ $(x - 4)$.
17. How many lb. per sq. in. is 5.4 tons per sq. ft.?



(Continued on page 214

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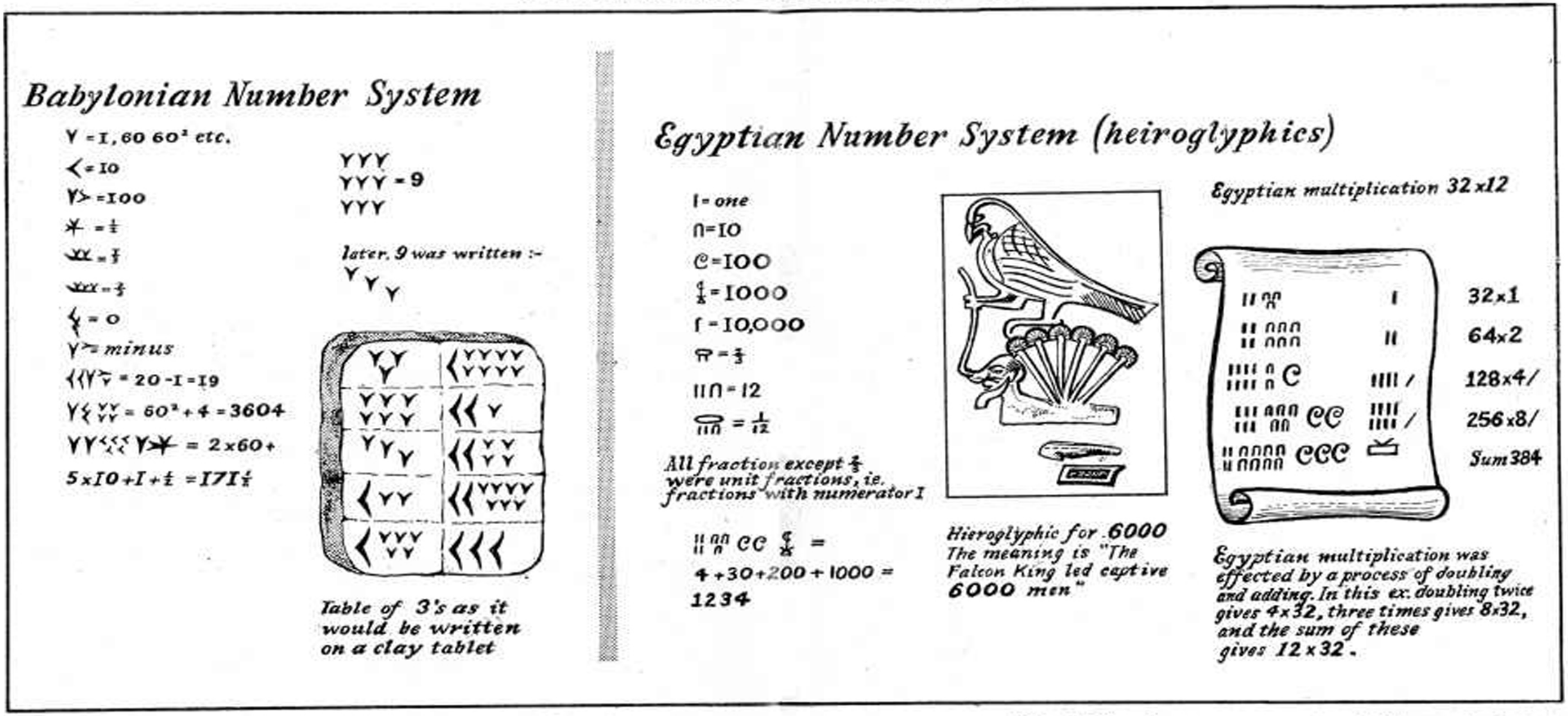


The early civilizations had systems of counting long before they wrote numerals and it was from the operation of counting that simple arithmetic evolved. The need to count was as natural to man as his need to communicate his thoughts in words. Our knowledge of early mathematics has been derived from documents which take us back about 5,000 years ; and from these we find evidence of the existence of well developed number systems in Babylon, Egypt and the Orient.

The principles on which these number systems were based are similar in many ways, though there is a wide variation in the type of symbols used. The similarity of these systems lies mainly in the use, in varying degrees, of the additive principle. This principle is the one used in Roman numerals, where a number symbol is written as many times as is necessary to add up to the number required. For example, a single stroke (1) denoting the number one would be written three times (111) to denote the number three.

You will see from the illustration that most of these early systems had a symbol for one which was used to compile numbers up to nine and then a new symbol for ten, so that numbers could be expressed as so many ones and

so many tens. This division into tens was most probably a natural consequence of man's use of his fingers for counting.

In the early Babylonian system we find a positional notation used to a certain extent. Our modern number system uses this principle ; for example, in the number 23 the position of the figure 2 tells us that it means two tens ; in the number 234, the position of the figure 2 tells us to read two hundreds. The Babylonians used this idea beyond the number 59, for the symbol for 60 was the same as the symbol for one. The position of this symbol could make it mean 1, 60, 60^2, etc., or $\frac{1}{60}$, $\frac{1}{60^2}$, etc.

This use by the Babylonians of the sexagesimal system, that is a number system based on powers of 60, may have arisen from their early belief that the year contained 360 days. Consequently, they divided a circle into 360 degrees and the fact that the radius can be stepped six times round the circumference gave the number 60 ($\frac{1}{6}$ of 360) a magic property. There is, however, no conclusive evidence for this belief.

With each of these early number systems the cumbersome notation made calculation, especially with fractions, a long and complicated process.

MATCH YOUR WITS

1. With six matches form 4 equilateral triangles, the sides of each being equal to the length of a match.

2. A man bought two horses for £80, and sold them for £80 each. The gain on one was 20% more than that on the other. What was the cost of each? R.H.C.

BINARY CROSS-FIGURE

Submitted by Mr. M. D. Meredith, Trinity School, North End, Croydon.

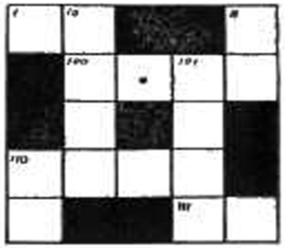
All numbers are expressed in the binary system.

ACROSS

1. H.C.F. of 110, 1111, and 10101.
100. One plus one half plus one quarter.
110. 110+11.
111. Rate per cent. per annum at which £10—1010s.—0d. earns 1/- simple interest in 1000 months.

DOWN

10. Double "1 across" doubled.
11. $\dfrac{a^{11}b^{110}}{a^{100}b^{101}}$ when $a=1000$ and $b=11000$.
101. Value of x if $\dfrac{x-10}{11}+\dfrac{x+100}{101}=110$
110. Last and least.



SERIOUSLY SPEAKING

Find the next number in the series

61, 52, 36, 94, . . .

and explain the rule for calculating the terms.

SUM AND PRODUCT

Which is it easier to find the value of

$1+2+3+4+5+6+7+8+9+0$

or $1\times2\times3\times4\times5\times6\times7\times8\times9\times0$? R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 28

SENIOR CROSS FIGURE No. 28

CLUES ACROSS: (1) 41; (3) 132; (6) 48; (7) 345; (8) 123456; (11) 456; (12) 765; (14) 78; (16) 256; (17) 84.
CLUES DOWN: (1) 441; (2) 182; (3) 1345; (4) 34; (5) 256; (9) 3456; (12) 72; (13) 65; (15) 84.

CHARLIE COOK'S DINNER GOES UNCOOKED

Charlie brought 4 half-crowns, 2 florins, and 12 sixpences.

FALLACY No. 27

The solution ignores the fact that the level of the trench is not raised one foot.

JUNIOR CROSS FIGURE No. 26

CLUES ACROSS: (1) 12; (3) 40; (4) 34; (5) 682; (7) 53; (9) 34; (11) 210; (12) 85.
CLUES DOWN: (1) 108; (2) 145; (3) 460; (6) 231; (8) 375; (10) 408; (11) 23.

APOLOGIES

The editor apologises for errors in the Cross-figure clues in issue No. 28.

Senior Cross Figure: 1 down should have read $(a+b)^n$ and 4 down should have read diameter instead of radius.

Junior Cross Figure: 3 across was badly worded. It should have read ½ (2 down+5) — 10.

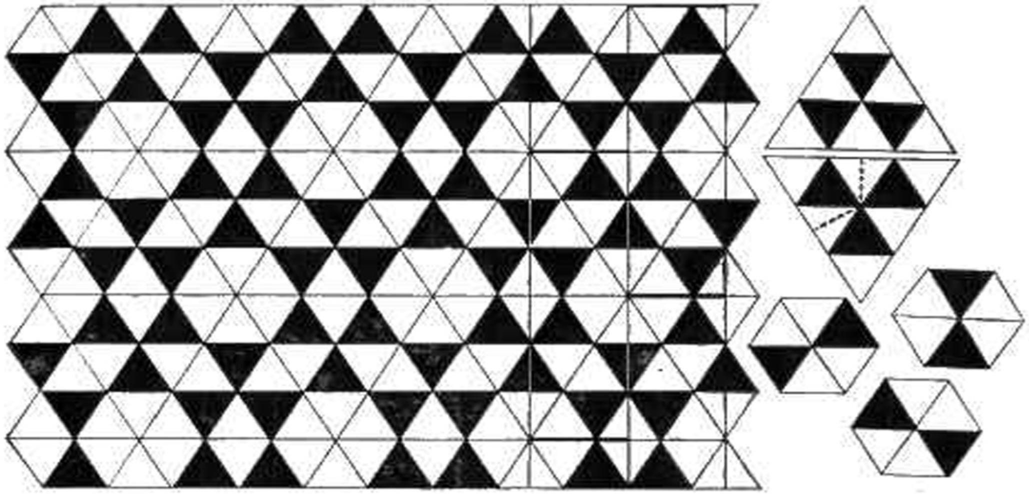
The picture of the Imperial yard was by courtesy of the *Radio Times Hulton Picture Library*. We apologise for having omitted this reference in the last issue. B.A.

18093 77344 40307 07469 21120 19130 20330 38019

MATHEMATICAL PIE

No. 29 *Editorial Offices:* 97 Chequer Road, Doncaster **FEBRUARY 1960**

PATCHWORK PATTERNS No. 2



This pattern of equilateral triangles in two colours was used for the floor of a Roman house in the first century. It is interesting to break the pattern down into repeats. It can be divided into parallelograms containing 12 light tiles and 6 dark tiles which can be fitted together without being turned, or into other shapes with more complicated outline, but all containing 18 tiles. It can be divided into triangles, containing 6 light tiles and 3 dark tiles, which have to be arranged in 2 different directions to make the pattern, or into hexagons, containing 4 light and 2 dark tiles, which are arranged in 3 different directions. These can be divided again into pentagons, or quadrilaterals or triangles, each covering 2 light and 1 dark tile, which arranged in 6 different directions fit together to form the pattern.

All repeating patterns can be divided into repeating rectangles. To do this, find 2 corresponding points of the pattern. If the line joining them is extended it will pass through more corresponding points equally spaced along the line, and there will be other lines parallel to this dividing the pattern into identical strips. These strips can then be divided into rectangles by drawing perpendicular lines. One way of dividing our pattern is shown by the lines on the right hand side. In this pattern the rectangles overlap like bricks in a wall. Designers would call it a half drop pattern.

C.V.G.

31501 97016 51511 68517 14376 57618 35155 65088

CUBIST ART

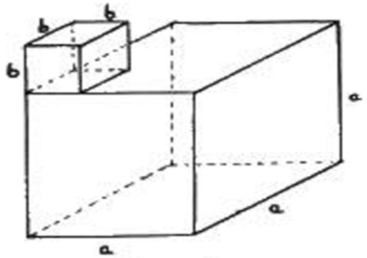


Figure 1

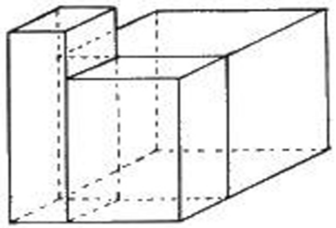


Figure 2

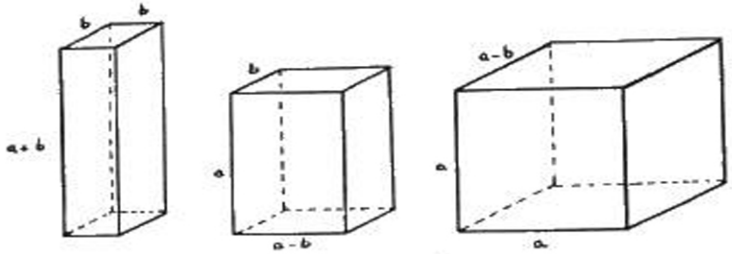


Figure 3

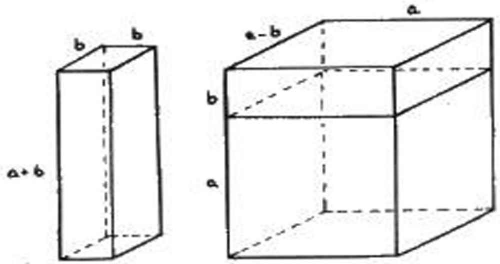


Figure 4

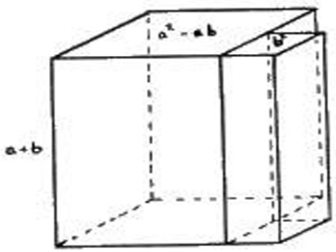


Figure 5

$(a+b)^3 = (a+b)(a^2 - ab + b^2)$

PIE and π (continued)

Following the note on π in Issue No. 26, we have received some interesting correspondence. Dr. Gosling, of Kingston, Ont., Canada, has sent information about his library of books on digital-computers—claimed to be the most complete in Canada, and M. Rigler, of Northbourne, Bournemouth, Hants., told us that he and other boys at his school have tried to use Klingenstierna's formula for π, but found the calculation hard going beyond the 15th decimal place !

We are full of admiration for youngsters with enterprise and cannot do better than quote Mr. Felton who wrote to us :—

> " M. Rigler should be encouraged—though perhaps not to carry on his calculations much further, or he and his friends will be busy for the next few centuries ! It must be appreciated that the amount of work necessary to work out a number like π to n decimals is roughly proportional to n^2 ; this means that to get to 10000 decimals would take about 500,000 times as long as getting to 15 decimals."

THE POWERS THAT BE

You can check that $2^4 = 4^2$.

I wonder if you can find another pair of numbers so that in a similar way $x^y = y^x$.

Apart from sheer guesses can you find any way of solving such a problem.

The Editor will award book tokens for any interesting solutions received before 1st June, 1960.

Suggested by Mr. J. W. Withrington, M.A., M.Sc., H.M.I.

PUZZLE CORNER

Contributed by Gillian Tyson, Port Erin, Isle of Man.

I have noticed a curious property of the squares of odd numbers which I hope you will find interesting enough to publish.

Square any odd number.

Divide the square by two.

Then the nearest integers above and below this answer, together with the odd number which we started, will be the lengths of the sides of a right-angled triangle.

Example : $15^2 = 225$
$225 \div 2 = 112\frac{1}{2}$
Nearest integers are 112 and 113
$112^2 + 15^2 = 113^2$

Can you explain why this method should be ?

STAMP COLLECTORS' CORNER No. 8

BLAISE PASCAL (1623–1662), was an exceptional boy. When he was 16 he discovered the theorem now known as Pascal's Theorem and wrote an " Essay on Conics " in which he deduced over 400 propositions in geometry as special cases of his theorem. At the age of 18 he invented and made the first calculating machine. Later he made important contributions to the theory of probability. At 31 he abandoned mathematics for theological and moral speculations.
C.V.G.

France 1944,
1·20 fr. + 1·80 fr. *black*.

WITHOUT COMMENT

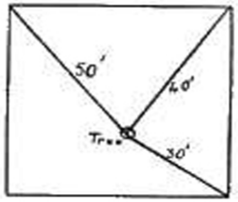
Col. Withycombe and his family are the only people who live on the island, which is 269 feet above sea level and 44 acres in diameter."

Extract from the *Nottingham Guardian Journal*, Tuesday, November, 1954.
Contributed by Mr. F. Goodliffe, Mapperley, Nottingham.

THE MATHEMATICIAN'S HUMOUR

Lord Kelvin (Professor William Thomson, 1824–1907), unable to meet his classes one day, posted the following notice on the door of his lecture room—" Professor Thomson will not meet his classes today."

The disappointed class decided to play a joke on the professor by erasing the "c". When the class assembled the next day in anticipation of the effect of their joke, they were astonished and chagrined to find that the Professor had outwitted them. He had erased the "l".



GEOMETRY IN A GARDEN

I own a square garden as shown in diagram, within the garden stands a tree 30′, 40′, and 50′ from three successive corners. How much land have I?

CARD TRICK

Two players in turn take a playing card from a pile and place the card on a tea tray. The one who first places a card so that it touches another card loses the game. How can the first player ensure that he wins?

Suggested by R. W. Payne, Mathematical School, Rochester.

MATHEMATICS OF A MILITARY MAN

" I'm very well acquainted, too, with matters mathematical,
I understand equations both simple and quadratical.
About Binomial Theorem I am teeming with a lot of news
With many cheerful facts about the square of the hypotenuse.

Major- General Stanley, Pirates of Penzance—By W. S. Gilbert.

MODERN GEOMETRY ?

When my sister discovered that she was taking the scholarship in February she developed an interest in geometrical instruments, this included the protractor.

After having had its functions explained to her in the simplest possible terms, she said, " Well, if a right angle is 90°, what's a left angle ? "

Sandra Shoham, 19, Asmara Road, London, N.W.2.

DO YOU KNOW ?

1. Which Mathematician was Master of the Mint ?
2. Who wrote " Angling may be said to be so like the Mathematics that it can never be fully learnt ? "
3. Which School had as its motto " Let none ignorant of Geometry enter my door ? "
4. What theorem in Geometry is known as " Pons asinorum ? "
5. What is the significance of the word " BODMAS " ?

57326 54893 82391 19325 97463 66730 58360 41428

Readers will be interested to know that Mr. Felton has found the cause of the disagreement between two formulae reported in our earlier note. He repeated the calculation and has provided us with the printed output charts from his Pegasus computer showing agreement between the Gauss and Klingenstierna formulae to about 10020 decimals. He also states :—

" Since this work was completed, M. Genuys, of the IBM—France company, has quite independently carried out a calculation (using Machin's formula) to 10000 places. I am happy to say that the result agrees with the first 10000 places of mine."

J.F.H.

" DO-IT-YOURSELF " PIE

It can be proved that $\frac{\pi}{4}=5 \text{ arc tan } \frac{1}{7}+2 \text{ arc tan } \frac{3}{79}$ and that (for such values)

$$\text{arc tan } x = \frac{x}{1+x^2}\left\{1+\frac{2}{3}\left(\frac{x^2}{1+x^2}\right)+\frac{2\cdot 4}{3\cdot 5}\left(\frac{x^2}{1+x^2}\right)^2+\frac{2\cdot 4\cdot 6}{3\cdot 5\cdot 7}\left(\frac{x^2}{1+x^2}\right)^3+\cdots\right\}$$

Evaluating $\frac{x^2}{1+x^2}$ for $x=\frac{1}{7}$, we have $\frac{2}{100}$ and for $x=\frac{3}{79}$, $\frac{144}{100000}$; the powers of 10 in the denominators cause the succeeding terms to become very small (the series is said to *converge* rapidly). Now try your luck !

J.F.H

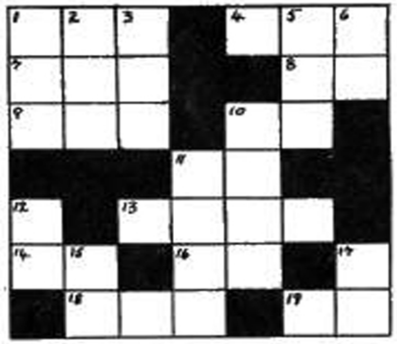
SENIOR CROSS-FIGURE No. 29

Submitted by Mr. F. G. Hewit, of Wrexham.

ABCDEFGH is a rectangular parallelepiped. All its edges and its diagonals are integers.

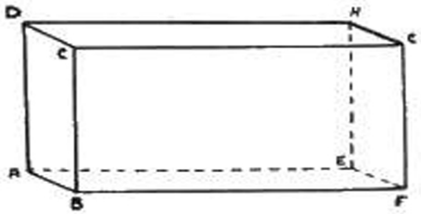
Across

1. BF^2
4. 11 down divided by 9.
7. BD^4
9. For fire, police, ambulance, phone - - -.
10. BD^2
11. BF
13. Number of square yards in 1 acre.
14. AD^2
16. AB^2 multiplied by 10.
18. DF multiplied by 33.
19. AB^4



Down

1. DF^2
2. The same as 18 across.
3. AB^2 multiplied by 51.
5. BD^3
6. AD^2 plus 1.
10. Number of lb. in one ton.
11. See 4 across.
12. AB^4
15. AD^3
17. AB multiplied by 7.



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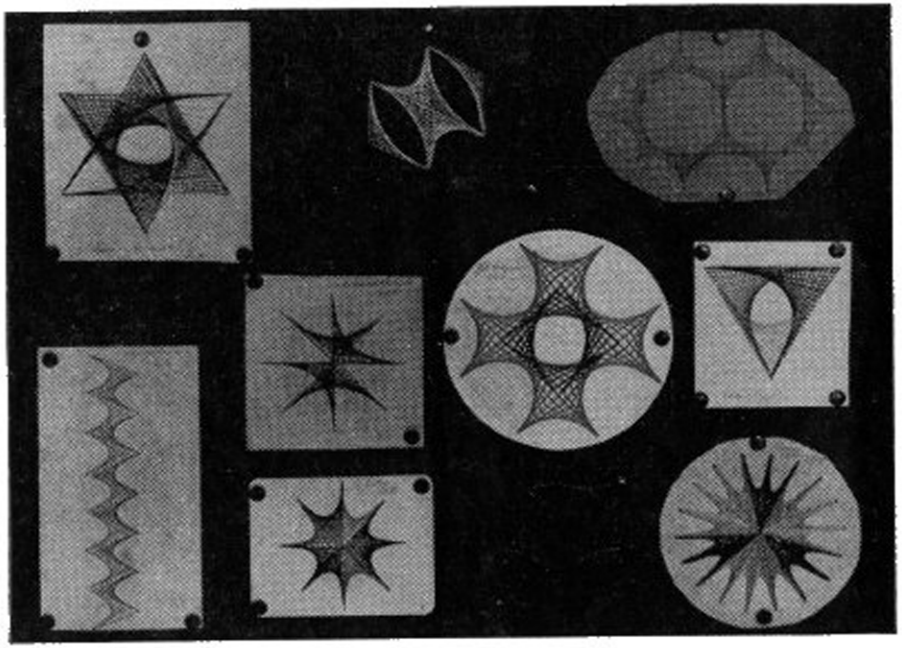
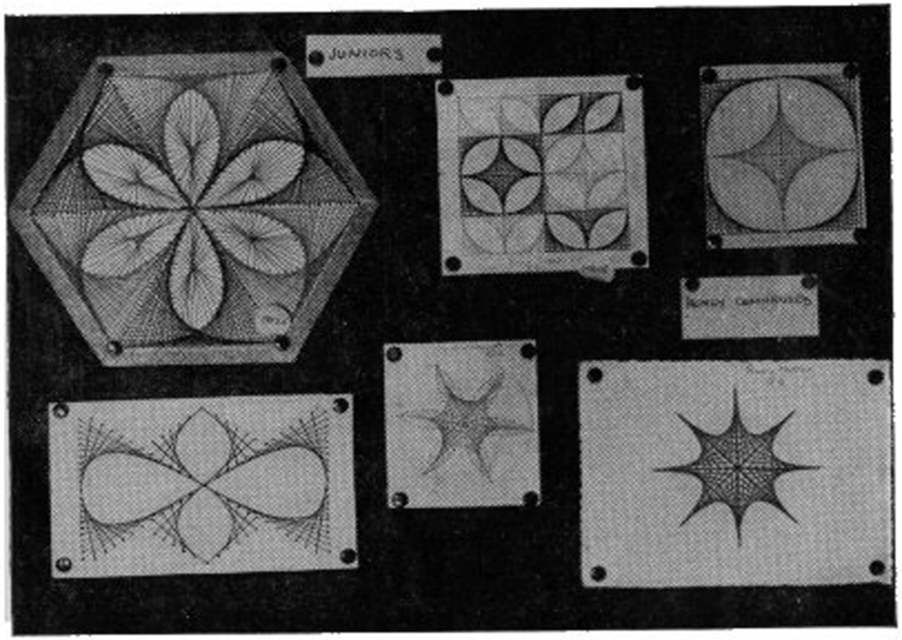
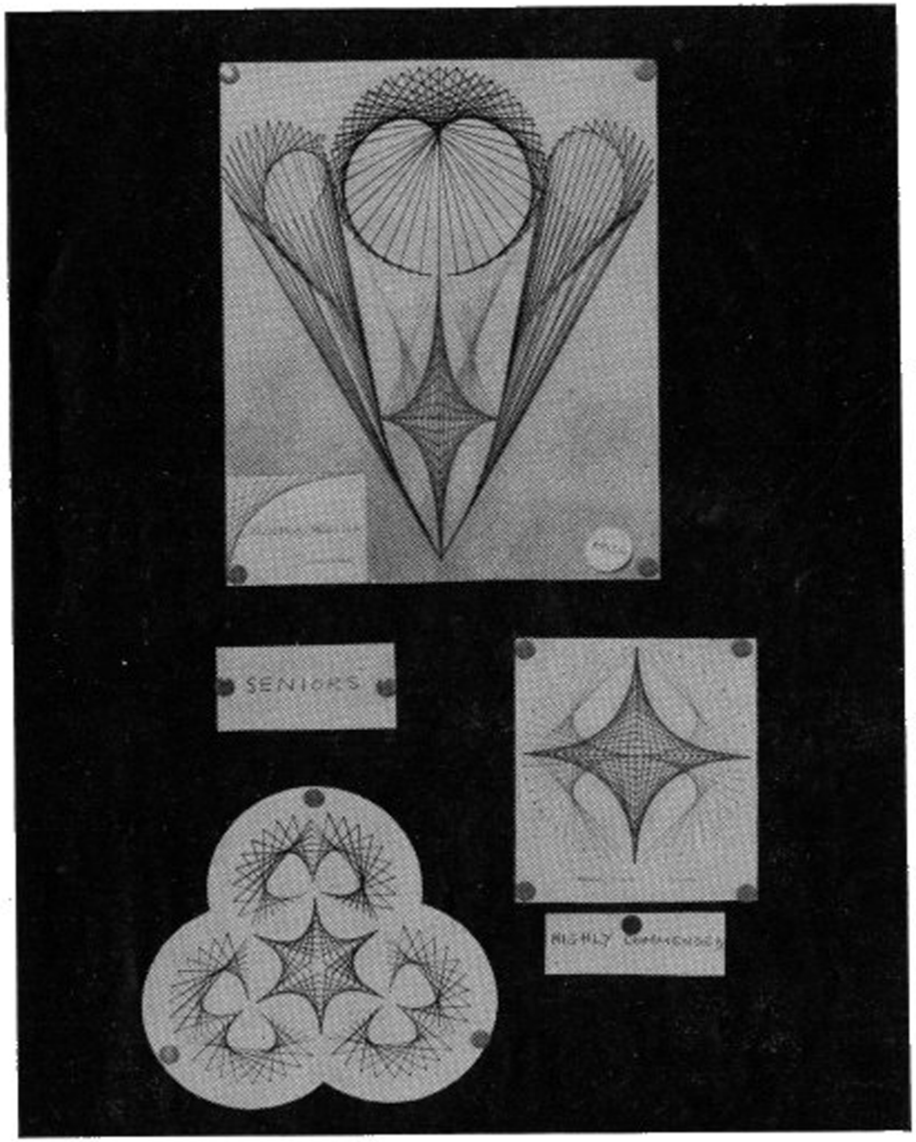


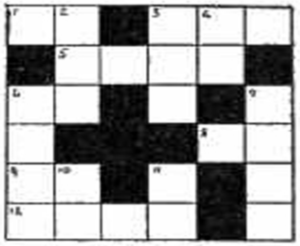




The photographs show the results of the work of the girls of Bournemouth School for Girls after the article on Mathematical Embroidery published in issue No. 25, October, 1958.

JUNIOR CROSS-FIGURE No. 27

Submitted by Suzanne Bendz and Sheila Walmark, Hazeldene School, Salcombe, S. Devon



CLUES ACROSS:

1. $3x+7-3y=19$. Find $x+2y$. $2x+y-2=15$.
3. The rate of interest on a loan is increased from 3% to $3\frac{1}{2}$%. The annual interest is raised by £1 7s. 6d. How much is the loan?
5. Number of square yards in an acre.
6. A boy receives 18 marks out of 60. What is his percentage?
8. Solve $x+31+7-8-x+2x=82$
9. Considered unlucky by some.
12. $780+340+60+120+2000$.

CLUES DOWN:

2. The second number in 12 across.
3. CMLXXX ÷ IV.
4. XXVIII + XII + XIV − III − I + XX.
6. 6,404 as a percentage of 18,760 correct to 2 places of decimals.
7. Twice 12 across.
10. Half the total external surface area of a cylinder, closed at one end and open at the other, with external dimensions—radius 3″, height 2″.
11. Solve $3x-10-x=30-2x$.

SOLUTIONS TO PROBLEMS IN ISSUE No. 29

SENIOR CROSS FIGURE No. 29

CLUES ACROSS: (1) 144; (4) 211; (7) 625; (8) 27; (9) 999; (10) 25; (11) 121 (13) 4840; (14) 16; (16) 90; (18) 429; (19) 81.
CLUES DOWN: (1) 169; (2) 429; (3) 459; (5) 125; (6) 17; (10) 2240; (11) 1899; (12) 81; (15) 64; (17) 21.

GEOMETRY IN A GARDEN.

By using two applications of the circle of Appollonius, it can be shown that the area of the garden is very approximately 3,200 sq. ft.

CARD TRICK.

Place the first card in the centre of the tray. Now wherever the opponent puts a card there will be a space symmetrically opposite in which the first player can put his next card. By continuing in this way, the first player must win.

DO YOU KNOW?

1. Sir Isaac Newton. 2. Izaak Walton. 3. Plato's school.
4. The base angles of an isosceles triangle are equal.
5. Brackets out, divide, multiply, add, subtract. This gives the order for simplifying arithmetic expressions.

PUZZLE CORNER.

Any odd number can be written as $(2n+1)$. Squaring gives $4n^2+4n+1$. Divide by 2, $2n^2+2n+\frac{1}{2}$. The two integers above and below are $2n^2+2n+1$ and $2n^2+2n$. Then

$$(2n^2+2n)^2+(2n+1)^2 = 4n^4+8n^3+8n^2+4n+1 \text{ and}$$
$$(2n^2+2n+1)^2 = 4n^4+8n^3+8n^2+4n+1.$$

MATCH YOUR WITS.

(1) Form a regular tetrahedron with the matches for the edges. (2) The horses cost £36$\frac{4}{11}$ and £43$\frac{7}{11}$

BINARY CROSS-FIGURE.

CLUES ACROSS: (1) 11; (100) 1.11; (110) 1001; (111) 11.
CLUES DOWN: (10) 1100; (11) 11; (101) 1011; (110) 10.

SERIOUSLY SPEAKING.

The next number in the series is 46. The rule for the series is reverse the digits of the perfect squares. The editor regrets the error in printing 36 instead of 63 in the series.

SUM AND PRODUCT.

The product is easier to evaluate as anything multiplied by zero is zero.

The editor apologises for the error in the caption of Cubist Art. It should read $a^3+b^3=(a+b)(a^2-ab+b^2)$.

B.A.

53249 44166 80396 26579 78771 85560 84552 96541

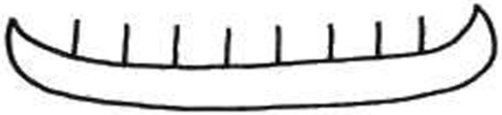
MATHEMATICAL PIE

No. 30 | *Editorial Offices:* 97 Chequer Road, Doncaster | MAY 1960

CONCRETE NUMBERS No. 2

The first step in the recording of numbers by our forefathers was dealt with in Part I (May 1959, No. 27) and the kind of pictures mentioned there would have been carved on wood, stone, etc., to record an item of historical interest.

In their endeavours to save time when recording, the primitive races automatically resorted to abbreviations and in the course of time the idea of number as such gradually became clearer and we can guess that the numbers in Part I would have eventually been written as follows :—



N. AMERICAN INDIAN

One of the obvious disadvantages of this symbolism is the amount of space which could be required for the number part of the story.

After this development, the story of writing numbers breaks up into two distinct directions. One part begins to develop into the recording of the spoken language, *i.e.*, the symbolism of writing words, *e.g.*, CUPTIE, whilst the other part deals with the development of the methods of writing "pure numbers," *e.g.*, 3, 5, the symbolism of arithmetic. R.H.C.

SQUARE IN THE FACE

By courtesy of the *Mathematics Students' Journal.*

If x represents a whole number that is also a perfect square, find a formula that will give the next larger square whole number.

76211 01100 44929 32151 60842 44485 96376 69838

MORE ABOUT TRIADS

We can call three numbers a Pythagorean triad if they have no common factor, and the square of one of the numbers is equal to the sum of the squares of the other two. For example, 3, 4, 5 form a Pythagorean triad, but 6, 8, 10 do not. Those of you who read the article on Triads in Pie No. 18 will see that any odd number or any number which is a multiple of 4 can represent one of the perpendicular sides of a right angled triangle whose sides are integers with no common factor.

For example, $11 = 11 \times 1 = (6+5)\ (6-5) = 6^2 - 5^2$

Therefore, squaring both sides, $11^2 = (6^2+5^2)^2 - (2\times6\times5)^2 = 61^2 - 60^2$

Hence 11, 61, 60 form a Pythagorean triad.

Again, $12 = 2\times2\times3$ so that $12^2 = 4\times2^2\times3^2$

Therefore, $12^2 = (2^2+3^2)^2 - (3^2-2^2)^2 = 13^2 - 5^2$

so that 12, 13, 5 form another Pythagorian triad.

Also $12 = 2\times6\times1$. Can you find the other triad to which 12 belongs?

It is not so easy to see what numbers can be the hypotenuse numbers of a Pythagorean triad. The answer is quite simple, although difficult to prove. It is this; a number can be the hypotenuse number of a Pythagorean triad if, and only if, all its prime factors leave a remainder 1, when divided by 4.

For example, 91 has a prime factor 7, which leaves a remainder 3 when divided by 4. Therefore, 91 is not a hypotenuse number, but

$65 = 5\times13$ and $65^2 = 63^2+16^2$ and 33^2+56^2

[as well as $(5\times5)^2+(5\times12)^2$ and $(13\times3)^2+(13\times4)^2$].

There is no simple way of finding the other two members of the triad when the hypotenuse number is given, but, if you are good at algebra, you will be able to do this problem—

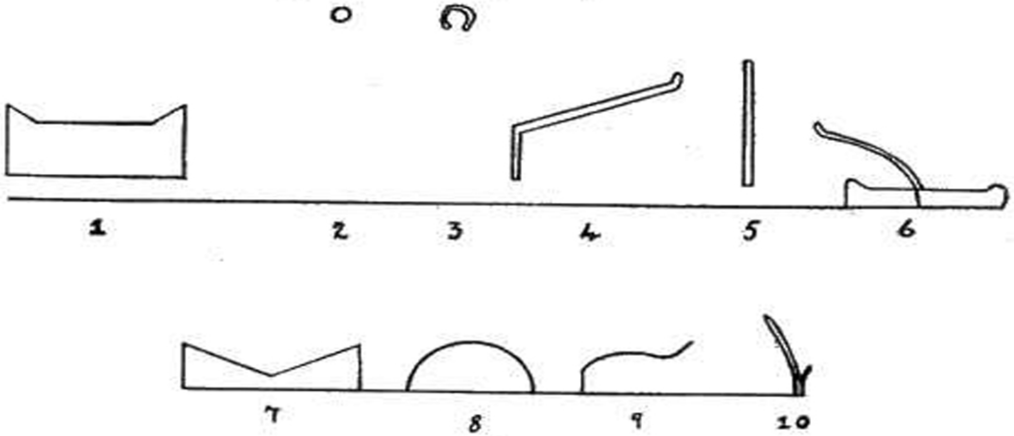
Let $a^2=b^2+c^2$ and $p^2=q^2+r^2$

Express $(ap)^2$ as the sum of two squares in two different ways. C.V.G.

ROUNDERS

Contributed by H. Bromby, Southampton Grammar School for Girls.

The following shapes are rotated through 360° about the axes shown. Name the familiar objects whose shape they trace out.



95228 68478 31235 52658 21314 49576 85726 24334

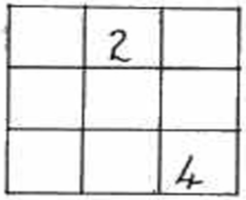


Figure 5

sequence principle. If you do this correctly, you will find that one cell always contains the number 6, whatever number you had put in the central cell. Which cell is it?

Question 8. (For those who like algebra). In Fig. 5, put x instead of 2, and y instead of 4, and prove that whatever number you put into the central cell, one of the other cells must contain $2y - x$.

Submitted by Canon Eperson, Bishop Otter School.

(to be continued)

CALCULATING PRODIGIES—No. 1.

George Parker Bidder, 1806-1878

George Bidder was one of a number of remarkable calculating prodigies who lived during the 19th century, and amazed audiences all over England with his exceptional powers in mental arithmetic.

The son of a stone-mason, he was born in Moretonhampstead, Devonshire, and, at the age of six, was taught to count up to 100. Using this limited knowledge he taught himself to add, subtract and multiply numbers less than 100 by making patterns with marbles and buttons, though he still remained ignorant of how to write numbers.

At 7 years old he had already gained a reputation in the village for his ability to calculate quickly. During the next two years his fame spread beyond the village and his father found it profitable to take him about the country to give public exhibitions. Finally, his father was persuaded to leave him in the care of some members of the University of Edinburgh and Bidder later graduated and became a civil engineer.

In spite of his very limited knowledge at the beginning of his career, Bidder learned quickly with practice. Most of the questions posed to him during his early years involved the mental addition and multiplication of large numbers, but by 1819 he was able to calculate square roots and cube roots of equally high numbers and could give almost immediate answers to problems on compound interest.

One of the questions posed when he was 9 years old was: "If the moon be distant from the earth 123,256 miles, and sound travels at the rate of 4 miles a minute, how long would it be before the inhabitants of the moon could hear the battle of Waterloo?" In less than one minute he gave the answer: "21 days, 9 hours, 34 minutes."

At 14 years old he was asked: "Find a number whose cube less 19 multiplied by its cube shall be equal to the cube of 6." The answer 3 was given instantly.

It is worth noting that with all these questions Bidder was quick to grasp the requirements of a spoken problem, but a written question took him much longer to understand, and he never wrote down any part of his calculation.

I.L.C.

36065 98764 86117 91045 33488 50346 11365 76867

MAGIC SQUARES No. 1

Magic Squares were known to the Chinese over 2,000 years ago, and even today they have a fascination for anyone interested in numbers. In mediaeval times magic squares were used as charms, for it was believed that they had magic powers to ward off the plague and other human ills: this superstition still persists, it is said, in some eastern countries, but their magic for us lies in their peculiar numerical properties. The sum of the numbers in each row, column and diagonal is the same.

8	1	6
3	5	7
4	9	2

Figure 1

Fig. 1 shows the numbers from 1 to 9 arranged in a square with nine cells: if you add the numbers in any row, column or diagonal you will get the same total of 15 every time. You will notice that each corner cell contains an even number, and that the central cell is occupied by the middle number of the sequence 1, 2, 3, 4, **5**, 6, 7, 8, 9. This is the only way in which these numbers can be arranged to form a magic square: 5 must be in the central cell, and starting with the number 2 in any of the four corner squares, the order of the numbers in the cells around the central cell is 27618394, either clockwise or anticlockwise.

Question 1. You can see in Fig. 1 that the numbers in the middle row 3, 5, 7, form a sequence (arithmetical progression); what other sequences can you find in the middle column, and the two diagonals?

15	2	13
8	10	12
7	18	5

Figure 2

Question 2. Fig. 2 shows another magic square made with nine other numbers: what is the magic total of each row, column and diagonal?

Question 3. Write down the nine numbers in ascending order of magnitude: in which cell is middle number of the series?

Question 4. What sequences (arithmetical progressions) can you find in the rows, columns or diagonals of Fig. 2?

9	1	8
5		

Figure 3

Question 5. It is possible to complete a magic square with nine cells, if the numbers in four cells are given, provided three cells are in a straight line, as the sum of these three numbers will give the magic total. Complete the magic square shown in Fig. 3 and verify that the number in the central cell is the middle number of the nine numbers arranged in order of magnitude, and that there are four sequences (arithmetical progressions)—in the middle row, middle column, and in both diagonals.

5		9
	6	

Figure 4

Question 6. Remembering that the numbers in each diagonal form sequences, complete the magic square indicated by Fig. 4.

Question 7. In Fig. 5 put any number you like in the central cell: (to avoid negative numbers, the number chosen should not be less than 7). Now complete the magic square, using the

26918 62056 47693 12570 58635 66201 85581 00729

STAMP CORNER No. 9

Nature and Nature's laws lay hid in night.
God said "Let Newton be!" and all was light.
ALEXANDER POPE

It did not last, the Devil howling "Ho!
Let Einstein be!" restored the status quo.
SIR JOHN C. SQUIRE

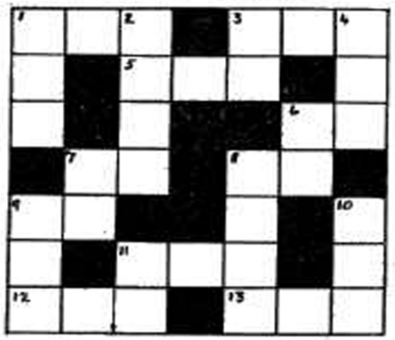
France
18 Franc Blue.

Israel
350 Brown.

C.V.G.

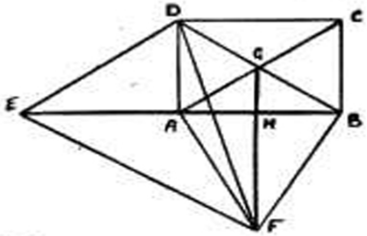
SENIOR CROSS-FIGURE No. 30

Submitted by Mr. W. T. G. Parker, The Grammar School, Minehead.



CLUES ACROSS:

1. The coefficients in order in the quadratic equation whose roots are $-\frac{2}{3}$ and -2.
3. DA.
5. The area of triangle DBF.
6. a times b, where a and b make $ax^3+bx^2+27x+18$ exactly divisible by $(2x+3)$ and by $(x+2)$.
7. $\frac{x}{4}-\frac{y}{2}=3\frac{1}{2}$ and $\frac{x}{9}+2y=16$. Find x.
8. y in 7 across.
9. y varies inversely as the square root of x and $y=30$ when $x=.04$. Find y when $x=.01$.
11. The volume, in cubic inches, of a pyramid on a rectangular base KLMN with vertex V vertically above a point P on KL, where PL=3″, LM=4″, VK=YM=13″.
12. The area of $\triangle$ DAB reversed.
13. The same as 1 across.



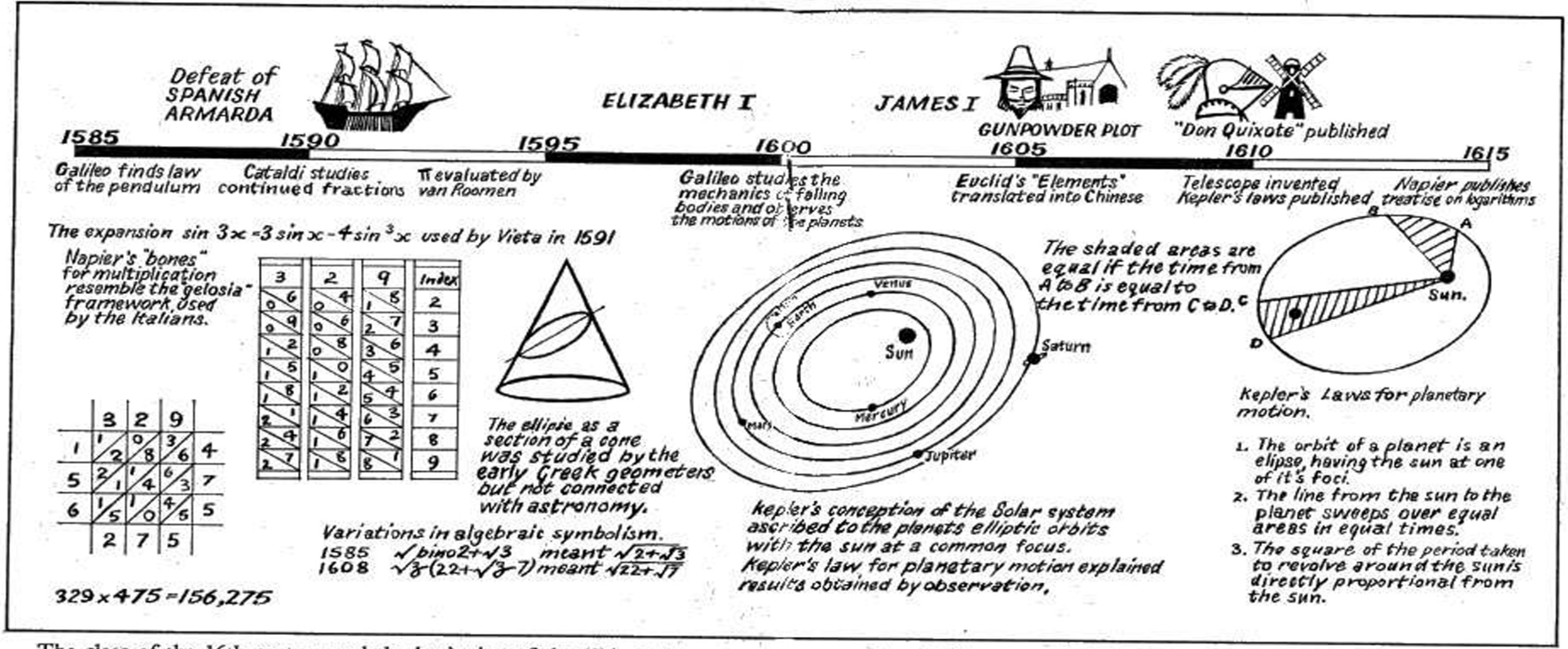
ABCD is a rectangle. AB=3 units, and DB=2DA. AE=AB, and ABF is an equilateral triangle.

CLUES DOWN:

1. The ratio 6.3 : 14.7 : 10.5 expressed as a ratio of smallest whole numbers.
2. DF.
3. The sum of the digits in 2 Down minus the sum of the digits in 4 Down.
4. DE.
6. The maximum value of $2x^3-3x^2-36x-19$.
7. $(2x+9)(2-x)$ when $x=-2$.
8. £10 is borrowed and interest is charged at 3% of the sum owing at the beginning of the year. If £1 is repaid at the end of each year, find, in pounds, the sum owing at the beginning of the sixth year.
9. The area of triangle DFC.
10. The number of tiles, 6 in. square, required to tile the walls of a bathtoom 7 ft. by 9 ft., to a height of 4 ft., the doorway being 3 ft. wide.
11. The height of the pyramid in 11 across, in inches.

41893 03968 64262 43410 77322 69780 28073 18915

TIME CHART 8



The close of the 16th century and the beginning of the 17th century marked a period in which computational techniques reached a new height; mathematicians were very much in demand and astronomy flourished throughout Europe.

Following closely on the introduction of decimal fractions by Stevin came the invention of logarithms by John Napier. Napier's system of calculation made it possible to reduce multiplication of numbers to the addition of corresponding numbers. The logarithms we use today are based on the same principle as Naperian logarithms but are much more simple in form. As with so many new developments in mathematics, the basic ideas underlying Napier's work were very simple and a number of other mathematicians were inspired to find ways of improving this new tool. Amongst these, two of the most noteworthy contributors were Henry Briggs (1556–1631), a professor of geometry at Gresham College, London, and Edmund Gunter (1581–1626) who is also remembered for his "Gunter's chain" which is used in surveying. Napier's contribution to the task of simplifying calculation came at a time when a great deal of arithmetic was being done in connection with astronomy. Kepler was studying the orbits of the planets, Galileo had begun to use a telescope to study the stars and German mathematicians had constructed trigonometric tables of considerable accuracy. In

view of this great activity it has been said that the invention of logarithms "by shortening the labours, doubled the life of the astronomer."

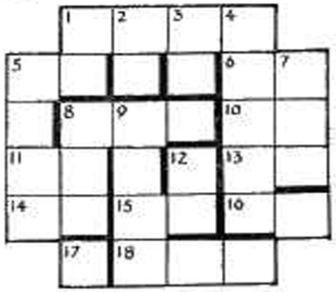
The revolution in astronomy due to the works of Copernicus, Tycho Brahe and Johannes Kepler gave man an entirely new vision of his place in the universe and helped to explain many of the phenomena which had puzzled earlier astronomers. Brahe (1546–1601) was a Danish astronomer who spent many years studying the motion of the moon and planets, and the last year of his life was spent in the observatory near Prague with Kepler as his assistant.

Kepler (1571–1630) was more mathematically inclined than Brahe and his first attempt to explain the solar system was made in 1596, when he believed he had discovered a relationship between the five regular solids and the number and distance of the planets. The publication of this theory brought Kepler much fame and at one time he tried to use an oval curve to represent the orbit of Mars. Further reflection brought forth the results which proclaimed his genius and which are now known as "Kepler's Laws." In finding the elliptic orbits of the members of the solar system Kepler bridged the gap between geometry and astronomy of the early Greeks. It has been said "if the Greeks had not cultivated conic sections, Kepler would not have superseded Ptolemy."

I.L.C.

JUNIOR CROSS-FIGURE No. 28

Submitted by Marion Mitchell, Form IV X, Bradford Girls' Grammar School.



CLUES ACROSS:

1. Last year + next — $\frac{1}{2}$ of this year.
5. $\sqrt{289}$.
6. A fifth of a furlong in yards.
8. Number of acres in half a square mile.
10. Number of chains in a mile.
11. Half of three times $(9-x)(6+2x)$ when $x=-2$.
13. Area of a triangle, base 20 inches, height 5 inches.
14. I spent $\frac{1}{4}$ of my money, and then $\frac{3}{5}$ of the remainder. I had 24/- left. How much had I at first?
15. Third prime number squared multiplied by the sixth prime number.
16. Tom has 4 times as many marbles as Dick, Jack has as many as Tom and Dick together, Sam has one less than Tom. 125 altogether, how many has Sam?
17. One eighth of half of 12^2.
18. Reverse a number and multiply the two together.

CLUES DOWN:

1. $\sqrt{729}$.
2. $10^2-(6^2-4^2-10)$.
3. A perfect square.
4. Number of ounces in a ton, reversed.
5. 103×11.
6. Number of gallons which can be held in a cistern four feet each way inside.
8. Number of farthings in a third of a guinea.
9. 18 cwt. 2 qr. 23 lb. in lb.
12. $7^2+(6^2-2^2-\sqrt{4})$.

SOLUTIONS TO PROBLEMS IN ISSUE No. 30

SQUARE IN THE FACE.

$(\sqrt{x}+1)^2 = x+2\sqrt{x}+1$.

ROUNDERS.

1. Cotton reel. 2. Inner tube. 3. Outer cover. 4. Flower pot. 5. Gramophone record. 6. Hand bell. 7. Diabolo. 8. Ball. 9. Vase. 10. Saucer.

SENIOR CROSS-FIGURE No. 30.

ACROSS: (1) 384; (3) 173; (5) 520; (6) 26; (7) 27; (8) 65; (9) 60; (11) 128; (12) 062; (13) 484.
DOWN: (1) 375; (2) 4577; (3) 10; (4) 346; (6) 25; (7) 20; (8) 6284; (9) 650; (10) 464; (11) 12.

MAGIC SQUARES.

Question 1. Sequences: 1, 5, 9 : 8, 5, 2 ; 4, 5, 6.
Question 2. Magic Total 30.
Question 3. 2, 5, 7, 8, **10**, 12, 13, 15, 18. 10 is in the central cell.
Question 4. Sequences: 8, 10, 12 : 2, 10, 18 : 15, 10, 5 : 7, 10, 13.

Question 5.

9	1	8
5	6	7
4	11	3

Question 6.

5	4	9
10	6	2
3	8	7

Question 7.

	2	
6		
		4

Question 8.

	x	
$2y-x$		
		y

To prove the result in Question 8 put m for any number in the central cell: then using the sequence principle, the first (top left corner) cell must contain the number $2m-y$, and the magic total, found by adding up the completed diagonal, is $3m$. So all the other cells can now be filled, by making the total of each row, column and diagonal $3m$.

In the answer to Question 3 notice the summetrical pattern formed by the numbers which do *not* appear

2 | XX | 5 | X | 7 8 | X | **10** | X | 12 13 | X | 15 | XX | 18

Note also that 10 is the average of all the numbers.

JUNIOR CROSS-FIGURE.

ACROSS: (1) 13; (3) 275; (5) 4840; (6) 30; (8) 26; (9) 13; (12) 3300.
DOWN: (2) 340; (3) 245; (4) 70; (6) 3413; (7) 6600; (10) 33; (11) 10.

B.A.

77917 45011 29961 48903 04639 94713 29621 07340

MATHEMATICAL PIE

No. 31 | Editorial Offices: 97 Chequer Road, Doncaster | OCTOBER 1960

If I have seen further than other men, it is by standing on the shoulders of Giants. Sir Isaac Newton.

What names would you suggest for the giants? B.A.

26654 08530 61434 44318 58676 97514 56614 06800

NUMBERS IN WORDS

Suggested by E. Rowland, Esq., Huddersfield.

Any number can be expressed in words, but generally this form is used only for small numbers : it is so much quicker and neater to use numerical symbols for the large numbers. Some new and amusing results can be obtained when numbers are written in words.

1. The eight letters in FORTY SIX are different but this is neither the largest nor the smallest number that can be expressed in words in which all the letters are different. What are the largest and the smallest numbers that you can find expressed in words in which all the letters are different?

2. Find a number which contains the same number of figures when expressed in the usual way as letters when expressed in words.

3. The following simple additions are obviously correct. It is also possible to replace each dot by a letter to form addition problems in words which are still correct. The first one becomes TWO+FIVE=7

(i)	. . . + ... = 7	(v)	. . . + ... + ... = 11
(ii)	. . . + ... = 8	(vi)	. . . + ... + ... = 12
(iii)	. . . + . . . + . . . = 9	(vii)	. . . + ... + ... = 13
(iv)	. . . . . + ... = 10	(viii)	. . . . . + ... + ... = 14

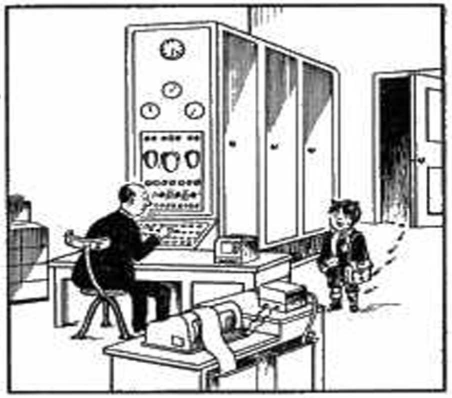
Paradoxes arise occasionally when numbers are written in words. The statement that there is a number which is " the least whole number which cannot be named in less than nineteen syllables" is paradoxical, because the phrase in inverted commas which has been used to define a number contains only EIGHTEEN syllables.

Can you find any more paradoxes?

MATHS

I like doing Maths.
Working sums and making graphs.
Finding the area of a square,
Giving it the utmost care.
Ruling a line, learning a table.
Doing it the best as I am able.
Algebra, geometry, fractions too
In maths there's certainly a lot to do.

Marilyn Boydle, North Manchester High School For Girls, Church Lane, Moston, Manchester, 9.



By courtesy of the "Daily Mirror."
"Have you finished my homework yet, Dad?"

ROUND AND ROUND

P is a point on a sphere of radius 6". A pair of compasses is opened to a radius of 4" and a circle centre P is drawn with the compasses on the sphere. What is the radius of this circle? What is the radius of the largest circle that can be drawn on the sphere?

70023 78776 59134 40171 27494 70420 56223 05389

FUN WITH NUMBERS—1

$$1 = 1$$
$$2+3+4 = 1+8$$
$$5+6+7+8+9 = 8+27$$
$$10+11+12+13+14+15+16 = 27+64$$

Deduce the next line and check the arithmetic. Then try to guess a general rule for the *n*th. line and prove it.

INTELLIGENCE TEST

A general knowledge quiz was set to five of the class, John, Brenda, Robin, Harold and Joyce. These were the questions

(1) What is the name of the nearest cinema?
(2) What is the name of the mayor.
(3) How many miles is it to London?
(4) What are the colours of the Rovers.
(5) What is the Christian name of a contrary young lady.
(6) How many centimetres in an inch?

These were the answers the master received.

John :	Astra	Painter	150	Red	Kitty	3
Brenda :	Ritz	Green	170	Blue	Mary	$2\frac{1}{2}$
Robin :	Grand	Watkins	140	Blue	Mary	3
Harold :	Gaumont	Painter	140	Yellow	Lucy	$2\frac{1}{2}$
Joyce :	Gaumont	Watkins	140	Yellow	Lucy	3

Each of the five has two correct answers and four incorrect and each question has been answered correctly by someone.

What are the correct answers.

R.H.C.

PROBLEMS

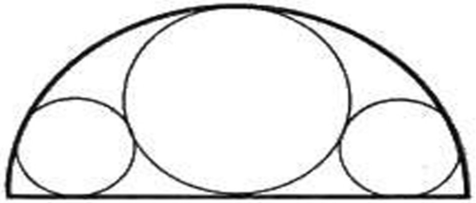
1. Two sides of a parallelogram are 7" and 9". If one of the diagonals is 8", how many inches are there in the other?

2. Find the smallest integral value of X that will make $1260X$ a perfect square.

3. An integer between 20 and 30 is added to its cube and the sum is 13848. Find the integer.

ANOTHER TIGHT FIT

Three circles, two of them equal are drawn in contact with a semicircle as shown. If the diameter of the semicircle is 12", what are the diameters of the smaller circles?



09907 96547 37612 55176 56751 35751 78296 66454

THE TWO BUCKETS

Two exactly similar buckets are as full of water as they can be, but one has a large piece of ice floating in it. Which weighs heavier?

THE POWERS THAT BE

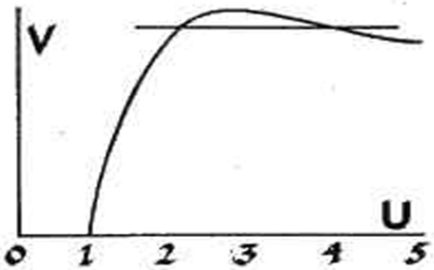


Figure 1. $v=\frac{\log u}{u}$.

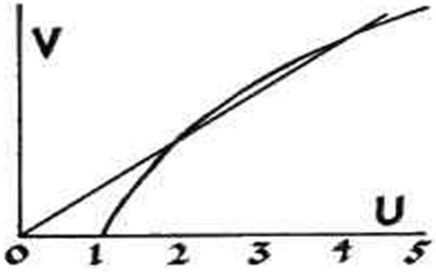


Figure 2. $v=\log u$.

Over a hundred readers sent solutions to the problem in Mathematical Pie No. 29 on finding pairs of numbers to satisfy $x^y=y^x$.

Those who tried a graphical solution changed $x^y=y^x$ to $(\log x)/x=(\log y)/y$. If the graph of $v=(\log u)/u$ is plotted, pairs values for x and y are given by the intersections of horizontal lines with the curve (Figure 1). Alternatively, pairs of solutions are given by drawing lines through the origin to insect the curve $v=\log u$ (Figure 2). It can be seen from the graphs, or proved in other ways, that the smaller of the two numbers must lie between 1 and e (2.718 . . .), so that there can be no integral solution other than 2 and 4.

To find rational solutions an algebraic approach is necessary. Let x be the smaller of the two numbers, and put $y=kx$. Then $x^{kx}=y^x$. Therefore $y=x^k$ and $kx=x^k$. Rearrangement gives $x=k^{1/(k-1)}$ and $y=k^{k/(k-1)}$. Pairs of values for x and y are given by substituting any value for k. To obtain rational values of x and y put $1/(k-1)=n$.

$$\text{Then } x=\left(\frac{n+1}{n}\right)^n \text{ and } y=\left(\frac{n+1}{n}\right)^{n+1}$$

Substituting $n=1$ gives $x=2$, $y=4$, $n=2$ gives $x=\frac{9}{4}$, $y=\frac{27}{8}$ and so on.

R. BOURKE

Book tokens have been sent to :—

A. Balfour (Edinburgh), C. H. Biggs (Tonbridge), J. Bonnici (Malta G.C.), B. K. Booty (Whitchurch), R. Bourke (Newcastle), H. L. Kotkin (Enfield), A. Lavington (Whitgift School), P. R. Mogridge (Exeter School), E. Violet (Wigton).

Ireland 1943 2½d Brown ½d Green

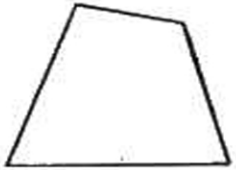
STAMP COLLECTOR'S CORNER No. 9

Sir William Rowan Hamilton, 1805—1865, was Ireland's greatest Mathematician. His first work was in the theory of optics. Later he investigated the fundamentals of algebra and created a new algebra which he called quaternions. His quaternions and his work in optics began the developments which lead to Einstein's theory of relativity and to modern quantum mechanics. In his old age, Hamilton became solitary and eccentric. One of his unusual habits was that of using mutton chops as book marks.

C.V.G.

61581 40025 01262 28594 13021 64715 50979 25923

MODERN GEOMETRY

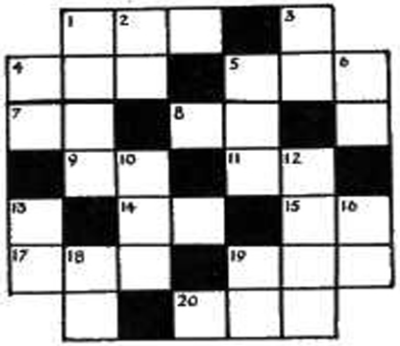


The sum of the inside angles of this quadrilateral is 360°. Without adding any more lines prove that the sum of the interior angles of a pentagon is 540°.

TIM-Ber!

Which is the greatest and which is the least of
(*a*) log (3+5), (*b*) log 5+log 3, (*c*) log (8 — 4), (*d*) log 8 — log 4.

SENIOR CROSS-FIGURE No. 31



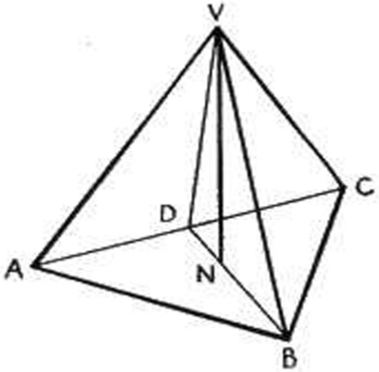
CLUES ACROSS:

1. △ VAC.
4. Four BD^2.
5. Sum of the lengths of the edges.
7. Twice DV.
8. Area △ BVD to nearest square unit.
9. Angle BVD to nearest degree.
11. 4DN reversed.
14. BN^2.
15. Angle BVN plus three.
17. Volume reversed.
19. Total surface area.
20. BN reversed.

CLUES DOWN:

1. Angle VDN.
2. Three VB to nearest unit.
3. Area VAD to nearest sq. unit.
4. Sum of sloping edges to nearest unit.
5. Area △ ABC.
6. $\frac{1}{2}(VB^2 - BN^2)$.
10. Rearrange 5 down.
12. VB.
13. Angle VBN.
16. Perimeter of VDB to nearest unit.
18. Angle AVC to nearest degree.
19. Perimeter of VDN plus one.

B.A.



VABC is a right tetrahedron. ABC is an equilateral triangle of side 12 units. VA=VB=VC. D is the mid-point of AC. VN is perpendicular to ABC. VN=12 units.

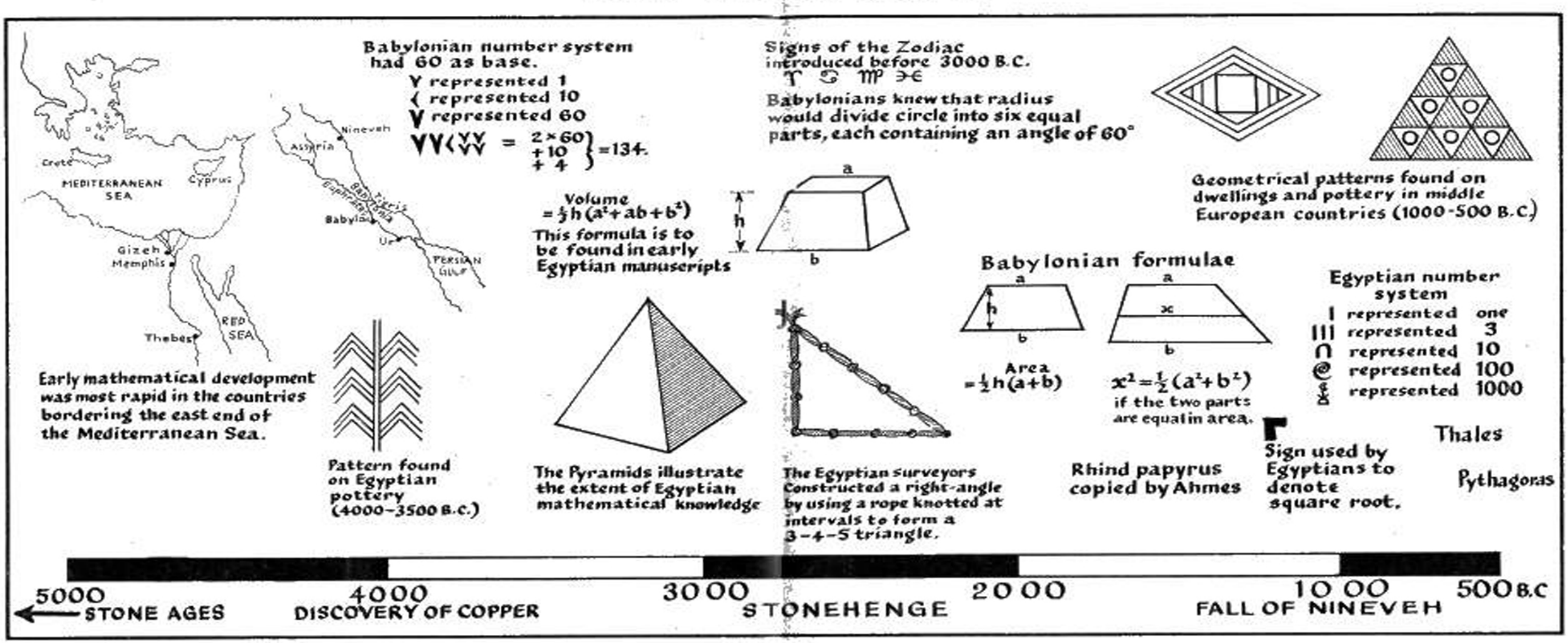
B.A.

AFTER THE 11 PLUS

Three boys in a secondary school and the headmaster all had their birthdays on the same day. The product of the ages of the boys and the headmaster was 2,652. The sum of the ages of the boys was exactly the age of the headmaster. How old was he?

94561 31407 11270 00407 85473 32699 39081 45466

TIME CHART No. I.



The beginning of mathematics came not with some sudden and startling discovery, but with a gradual conception of number and form. Looking back through the last 7000 years of history we find that wherever there was culture, even of the most primitive kind, there was also mathematics. The cave paintings in France and Spain show a remarkable understanding of form, and the pyramids in Egypt, dating back to about 3000 B.C., illustrate an understanding of geometrical design and construction.

In these early times the Egyptians and Babylonians were perhaps the most advanced people in the realm of mathematics. While many tribes were still at the stage of simple counting, these two great civilizations had well-developed number systems and were able to write and calculate with fairly high numbers.

Most of our knowledge concerning Egyptian mathematics has come from the Rhind papyrus. This is believed to have been written before 3000 B.C. and is known to have been copied by the scribe Ahmes in about 1700 B.C. The papyrus is one of the earliest mathematical texts and contains problems in calculation which include work with fractions. By this time the Egyptians also had a system of units of measurement which included measurement of area as well as length. After many years of observation of the sky, the priests of Egypt were able to compile star catalogues and astronomy was being studied seriously before 1000 B.C.

During these early centuries similar progress was also being made by the Babylonians. Their number system, though differing in form from that of the Egyptians, was equally well developed. As far back as 1800 B.C. the Babylonians had compiled tables for multiplication and division, and primitive ideas of geometry and algebra followed very quickly. Their astronomy seems to have developed from the study of astrological omens and, by the 8th century B.C., the court astronomers had compiled a calendar of eclipses of the moon.

Throughout these years of mathematical development there was very little exchange of knowledge between the Egyptians, Babylonians, Hindus and Chinese, each pursuing their own line of cultural progress. The credit for drawing together this diversity of knowledge belongs to the Greeks. Whereas the Egyptians and Babylonians studied mainly for utilitarian purposes, the Greeks were rapidly developing into a nation of philosophers. One of the first Greek philosophers was Thales of Miletus, whose pupil, Pythagoras, spent a number of years travelling in the Mediterranean countries learning from the priests and scribes. After his travels Pythagoras, like Thales, founded his own school and his work added much to the existing fund of mathematical knowledge.

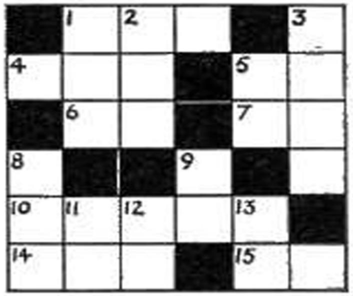
I.L.C.

46458 80797 27082 66830 63432 85878 56983 05235 80893 30657 57406 79545 71637 75254 20211 49557

JUNIOR CROSS FIGURE No. 29

Submitted by Maureen Hobbs, Hazeldene School, Salcombe.

Decimal parts of answers are written as numbers without indicating the decimal point.



CLUES ACROSS :

1. π to 2 decimal places.
4. Eight score.
5. 3^3.
6. $\frac{1}{2}$ as a percentage.
7. Area of a square of side 6 inches.
10. 10 nautical miles per hour expressed as feet per hour.
14. Number of yards in $\frac{1}{2}$ mile.
15. Simple Interest when the principal is £640 and the Amount is £696.

CLUES DOWN :

1. Number of days in one year.
2. Number of cm. in one metre.
3. Number of yards in one mile.
5. $\sqrt{529}$.
8. The duty on a watch of value £4 when the customs charge is $33\frac{1}{3}\%$ of the value (in £ s. d.).
9. Percentage equivalent of 7 marks out of 10.
11. $\frac{8}{100}$ as a decimal.
12. $\frac{4}{5}$ as a percentage.
13. 5 metres as a decimal of 1 Hm.

SOLUTIONS TO PROBLEMS IN ISSUE No. 31

NUMBERS IN WORDS

(ii) ONE + SEVEN, (iii) ONE + TWO + SIX,
(iv) THREE + SEVEN, (v) TWO + FOUR + FIVE
(vi) ONE + FOUR + SEVEN, (vii) TWO + THREE + EIGHT
(viii) THREE + FOUR + SEVEN.

ROUND AND ROUND.

Radius of the first circle is $\frac{3}{5}\sqrt{2}$. The largest circle that can be drawn on a sphere is a great circle ; the radius of the compasses must then be $6\sqrt{2}$.

MODERN GEOMETRY

If a point on one side is taken as a vertex, the angle at that vertex will be 180°. Hence the sum of the angles of a pentagon must be 360° + 180° or 540°.

TIM-Ber

In order from greatest to least :— b, a, c, d. The values are log 15, log 8, log 4, and log 2.

SENIOR CROSS-FIGURE No. 31

ACROSS : (i) 749 ; (4) 432 ; (5) 677 ; (7) 25 ; (8) 62 ; (9) 46 ; (11) 41 ; (14) 48 ; (15) 33 ; (17) 052 ; (19) 287 ; (20) 396.
DOWN : (1) 7354 ; (2) 42 ; (3) 37 ; (4) 42 ; (5) 624 ; (6) 72 ; (10) 642 ; (12) 1386 ; (13) 60 ; (16) 37 ; (18) 51 ; (19) 29.

AFTER THE 11 PLUS

The Editor apologises for the error in this problem. The product of the ages of the boys was 2,652. Their ages were 12, 13, 17 and the headmaster was 42.

THE TWO BUCKETS

By the Principle of Archimedes, the weights of the buckets are the same.

FUN WITH NUMBERS—I

17 + 18 + 19 + 20 + 21 + 22 + 23 + 24 + 25 = 64 + 125.
$[(n-1)^2 + 1] + [(n-1)^2 + 2] \ldots\ldots\ldots\ldots + n^2 = (n-1)^3 + n^3$.

INTELLIGENCE TEST

(1) Astra ; (2) Watkins ; (3) 150 ; (4) Yellow ; (5) Mary ; (6) $2\frac{2}{3}$.

PROBLEMS

(1) 14. (2) 35. (3) 24.

ANOTHER TIGHT FIT

The diameter of the smaller circles are each 3 inches.

JUNIOR CROSS-FIGURE No. 28

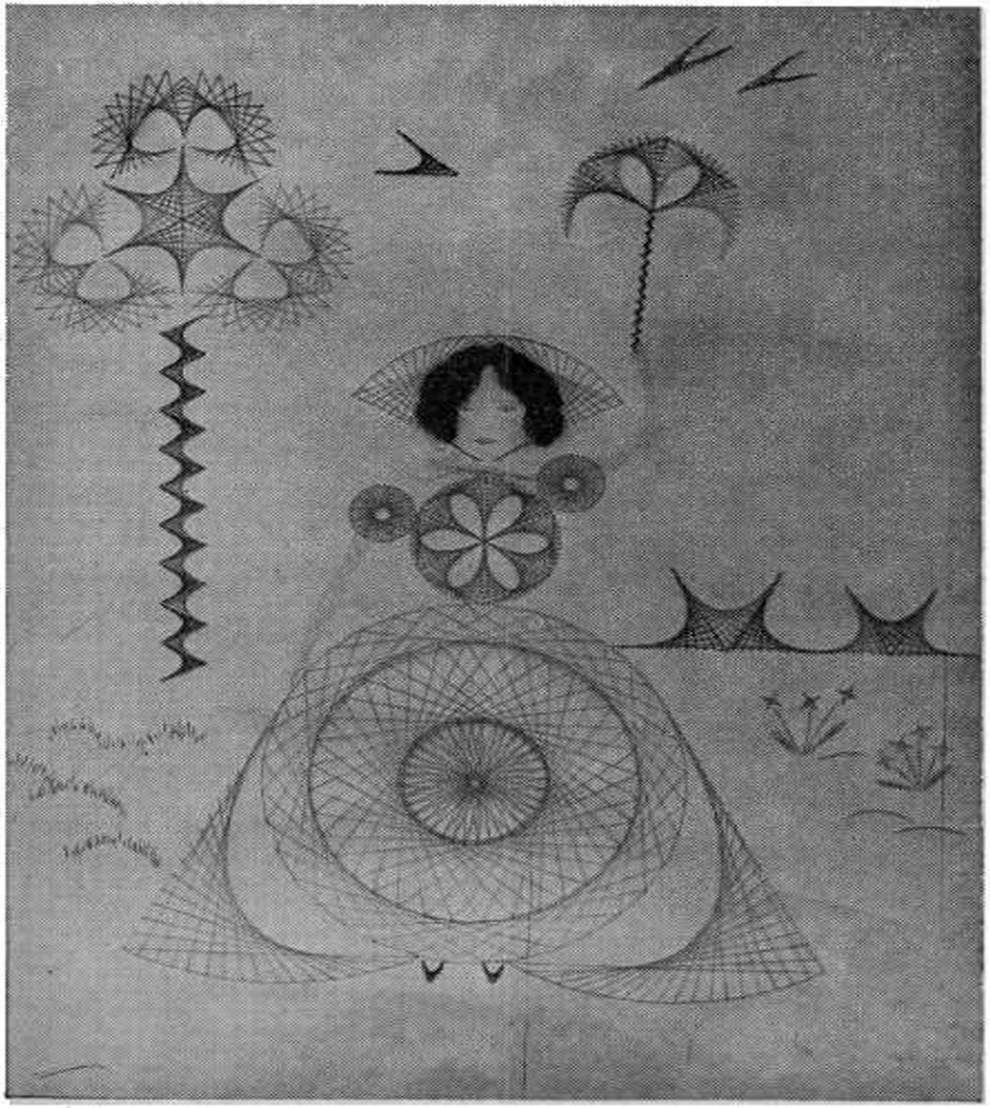
ACROSS : (1) 2940 ; (5) 17 ; (6) 44 ; (8) 320 ; (10) 80 ; (11) 33 ; (13) 50 ; (14) 36 ; (15) 99 ; (16) 35 ; (17) 9 ; (18) 574.
DOWN : (1) 27 ; (2) 90 ; (3) 49 ; (4) 04853 ; (5) 1133 ; (7) 400 ; (8) 336 ; (9) 2095 ; (12) 79.
The clue 6 down should have been labelled 7 down.

B.A.

25602 90228 47210 40317 21186 08204 19000 42296

MATHEMATICAL PIE

No. 32 — *New Editorial Offices : (See Issue No. 33)* — FEBRUARY, 1961



PARABOLA JAEN by VIb, Herts and Essex High School

43751 89573 59614 58901 93897 13111 79042 97828

SPOTS BEFORE THE EYES

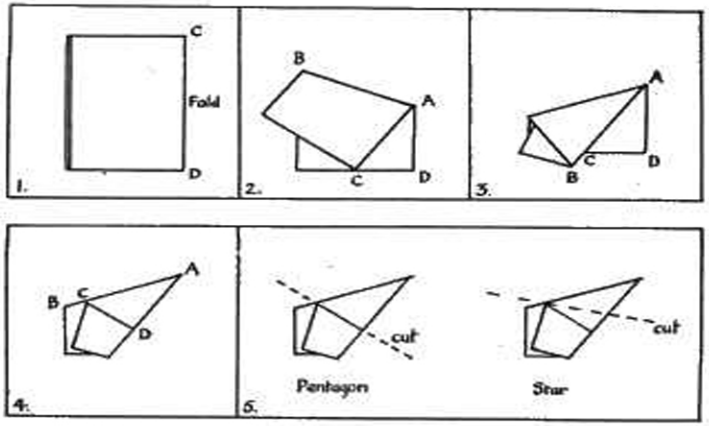
A set of dominoes consists of a set of possible pairs of the numbers 0 to 6. (*a*) How many dominoes are there in a set ? (*b*) What is the total of their spots ? (*c*) Ask four people each to take seven dominoes and add up the value of their spots. The sum of their total should come to (*b*) but you will be amazed at the number of times someone in the four will have made a mistake. Try it on your friends.

R.H.C.

PAPER-FOLDING No. 1

Pentagon and 5-pointed Star

To make a pentagon or a five-pointed star by folding start with a rectangular sheet of paper 5 inches long and $3\frac{1}{2}$ inches wide. A larger sheet may be used so long as you keep the sides in this same ratio.



1. Fold once parallel to the shorter sides.
2. Fold so that C coincides with the middle point of the lower edge and crease along AB.
3. Fold AB onto AC, thus bisecting the angle BAC.
4. Fold the flap ADC behind and turn over.
5. Now that the folding is complete a straight cut along DC will make a regular pentagon; or an oblique cut through C will make a five-pointed star.

I.L.C.

CENTURY MAKERS

In how many ways can 4 nines be arranged to make 100 ?

A FIELD STUDY

The diagonal of a rectangular field is 125 yards and the perimeter is 322 yards. Find the area of the field.

J.G.

Now let us use U for " umpteen," then to Alice ab stands for $a \times U + b$. Using this notation we can re-examine Alice's system of multiplication :

$4\times5=12$, this means $4\times5=1\times U+2$. Alice's U stand for 18 in our system.

$4\times6=13$, this means $4\times6=1\times U+3$. Alice's U stands for 21 in our system.

You will notice that like Topsy " umpteen " has " growed," it has increased by three so that in the next multiplication " umpteen " will have increased by a further three and will be equal to 24 in our system. This means that in the next multiplication " umpteen " will be 27, in the next 30, and in the next 33. We know that

$4\times5=18+2$	To Alice this would be	$4\times5=12$ " umpteen " is 18
$4\times6=21+3$		$4\times6=13$ " umpteen " is 21
$4\times7=24+4$		$4\times7=14$ " umpteen " is 24
$4\times8=27+5$		$4\times8=15$ " umpteen " is 27

The general statement of these results is $4\times n=3(n+1)+(n-3)$ where umpteen is $3(n+1)$ and the number of units is $(n-3)$. You will see that only when n is 3 can Alice have no units. When n is 3, Alice says $4\times3=10$ and " umpteen " is 12.

The results are as follows

$4\times1=4$ " umpteen " is 6

$4\times2=8$ " umpteen " is 9

$4\times3=10$ " umpteen " is 12

. . . .

$4\times8=15$

$4\times9=16$

However what does Alice write for our ten ? To her, 10 means " umpteen." Suppose for our ten she wrote *a*, for eleven *b*, twelve *c*, thirteen *d*, fourteen *e*, her multiplication tables continues :—

$4\times a = 17$

$4\times b = 18$

$4\times c = 19$

$4\times d = 1a$

$4\times e = 1b$ and so on.

Poor Alice ! Will she ever reach 20 ? It is not surprising that with so much ambition, distraction, uglification, and derision she found herself shrinking and stretching into so many sizes in Wonderland.

" Mock Turtle."

TREASURE HUNT No. 1

An equilateral triangle has 4″ sides. A point P is 2″ from one side and 2″ from another. By drawing or calculation find the possible distances of the point P from the third side.

J.G.

POINTS OF VIEW

(Adapted from Le Facteur X).

A large isosceles triangle has been painted on a school playground. Show all the viewpoints on the school playground from which the two equal sides subtend equal angles.

ANOTHER MAGIC SQUARE

1			4
	6	7	
8			5
	3	2	

Placez dans chaque case libre les nombres 9, 10, 11, 12, 13, 14, 15, et 16 de façon à former 34 dans tous les sens. J.F.H.

No Place for Squares !

Suggested by Canon Eperson, Bishop Otter College, Chichester.

Instead of squares, place equilateral triangles on each side of the right-angled triangle. Show that the sum of the two smaller triangles is equal to the triangle on the hypotenuse by cutting them into pieces and fitting these into the large one. Bogey is 6 pieces ; can you improve on this by managing with fewer ? J.F.H.

CURIOSER AND CURIOSER

Lewis Carrol, in real life the Reverend Charles Lutwidge Dodgson, was a lecturer in Mathematics at Christ Church, Oxford. It is not surprising that " Alice's Adventures in Wonderland " includes many references to Mathematics. Perhaps it is not realised that what appears to be nonsense has a sound mathematical basis and has application in the world of today.

" Let me see ; four times five are twelve, and four times six are thirteen, and four times seven is ---- oh, dear ! I shall never get to twenty at that rate ! "

Let us examine this statement a little more closely. At first it seems illogical, but let us translate it into the language of Mathematics.

$4\times5=12 \qquad 4\times6=13 \qquad 4\times7=--$

Now discard all previous ideas. What does 12 really mean? In our system of numbers it stands for one ten and two units, where ten is the base of our number scale, but in Mathematics there are umpteen such scales. So to Alice 12 meant " umpteen " and two units, where umpteen was changing all the time in an orderly manner.

In our system, the decimal system, four times five is twenty, that is two tens. In figures this is $4\times5=2(10)+0$. All our numbers are understood to be of the following form :—

ab stands for $a\times10+b$, i.e., *a* tens and *b* units
e.g., 32 stands for $3\times10+2$, i.e., 3 tens and 2 units
25 stands for $2\times10+5$, i.e., 2 tens and five units
10 stands for $1\times10+0$, i.e., 1 ten and 0 units.

ESPECIALLY FOR THE GIRLS

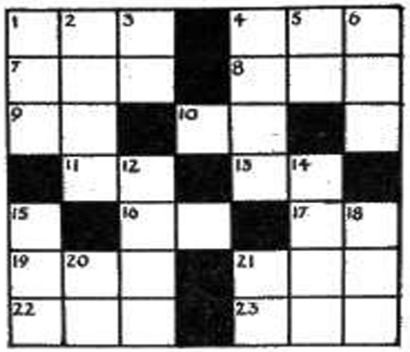
Reproduced by permission of the "Mathematics Teacher"

(1) Draw a segment *AB*, $1\frac{1}{2}''$ long, in the centre of the paper.
(2) Construct a perpendicular bisector of *AB* intersecting it at *O*. Call it *XY*.
(3) Make *OY* $2\frac{1}{2}''$ long.
(4) With *A* and *B* as centres and a radius of $\frac{7}{8}''$ construct arcs of circles intersecting *XO* at *C* and *AB* extended at points *M* and *N* respectively.
(5) Construct a perpendicular bisector of the distance *MY* and let it intersect the arc *CN* at *D*.
(6) Using *DY* as a radius and *D* as a centre construct an arc from *M* to *Y*.
(7) In a like manner construct an arc from *N* to *Y*, using a point on arc *MC* as centre.
(8) When the figure is completed write an appropriate saying on it.

SENIOR CROSS FIGURE No. 32

VABCD is a right rectangular based pyramid. AB = 8, BC = 6, VN = 12 units.

Give irrational answers correct to 3 significant figures. Angles to be given in degrees and minutes.

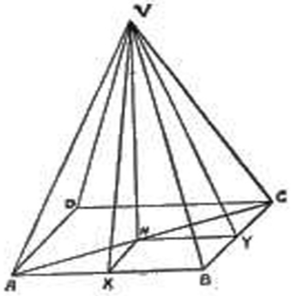


CLUES ACROSS :

1. VX.
4. Angle VAB.
7. 3 consecutive even numbers.
8. AB^3+BY^3.
9. Area of ABC.
10. XY^2.
11. Twice AC.
13. BY^2.
16. One third VX^2.
17. One half of the volume of VABcD.
19. Area Rectangle ABCD + △VAB.
21. Cos VBC reversed.
22. Total surface area.
23. $AB^3 - BC^3 + 11$.

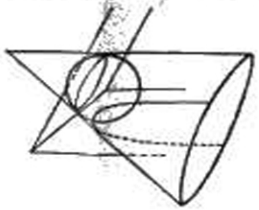
CLUES DOWN :

1. Angle XVN.
2. Angle BVC.
3. Area ABCD.
4. Angle VXN.
5. AC + CV.
6. Area VAB reversed.
12. Angle AVB reversed.
14. Angle YVN.
15. Volume of pyramid.
18. VY.
20. Area of VBC + VAD — 4.
21. VA.

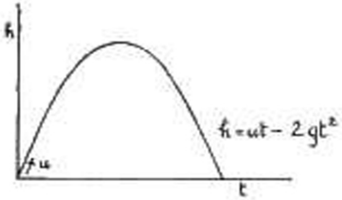


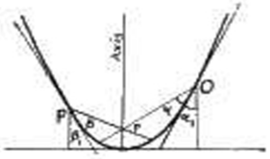
B.A.

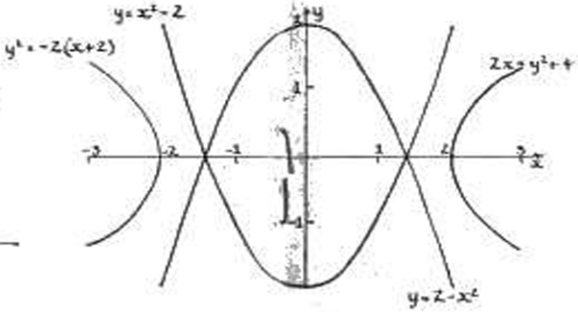
PARABOLIC ARCHES

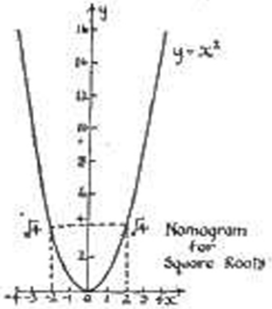


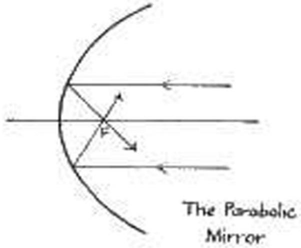
SUSPENSION BRIDGE







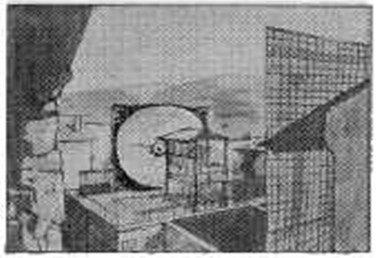




PARABOLIC ARCS OF SPARKS

THE PARABOLA

The parabola belongs to the family of curves known as conics because they can be obtained by slicing through a cone in various ways. It is probable that these curves – the circle, ellipse, parabola, hyperbola – were first studied by Menaechmus (B.C. 350), but they were given their names by the famous Apollonious. Archimedes also left some work on the parabola in which he used methods that anticipated the Integral Calculus. Descartes discovery of co-ordinates greatly simplified the study of curves.



SOLAR FURNACE

A conic.

If a cone is cut by a plane, the boundary of the cut surface or surfaces is called a conic section, or just a conic. If the plane is parallel to its sloping edge, the conic is a parabola. The Greeks studied these curves.

Axis, directrix, and focus.

Two important straight lines are associated with the parabola—the axis and the directrix. The axis divides the parabola into congruent halves. The focus is a point on the axis such that every point on the parabola is the same distance from it as it is from the line known as the directrix, which is at right angles to the axis. The diagram shows a construction for finding the focus *F*, of a given parabola.

Graphs and Parabolas.

Any quadratic expression plotted in the usual way as a graph results in a parabola. In the centre diagram, it is shown that interchange of x and y causes the direction of the parabola to alter. A convenient nomogram for finding squares and square roots can be obtained by plotting $y = x^2$ to suitable scales. The symmetrical curve on the right shows that a square root may be positive or negative.

Projectiles and Parabolas.

Galileo and Newton showed that the laws governing the movement of projectiles, under the influence of gravity, can be expressed as quadratic expressions. Some projectiles move along parabolic paths. The illustration at the bottom left shows the paths of sparks flying from a blacksmith's anvil.

85306 14228 81375 85043 06332 17518 29798 66223

Architecture and Parabolas.

A beam, when uniformly loaded horizontally, adjusts itself to a parabolic shape under the stress of the load. The two illustrations at the top of the block show how this property is used in civil engineering to obtain maximum strength.

Reflection and Parabolas.

A parabola rotated about its axis forms a parabolic surface. Such a surface shows remarkable properties if it is made into a mirror. If a source of light is placed at the focus, a parallel beam of light emerges. Car headlights, searchlights and beamed radio transmission are examples of the use of this principle. A parallel ray of light, *e.g.*, sunlight, falling on a parabolic mirror is concentrated, after reflection, on the focus. The illustration at the bottom right shows the parabolic mirror of a solar furnace, for obtaining very high temperatures, which the French have built in the Pyrenees. Metals contained in a crucible placed at the focus may be melted easily. The energy obtained from the sun is cheap but the reflecting mirror is very expensive.

71721 59160 77166 92547 48738 98665 49494 50114

O, O, ANTON I O!

(A puzzle rhyme attributed to mathematician Dr Whewell).

You 0 a 0, but I 0 thee,
O 0 no 0, but O 0 me;
Then will you 0 no 0 be
But give 0 0 I 0 thee. J.F.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 31

A FIELD STUDY

Let the length and breadth be x yards and y yards.

Then $x^2+y^2 = 125^2$ and $2(x+y) = 322$

$$x+y = 161$$

Squaring $(x+y)^2 = 161^2$

$$x^2+2xy+y^2 = 161^2$$

$$2xy = 161^2 - 125^2$$

Area $= xy = 4968$ sq. yd.

SENIOR CROSS FIGURE No. 32

Across: (1) 124; (4) 725; (7) 468; (8) 539; (9) 24; (10) 25; (11) 20; (13) 81; (17) 81; (19) 975; (21) 132; (22) 223; (23) 307.

Down: (1) 142; (2) 2642; (3) 48; (4) 7558; (5) 23; (6) 594; (12) 0553; (14) 1830; (15) 192; (18) 127; (20) 72; (21) 13.

POINTS OF VIEW

The equal sides subtends equal angles at every point on the altitude to the third side of the triangle.

ANOTHER MAGIC SQUARE

Reading the rows from left to right, they are 1, 15, 14, 4; 12, 6, 7, 9; 8, 10, 11, 5; 13, 3, 2, 16.

TREASURE HUNT No. 1

There are four positions for the point P. The distances are $4+2\sqrt{3}$, $4-2\sqrt{3}$, and $2\sqrt{3}-2$ for two of the positions.

JUNIOR CROSS FIGURE No. 29

Across: (1) 314; (4) 160; (5) 27; (6) 50; (7) 36; (10) 60800; (14) 880; (15) 56.

Down: (1) 365; (2) 100; (3) 1760; (4) 23; (8) 168; (9) 70; (11) 08; (12) 80; (13) 05.

The frontispiece of issue No. 31 of Sir Isaac Newton and his giants brought no replies from readers. The giants were Galileo, Copernicus, and Kepler, Euclid, Archimedes, Menaechmus and Eudoxus. B.A.

INTELLIGENCE TEST

An alternative solution to the Intelligence Test has the colours of the Rovers, Red, and the name of the cinema, Gaumont.

The arithmetic book I had when I was at school was full of problems like this:—

"The vertical height of a frustum of a cone is 8 in. and the radii of the ends are respectively 7 in. and 3 in. Find the area of its curved surface?"

If your arithmetic book is like mine this nomogram will help to check your homework.

This is how to use it to answer the question. Find the mark 8 on the h-scale, and find the point where the curves numbered 7 and 3 intersect. Place a ruler between these points and read off the value at its intersection with the A-scale.

If the measurements of the radii are not whole numbers you must estimate the position of the curves.

The nomogram can also be used to find their curved surface areas of cylinders and cones, the areas of circles, and the area between two concentric circles.

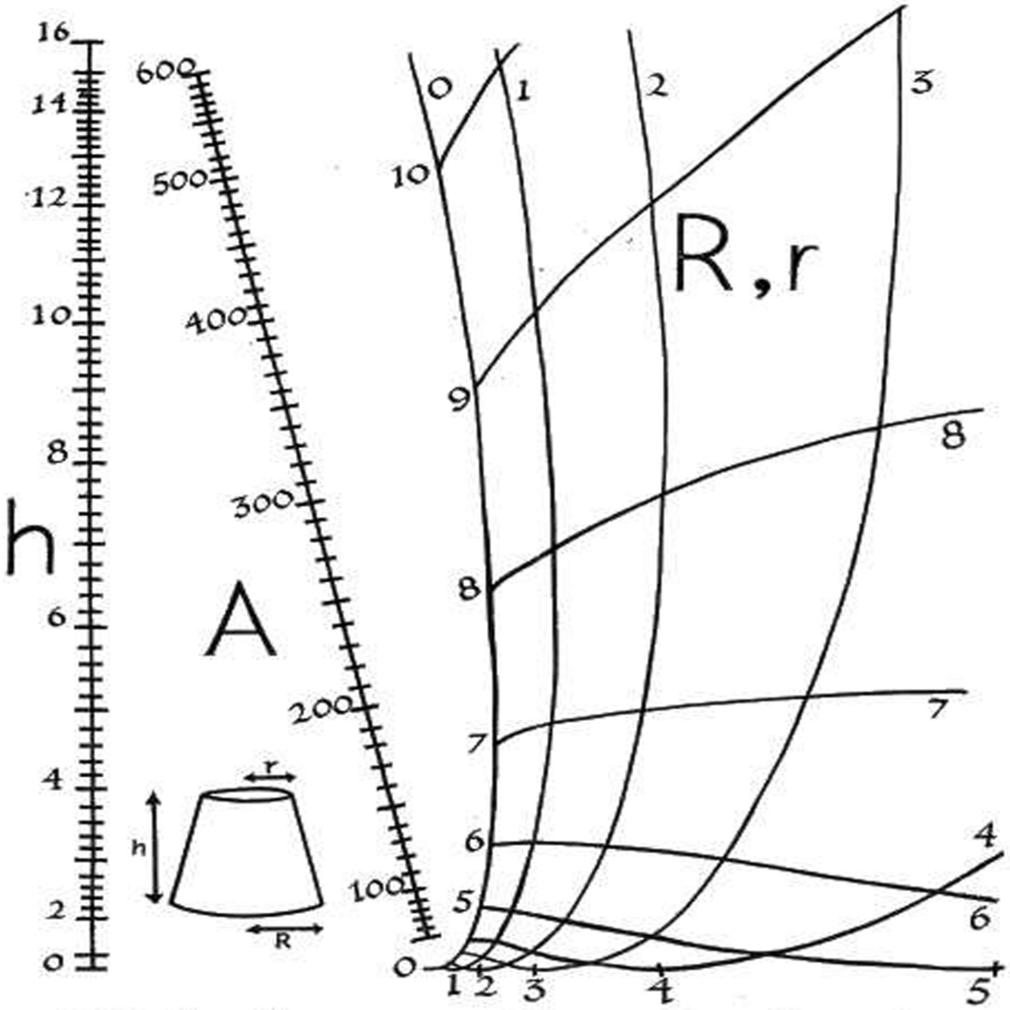
Only a limited range of values of the variables can be represented on a nomogram. If you wish to use the nomogram for larger or for smaller values, all the lengths can be multiplied by the same factor. C.V.G.

23828 06899 64004 82435 40370 14163 14965 89794

MATHEMATICAL PIE

No. 33 — *Editorial Offices:* See Issue No. 34 — MAY, 1961

THE AREA OF THE CURVED SURFACE OF A FRUSTUM OF A CONE



To find how this nomogram can help you with your homework, turn to the back page.

61711 96377 92133 75751 14959 50156 60496 31862

MATH MAGIC

Prepare six pieces of card and write on each of the backs one of the following numbers 16, 13, 49, 85, 98, 77. Write a letter on the front of each card as follows ;

A on the reverse side of 16
N on the reverse side of 13
G on the reverse side of 49
L on the reverse side of 85
E on the reverse side of 98
S on the reverse side of 77

Place your six letters on the table face up. Turn your back and ask one of your friends to choose a card, see what number is on the other side of the card. Tell him then to shuffle the six cards and hand them to you. Now produce a pencil and whilst you are tapping the cards with it ask your friend to *spell* out his number silently letting each tap represent a letter in the spelling of his selected number. Ask him to advise you when he has spelt out all the letters and when he has stopped your pencil will be resting on the number selected.

Secret. Lay out the cards to spell the word ANGLES. Take your pencil and tap anywhere for the first six taps but be sure that your seventh tap is on the letter *A*. The eighth must then land on *N*, the ninth on *G* and so on, and your last tap will then be on the back of the correct card number. Can you find out the real reason why the trick works?

3 SECOND QUIZ

You are allowed 3 seconds to write down the answers to each of the following questions.

(*a*) If 50 articles cost 50/- what is cost of each?
(*b*) If 100 articles cost 50/- what is cost of each?
(*c*) If 75 articles cost 50/- what is cost of each?

Look before you leap.

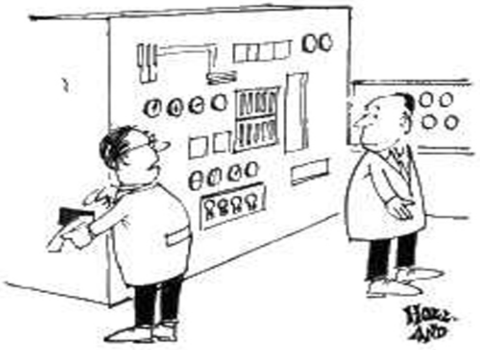
STAMP COLLECTOR'S CORNER No. 20

East Germany, 1950
Grey

Leonard Euler, 1707–1783, was born in Switzerland, but most of his active life was spent in Berlin and St. Petersburg (Leningrad). He was a most prolific writer. One of his first achievements was a solution of the " problem of three bodies " in astronomy. Newton had proved that a single planet will move in an ellipse round the sun, but the problem of sun, planet, and moon is much more difficult. Euler found a method of predicting the position of the moon by a series of approximations. His method was used in the preparation of nautical almanacs. He also investigated the strength of beams and struts, reformed the Russian system of weights and measures and devised a theory of investment which gave a sound basis for pension schemes, as well as making great advances in pure mathematics.
C.V.G.

94726 54736 42523 08177 03675 15906 73502 35072

I do hate sums. There is no greater mistake than to call arithmetic an exact science. There are permutations and aberrations discernible to minds entirely noble like mine ; subtle variations which ordinary accountants fail to discover ; hidden laws of numbers which it requires a mind like mine to perceive. For instance, if you add a sum from the bottom up, and then again from the top down, the result is always different.
Mrs. La Touche (19*th century*)



It says " Pass the log tables, please "
Reproduced by permission of " New Scientist "

DO YOU KNOW ?

Question : The sun and moon are the most prominent of the visible heavenly bodies. Which appears to the eye to be the larger?

Answer : They appear to be almost the same size, but the sun appears to be slightly larger.

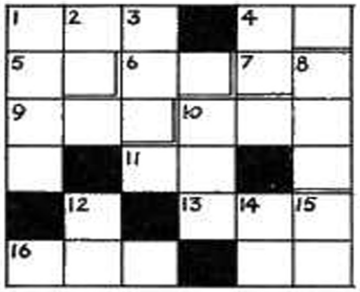
The sun covers a part of the sky that makes an angle of 32 minutes at the eye ; the moon an angle of 31 minutes.

" DO YOU LIKE IT ? "

Who was the first space man, according to Shakespeare to go into orbit round the Earth and what must have been his least average speed for the journey.
R.H.C.

JUNIOR CROSS FIGURE No. 30

Submitted by Ann Grigg, Hazeldene School, Salcombe, Devon.



ACROSS :

1. Area of the four walls of a room, 12 ft. by 10 ft. by 6 ft. high.
4. 27 apples cost 4/6d. Find the cost of 15 apples in pence.
5. ¼ of 336.
6. ½ of this number is 8.
7. 9^2.
9. 1 gross.
10. $\sqrt{11236}$.
11. No. of yd. in a chain.
13. No. of lb. in a cwt.
16. 27.01 × 5 correct to the nearest whole number.

DOWN :

1. 1 ton 5 cwt. 16 lb in lb.
2. 2^6.
3. $\sqrt{197136}$.
4. 646 ÷ 17.
8. Square of a prime number.
10. Square of another prime number.
12. ⅓ of 100 correct to the nearest whole number.
14. $\sqrt{100}$.
15. No. of lb. in ¼ cwt.

29741 67729 47867 24229 24654 36680 09806 76928

A MATHEMATICAL COLLECTION

When you are tired of collecting match-box labels, stamps, bus tickets, etc., try making a mathematical collection. This could take many forms—

items in everyday use with the shape of all the regular figures (two dimensional), *e.g.*, a Russian medal for a pentagon,

cartons and containers in common usage representing the regular and prismatic solids, *e.g.*, tetrahedron milk pack,

a collection showing mathematics at work in nature—three leafed clover, six pointed snow crystals, etc.

mathematics in the news—newspaper and magazine items which include mathematical words and prefixes, *e.g.*, " A Hyperboloid Over Your Head," a reference to the timber-shell roof going up at Oxford Road Station, Manchester.

As with all collections only one of each type is collected. Duplicates should be used for swopping with your friends, particularly if you get a spare Great Rhombicosidodecahedron !

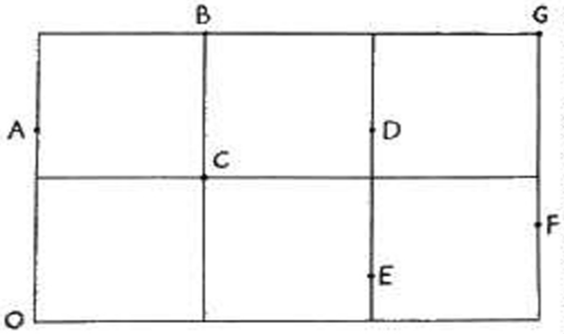
Prize for best list submitted by 15th August. I.L.C.

AN ODD QUESTION

Which is the smallest number which, when divided by 3 leaves a remainder of 1, and when divided by 5 and 7 will leave 3 and 5 respectively?

USING THE HEAD AS WELL AS THE FEET

(From Le Facteur X)



In a novel sort of race, the runners were allowed to choose their own route which, starting from point O, should bring them back to the start as soon as possible. En route, they had to collect tickets from the control points A, B, C, D, E, F and G as shown on the plan above, but not necessarily in strict alphabetical order. Each runner had to keep to the lines. The points A, D, E and F divide the sides of the small squares in the ratio 2 : 1. What is the shortest route? For simplicity you may take the sides of the small squares as 60 yards each. J.F.H.

IT'S ALL GREEK

$\beta\ \theta\ \delta\ \gamma\ \mu$ = TACIT
$\pi\ \beta\ \delta\ \gamma\ \gamma$ = AMISS
$\delta\ \sigma\ \lambda\ \gamma\ \gamma$ = SAILS
$\mu\ \delta\ \pi\ \lambda\ \mu$ = LEAST
$\gamma\ \delta\ \lambda\ \beta\ \gamma$ = CLASS

A spy tried to memorise some code words. On writing them down, he got them in the wrong order as above. Find the correct order.

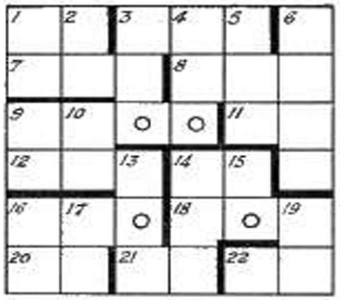
56951 62396 58645 73021 63159 81931 95167 35381

IT'S A TWIST

With one line divide a rectangle 16 cm. by 9 cm. into two parts which can be fitted together to make a square of side 12 cm. C.V.G.

SENIOR CROSS FIGURE No. 33

The number of figures to be inserted for each clue is indicated by the thick lines which act as stops. Small circles indicate zero.



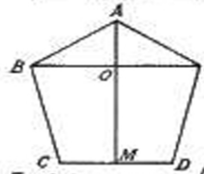


FIG. 1, SYMMETRICAL, NOT REGULAR.

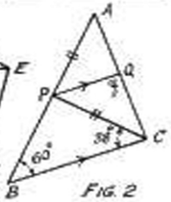


FIG. 2

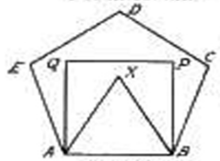


FIG. 3 THREE OF THE SIMPLEST REGULAR FIGURES.

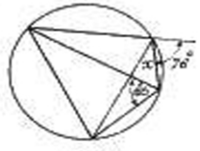


FIG. 4

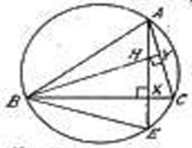


FIG. 5. AX, BY ARE ALTITUDES.

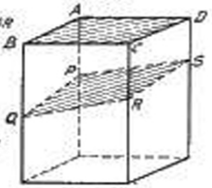


FIG. 6

ACROSS :

1. abc if $bc=7$, $ca=3$, $ab=21$.
3. abc if $a^2+b^2=106$, $b^2+c^2=74$, $c^2+a^2=130$.
7. abc if $a+b=24$, $b+c=10$, $c+a=28$.
8. Total surface area of 3 identical cubes is 648 sq. ins. What is the sum of their volumes?
9. Area of the pentagon (Fig. 1) $AO=12$ in., $AM=32$ in., $BE=70$ in., $CD=28$ in.
11. Sum of all the prime factors of 37037.
12. Angle PXE (Fig. 3).
14. Angle x (Fig. 4).
16. This number increased by 20% and then by 25% is 330.
18. Angle y (Fig. 2) $AP=PC$, PQ parallel to BC.
20. (Fig. 5). Angle HBE when angle $BAC=63°$ and angle $ABC=69°$.
21. A, B, C, D, are four points in order on a line. $AC=52\frac{1}{4}$ in., $BD=46\frac{3}{4}$ in., $AD=84$ in. What is BC?
22. $x+y$ when $x^2-xy+y^2=441$ and $xy=45$.

DOWN :

1. (Fig. 6). A rectangular beam is sawn across giving a section PQRS. $AP=33$ in., $BQ=56$ in., $CR=47$ in. What is DS?
2. Number of sides of a regular polygon whose internal angles are $154\frac{2}{7}°$.
3. $\dfrac{a(a^3+1)+(a+1)^2+(a+1)}{a+1}$ when $a=7$.
4. Perimeter of Fig. 1. (See 9 across).
5. abc if $a+b-c=14$, $a-b+c=12$, $b+c-a=0$.
6. Value of C when $A=ax+b$, $B=aA+b$, $C=aB+b$ and $a=10$, $b=8$, $x=8$.
9. Value of x when $a^2+(c-x)^2=b^2+x^2$, $a=11$, $b=1$, $c=10$.
10. Angle ACP (Fig. 3).
13. Can be written as $9a^2+1$ or as $90a+1$.
14. abc if $b=2a+1$, $c=2b+1$, $a+b+c=25$.
15. abc if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{9}$ and $ab+bc+ca=10$.
16. There are 52 animals (sheep and goats). For every 7 sheep there are 6 goats. How many sheep are there?
17. How many goats?
19. Area of rhombus with diagonals 4 in. and 7 in.

J.G.

83540 56704 03867 43513 62222 47715 89150 49530

More Mathematical Patterns

Figure 1 shows a familiar pattern treated in a different way. Alternate quadrilaterals are filled in to make a chequered pattern which suggests strings of beads on curves which run diagonally across the quadrilaterals. Patterns of this sort can be made with tangents to any curve.

The other patterns are made by two sets of lines or circles constructed according to some simple rule. In Figure 2 lines are drawn from the ends of a diameter of a circle to 18 points equally spaced round the circumference. The intersections lie on a set of circles and on a set of hyperbolae. Sixth-formers can work out the equations.

The basis of Figure 3 is equally spaced points on each of two lines. The points of one line are joined to one fixed point, and the points on the other line to another fixed point. In Figure 4, equally spaced points on one line are joined to each of two fixed points.

In all these patterns the lines can be extended indefinitely in either direction. In making a design it is important to decide where not to draw the line. Quite different effects are produced by filling a space, as in Figure 4, and by finishing at a string of quadrilaterals, as in Figure 3.

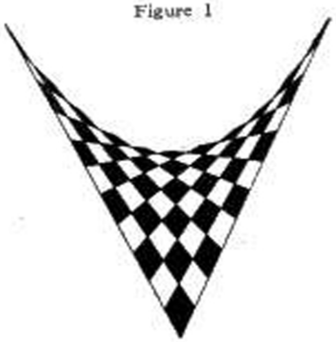
In Figure 5 a network of ellipses is suggested by two sets of parallel lines, the lines being drawn through points equally spaced on two semi-circles (see Figure 6). In this pattern there are the same number of divisions on each circle. Intricate patterns of Lissajous's figures are produced if there are different numbers of points on the two circles.

In Figure 7 a set of parallel lines and a set of concentric circles suggest confocal parabolas. In Figure 8, two sets of concentric circles make confocal ellipses and hyperbolae. It is said that this pattern, formed by ripples on the college pond, set Thomas Young searching for the optical interference patterns which proved the wave theory of light. The pattern is still of importance in one aircraft guidance system.

C.V.G.

The Editor will be pleased to see photographs of any embroidery based on these or similar patterns. Boys who do marquetry work should also be able to find inspiration here.

Figure 1



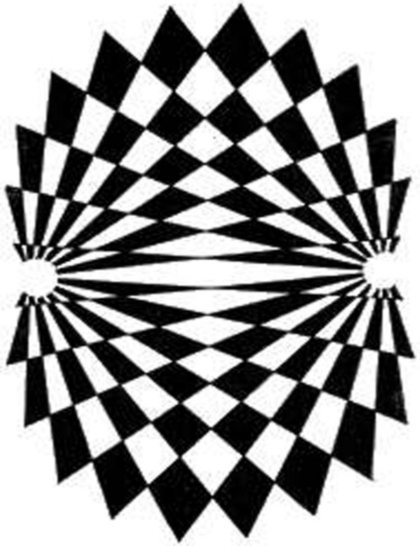


Figure 2

Figure 3

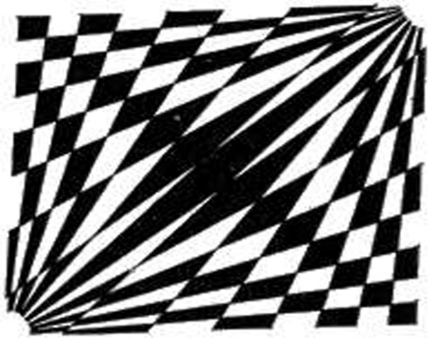


Figure 4

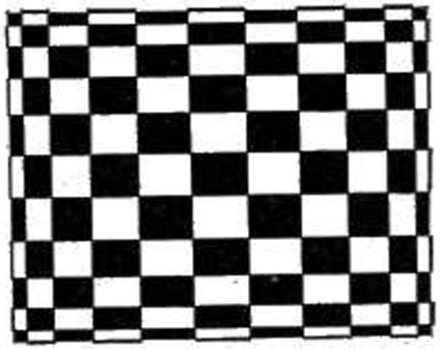


Figure 5

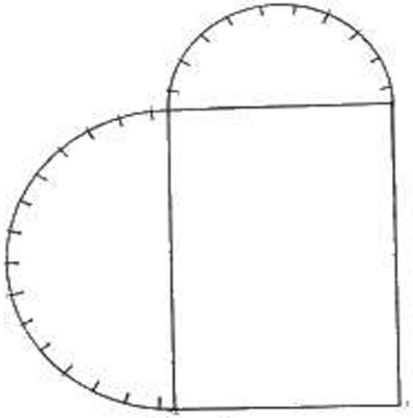


Figure 6

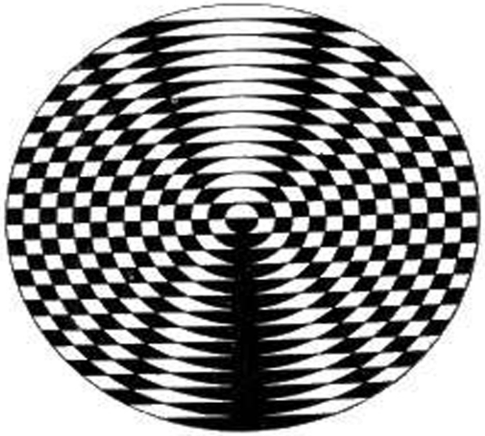


Figure 7

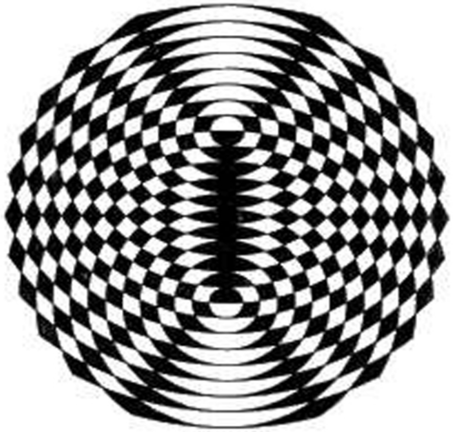


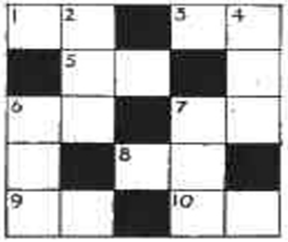
Figure 8

98444 89333 09634 08780 76932 59939 78054 19341

44737 74418 42631 29860 80998 88687 41326 04721

JUNIOR CROSS FIGURE No. 31

Submitted by R. C. Bulpin, The Grammar School, Minehead.



CLUES ACROSS:

1. "Top of the house."
3. Perfect square.
5. Simple interest on £600 for 3 years at $3\frac{1}{2}\%$ p.a.
6. $2x+y=18$ Find $3(x+1)$.
   $x-y=3$
7. Express 36 as a percentage of 90.
8. $\frac{1}{2}$ (6 across + 4) × 4.
9. CDVIII ÷ VI.
10. x^2-2y (See 6 across).

CLUES DOWN:

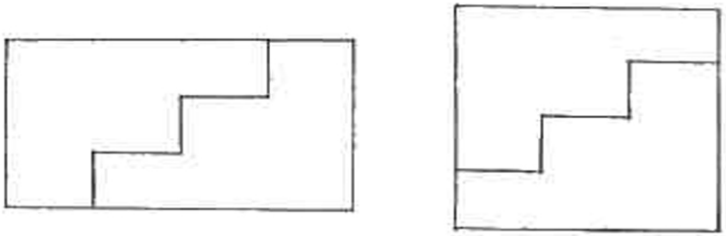
2. Number of half crowns in £120 10s. 0d.
4. Internal angle of a regular nonagon.
6. $8(\frac{4}{9} \times \frac{2\frac{1}{4}}{5} \div \frac{1}{16})$.
7. Average speed of a plane which travelled 1,624 miles in $3\frac{1}{2}$ hours.

SOLUTIONS TO PROBLEMS IN ISSUE No. 33

3 SECOND QUIZ

(*a*) 1/-; (*b*) 6d.; (*c*) 8d. not 9d. which is the usual quick answer.

IT'S A TWIST



SENIOR CROSS FIGURE No. 33

ACROSS: (1) 21; (3) 315; (7) 441; (8) 648; (9) 1400; (11) 68; (12) 159; (14) 39; (16) 220; (18) 101; (20) 84; (21) 15, (22) 24.
DOWN: (1) 24; (2) 14; (3) 310; (4) 160; (5) 546; (6) 8888; (9) 11; (10) 458; (13) 901; (14) 315; (15) 90; (16) 28; (17) 24; (19) 14.

AN ODD QUESTION

The smallest number is 103.

USING THE HEAD AS WELL AS THE FEET

Collecting tickets in the order *OACBGFDEO* requires covering seven hundred yards. Return to the centre line after visiting *A* and *D*. Continue down past *E* and return to *O* along the bottom line.

IT'S ALL GREEK

The correct order was Least, Class, Amiss, Tacit, and Sails.

DO YOU LIKE IT

'A Midsummer Night's Dream.' Puck circled the earth in 40 min. His average speed must have been approximately 37,700 miles per hour.

JUNIOR CROSS FIGURE No. 30

ACROSS: (1) 264; (4) 30; (5) 84; (6) 40; (7) 81; (9) 144; (10) 106; (11) 22; (13) 112; (16) 156.
DOWN: (1) 2816; (2) 64; (3) 444; (4) 38; (8) 169; (10) 121; (12) 33; (14) 10; (15) 28.

O, O, ANTON I O

You sigh for a cipher, but I sigh for thee,
O sigh for no cipher, but O sigh for me;
Then will your sigh for no cipher be,
But give sigh for sigh, for I sigh for thee.

31666 36528 61932 66863 36062 73567 63035 44776

MATHEMATICAL PIE

No. 34 — *Temporary Editorial Address:* 135 Peartree Avenue, Southampton — OCTOBER, 1961

CLOCK ARITHMETIC No. 1

Addition and Subtraction

You will all know that $4+5=9$ and will readily agree that the process can be illustrated by using a ruled scale as shown in Fig. 1.

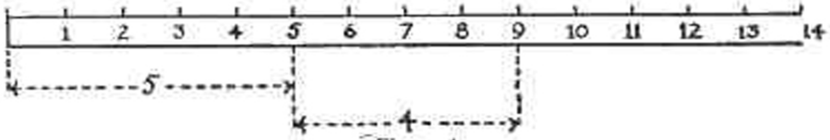


Figure 1

Suppose we now try to do this addition on a circular scale instead of a linear one. Such a circular scale will look like a clock face and let us take first one which has only six numbers on it, as in Fig. 2.

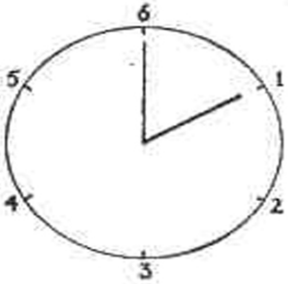


Figure 2

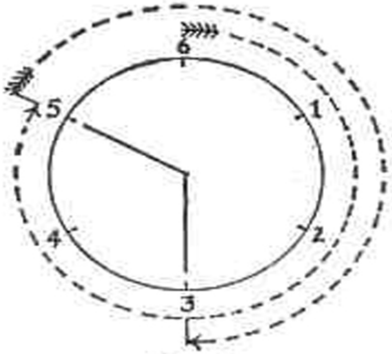


Figure 3

Then $4+5$ on this kind of scale would be done as shown in Fig. 3. This clock face arithmetic therefore gives $4+5=3$. Similarly, $3+4=1$. Can you give the clock face values of $5+2$, $3+2$, $2+2$, $4+4$, $6+3$? There is no 0 in this set of numbers. What number performs the function of zero in this arithmetic?

+	1	2	3	4	5	6
1						
2			5 [b]			
3						
4						
5				3 [a]		
6						

Figure 4

Now complete the addition table as shown in Fig. 4.

$5+4=3$ (see *a*)
$2+3=5$ (see *b*)

Notice that the first number in the addition sum is always indicated in the extreme left hand column and the second number in the top line, the answer being found in the body of the table.

09243 23789 69070 69779 42236 25082 21688 95738

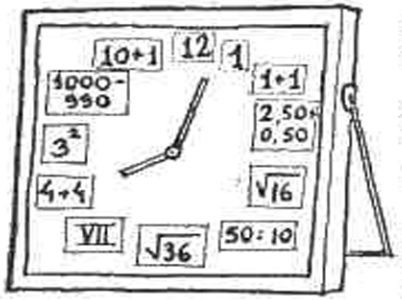
This game is known by mathematicians as addition in Modulo 6 arithmetic (No number larger than 6 appears in the table). Can you now make up an addition table for Modulo 12 arithmetic, *i.e.*, using the normal clock face numbers 1 to 12? This time some of your sums may include 7, 8, 9, 10, 11 and 12, which of course were not allowed in your first game.

Another piece of research is to make up similar tables for games of Modulo arithmetic for all Modulo arithmetics up to 12.

Problem.

Can you now use these tables to do subtraction and then make up subtraction tables for each Modulo arithmetic.

Example.



Reproduced by courtesy of *Punch*, 2nd, December 1959.

Modulo 6 subtraction $5 - 2 = 3$ and $1 - 3 = 4$. Do you notice anything queer about the answers you get?

But what use can be found for Modulo arithmetic? It is strange that you should have been using it but have not noticed it.

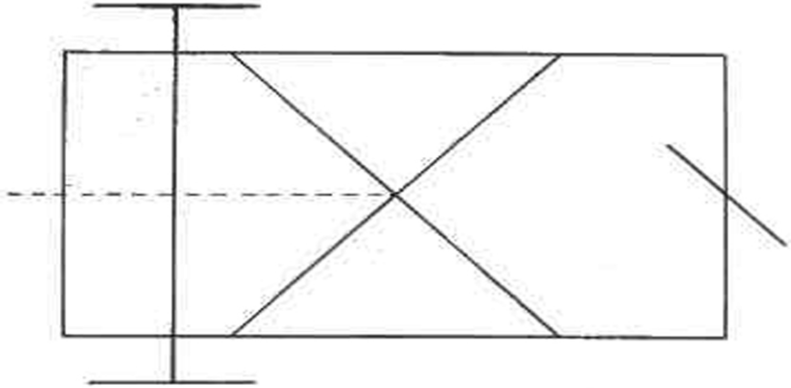
Example in Modulo 7.

In counting the days of the week, if 1 indicates Sunday, 2 indicates Monday, 3 Tuesday, etc., then five days after Tuesday is $3+5=1$, *i.e.*, Sunday.

Can you find an example in Modulo 12 arithmetic?

In one of our future issues we will deal with the problem of multiplication in Modulo arithmetic.

RUNNING IN CIRCLES



The diagram shows the skeleton chassis of a tricar. The front wheel is turned through 30° from the normal position. What are the radii of the circles described by the three wheels when the car is set in motion, given the distance between the axles is 12 feet, the distance between the wheels on the axle is 5 feet, and the diameter of the wheel is 2 feet? J.G.

37986 23001 59377 64716 51228 93578 60158 81617

JOHNNY IS NO SQUARE

John wrote down the cube of his age in years. He then subtracted his age from this figure and obtained 4080. How old is he?

FUN WITH NUMBERS—2

$2\times2 - 1\times3$
$3\times3 - 2\times4$
$4\times4 - 3\times5$
$5\times5 - 4\times6$
$6\times6 - 5\times7$

What do you notice about these expressions? Deduce the *n*th. line.

£ s. d.

What like fractions of a pound, a shilling and a penny when added together will make exactly one pound? R.H.C.

GOALS, MORE OR LESS

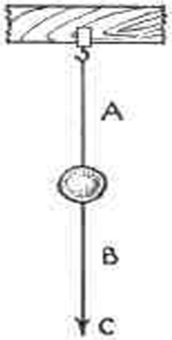
If the star performer in the school eleven had scored two more goals than he actually did, his average would have been 3 goals per match. With a score of two goals less, the average would have been 2. What was his total score? J.F.H.

FLOOR SPACE

Show how you can cover half a square two feet on each side and still have a square which is two feet from top to bottom and two feet across.

IT'S NOT WHAT YOU DO (IT'S THE WAY THAT YOU DO IT)

(Adapted from Le Facteur X)



The diagram represents a heavy ball of metal suspended by means of a wire *A* of uniform thickness. From a point diametrically opposite to the point of suspension, a piece of similar wire *B* hangs underneath the ball. What will happen when

(*a*) a gradually increasing downward pull is applied at *C*?

(*b*) a very sudden and powerful downward jerk is applied at *C*?

J.F.H.

SHAKESPEARE AS MATHEMATICIAN, RE SIGNS

Bolingbroke : Are you contented to resign the crown?
Richard II : Ay, no ; no, ay ; for I must nothing be ;
Therefore no no, for I resign to thee.

Shakespeare seems to have known the rule of signs, *viz.* : $2 - 2 = 0$; $-2 + 2 = 0$; and two minus signs operating on each other give a plus.

12019 05166 09628 04909 26360 19759 88281 61332

The reason is quite simple. It is a question of cost. If a printer is to be able to print all the numbers from 1 to 999 he needs three of each of the figures from 1 to 9 (He could leave spaces for the zeros). In the scale of two we write one thousand and twenty-three as 111,111,111. To print numbers up to this needs nine figures, but since the only figure used is 1 we need only nine altogether. The same comparison holds for all large numbers. Using the scale of ten the printer needs a stock of type three times as great as the stock needed for the scale of two.

In a computor some piece of apparatus is needed to represent each of the digits to be used in its calculations. Therefore a computer using scale of ten is three times as big as a computer using scale of two. As large computers cost hundreds of thousands of pounds this represents an enormous difference in cost even though a few hundred pounds has to be spent on an input machine to change numbers from scale of ten to scale of two, and an output machine to change numbers from scale of two to scale of ten.

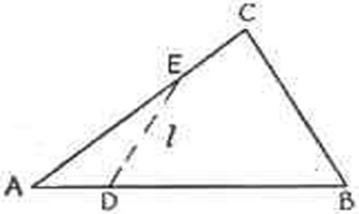
C.V.G.

Claudius said he could take one from four and have five left. Thereupon Julius showed how he could take eleven from twenty nine and have twenty left.

DIGITAL COMPUTERS

TRY ANGLES

(adapted from Le Facteur X)



The owner of a triangular field has a fence of length l with which he wants to divide the field into two parts as shown. He further requires that the portions AD and EC shall be of equal length. How is he to determine point D?

J.F.H.

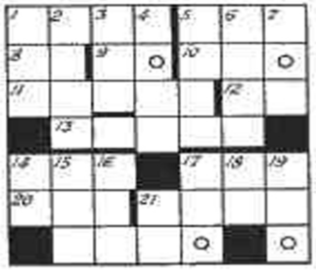
NOT VERY OBVIOUS

Two right angled triangles have equal hypotenuses and equal perimeters. Are they congruent?

27777 10386 38773 43177 20754 56545 32207 77092

SENIOR CROSS FIGURE No. 34

A small circle indicates zero. Fractions should be decimalised. The number of significant figures is indicated by the double lines.



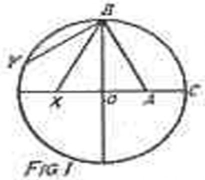


FIG 1

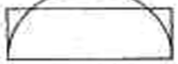


FIG 2

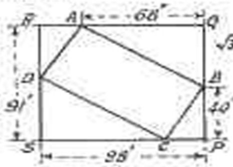


FIG 3

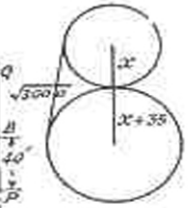


FIG 4

FIG 5

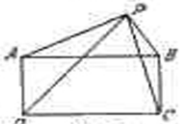


FIG 6

ACROSS:

1. A number which is a perfect square, whose four digits in order may be written $x+1$, $x-1$, $x-1$, $x+2$.
5. Stable hands in 44 ft.
8. The angle BOY (Fig. 1). Diameters perpendicular. $OA=\frac{1}{2}OC$, $AX=AB$, $BY=BX$. (Calculation or scale drawing).
9. A lady kept 84 pets—cats and goldfish. One day each cat ate 2 goldfish and then there were only 12 left. How many goldfish originally?
10. A price in shillings which, when increased by a quarter of itself, further increased by 15%, then reduced by 3s. in the £, is £39 2s.
11. Recurring figures in the decimalised $\frac{10}{21}$. (Exclude zero).
12. How many cats? (See 9 across).
13. $(a+b)^2-(a-b)^2$ if $a=222$, $b=111$.
14. $(x^2+y^2)(1+k^2)$ if $x-ky=14$, $kx+y=13$.
17. A floor is tiled with 6 in. square tiles. A circle of 4 ft. radius is drawn with its centre in the middle of the floor at the junction of 4 tiles. How many complete tiles are enclosed?
20. $\dfrac{a^3-b^3}{a-b}$ if $a+b=30$, $ab=48$.
21. (Fig. 2). Rectangle and semicircle of equal areas. Radius of semicircle is 10 in., find the height of the rectangle. (Take $\pi=3{\cdot}1416$).
22. (Fig. 3). Area of rectangle ABCD inscribed in rectangle PQRS.

DOWN:

1. Becomes a perfect square if you add 10 or subtract 33.
2. Less than 53×53, but leaves no remainder when divided by 53, and leaves a remainder 53 when divided by 7×53.
3. Sum of the areas of all the different rectangles (including squares), whose adjacent sides may be any of the numbers from 1 to 6.
4. Two numbers, the larger first. They are the radii of the circles in Fig. 4. One radius is 35 in. longer than the other and the common tangent is $\sqrt{3000}$.
5. Freezes when divided by 53. Boils when divided by 8.
6. (Fig. 5). Diameter of a semicircle touching the shorter sides of a 3", 4", 5" triangle. (First 3 dec. pl.).
7. Sum of the 3 altitudes of an isosceles triangle whose sides are 75, 75, 90.

14. $a-b$ if $a^2-ab=ab-b^2=228$.
15. Three consecutive digits in descending order. Reverse and add to the original and the result is 1110.
16. Two numbers. The values of a and b if the roots of

$$x^2-bx+a=0 \quad \text{are} \quad \frac{22\pm\sqrt{464}}{2}.$$

17. pv. When p is reduced by 3 and v is increased by 3, pv is unaltered. If p is increased by 3 and v is reduced by 2, pv is again unaltered.
18. (Fig. 6). Sum of the areas of the triangles APD and BPC. The rectangle is 15 in. by 10 in.
19. This number increased by 25% three times is $859\frac{3}{8}$.

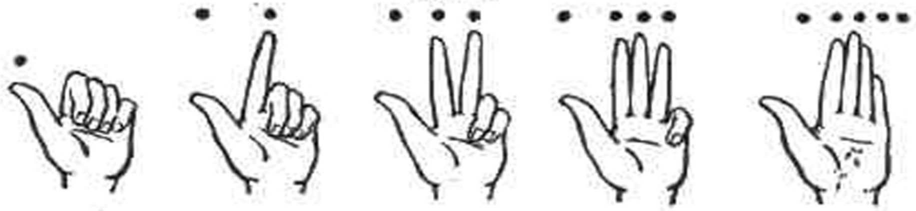
J.G.

55782 97352 33446 04281 51262 72037 34314 65319

MISSED CHANCES

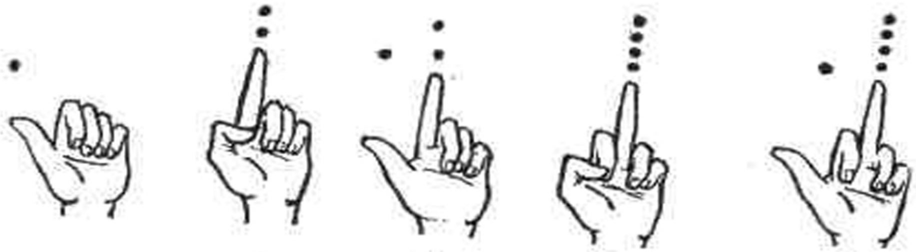
People who count on their fingers are usually very young or not very bright. Whatever the other cavemen may have thought of him, the first man to count on his fingers was probably not very bright by our standards, but he has had a tremendous influence on the mathematics that have come after him. He decided that when he thought of one thing he would stick up his thumb, and that when he thought of two things he would hold up a finger as well, and that when he thought of three things he would hold up another finger. Like this :—

Figure 1



Now if he had been a little brighter he might have decided to stick up his thumb when he thought of one thing, but to stick up his finger instead when he thought of two things. Then when he thought of three things he would stick up his finger and thumb. Like this :—

Figure 2

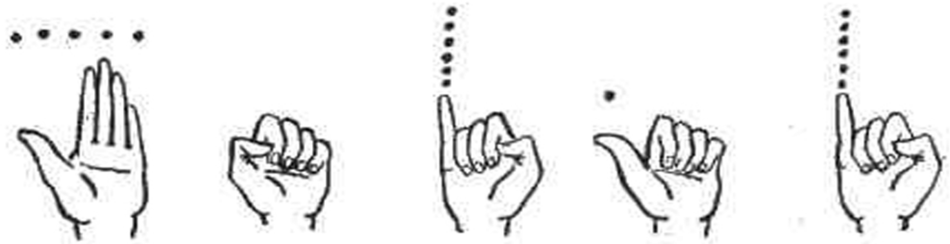


Then instead of being able to count up to five on the fingers of one hand, up to ten on both hands, and up to a score if he used his toes as well, he could have counted up to thirty-one on one hand, up to one thousand and twenty-three using both hands, and to over a million if he used his toes.

Of course primitive men never had that many of anything so it did not matter at first. If the first man who had to count above five had thought much about large numbers he might have speeded up the progress of arithmetic by thousands of years. If, when he had used up the fingers of one hand, he had stuck up the thumb of his second hand and put down the fingers of the first hand, and then started again on the first hand, as in *figure 3*, he would have been able to count up to thirty-five on his fingers and would have made the important discovery that one sign can stand for a group of things.

77774 16031 99066 55418 76397 92933 44195 21541

Figure 3



When men started doing calculations with pebbles they represented thirty-two like this

0000000000
0000000000 0
0000000000 0

It took many centuries before any one thought of this

0 0
0 0
0

where one large pebble represents ten small ones. The first man to do this probably said that the pebbles represented three men and two fingers.

Less than a thousand years ago people in Europe still wrote XXX11, where X stands for two hands and 1 stands for one finger.

However someone in India realised that a pebble in the second column could represent ten units without having to be a different size of pebble. So he represented thirty-two like this :—

0
0 0
0 0.

Still later more easily written symbols were invented for and a symbol for an empty column, and the scale of ten was perfected.

0
0 0
o **0** 0
0 **0** 0

and so on,

Our first cave man might have invented the scale of two in which the number up to twelve would be written like this :—

1 10 11 100 101 110 111 1000 1001 1010 1011 1100.

Our second cave man might have invented the scale of six with the numbers written like this :—

1 2 3 4 5 10 11 12 13 14 15 20.

In the scale of ten 111 means One (ten × ten) and one (ten) and one. In the scale of six 111 means one (six × six) and one (six) and one. In the scale of two it means one (two × two) and one (two) and one.

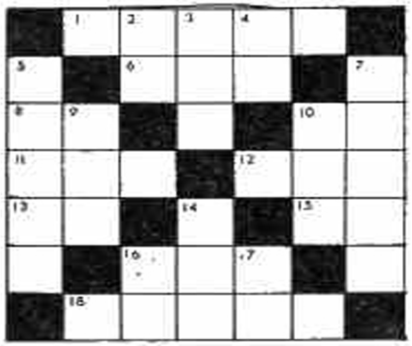
The readers of MATHEMATICAL PIE have become used to the scale of ten, but some of them may have found it hard at first. Because our first mathematician was not very bright every child learning arithmetic has to learn 81 facts like this :—$3 \times 4 = 12$ and 81 facts like this $3 + 4 = 7$. With the scale of two the only figures would be 0 and 1, and the only fact to learn would be $1 + 1 = 10$.

Perhaps in the long run it was a good thing that the first mathematician missed his chance. Arithmetic would have been so easy that it would not have challenged the intelligence. Certainly there would have seemed no need for logarithms, slide rules and calculating machines, so that it is perhaps rather odd that large electronic calculating machines work in the scale of two.

34189 94854 44734 56738 31624 99341 91318 14809

JUNIOR CROSS FIGURE No. 32

Submitted by J. Aust, St. Andrew's School, Woking.



ACROSS :

1. 1 mile in Km.
6. 8s. $5\frac{3}{4}$d. in farthings.
8. $a = \frac{1}{2}$, $b = -2$, $c = 3$, find $(ab)^4 + c^3 - b^2 - 2a$.
10. $x + y = -40$, $5x - 3y = +8$, find x
11. $23^2 + 16^2 + 13^2 - 29$.
12. Find y in 10, multiply by -6 and add 44.
13. A man was fined £4 10s. for speeding—if the rate was 9/- for every 3 m.p.h. exceeding 30 m.p.h., what speed was he doing?
15. Fred has x marbles and Tom has 7 more than Fred. Jill has 3 less than Fred and James has 27 less than Fred. Tom + Jill + James have 57 more marbles than Fred—how many has Fred? Reverse it.
16. Number of right angles in the sum of the interior angles of a 64 sided polygon.
18. Volume of a pipe, radius 3.162 in. and 1225 in. long, to the nearest whole number.

DOWN :

2. A man goes for 2 hr. at 20 m.p.h. and $1\frac{1}{2}$ hr. at 16 m.p.h. How many miles does he go?
3. $20^2 - 21^2 - 7^2 + 3^2 + 1$ reversed.
4. $(19 \times 18) - (13 \times 19)$.
5. 12/$9\frac{1}{2}$d. as a decimal of £1 to 5 decimal places.
7. 1.53×97.
9. How many sheets in 5 quires of paper?
10. 968 f.p.s. = ? m.p.h.
14. £1/5s.$\times\frac{3}{4}$, answer in pence.
16. An isosceles triangle has the vertical angle = 144°, what is the size of one of the base angles?
17. A man sold a car for £43/4s., making a profit of 8%. Find the cost price in £.

SOLUTIONS TO PROBLEMS IN ISSUE No. 34

RUNNING IN CIRCLES

Front wheel 24 ft., inside rear wheel $(12\sqrt{3} - 2\frac{1}{4})$ ft., outside rear whee $(12\sqrt{3} + 2\frac{1}{4})$ ft.

SENIOR CROSS FIGURE No. 34

The clue to 14 down should have read "$a^2 - ab = 1673$, $ab - b^2 = 228$."

ACROSS : (1) 4225, (5) 132, (8) 72, (9) 60, (10) 640, (11) 47619, (12) 24, (13) 98568, (14) 365, (17) 174, (20) 852, (21) 7854, (22) 4250.

DOWN : (1) 474, (2) 2279, (3) 266, (4) 5015, (5) 1696, (6) 3428, (7) 204, (14) 38, (15) 654, (16) 522 (17) 180, (18) 75, (19) 440, (21) 75.

MISSED CHANCES

The editor apologises for the omission of a 1 on line 4, page 266. The number should have read 1, 111, 111, 111.

CLAUDIUS

4 = IV, 1 = I, 5 = V
29 = XXIX, 11 = XI, 20 = XX.

NOT VERY OBVIOUS

The two triangles must be congruent.

JOHNNY IS NO SQUARE

Johnny is 16.

FUN WITH NUMBERS—2

The nth line is $(n+1) \times (n+1) - n \times (n+2) = 1$. Every line has the value 1.

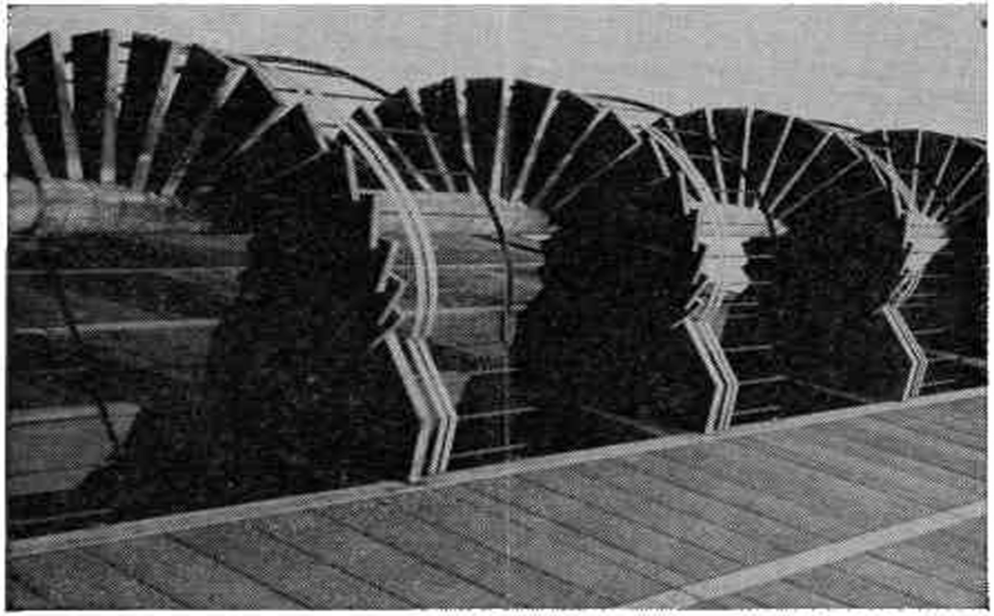
£ S. D.

The fraction is $\frac{240}{253}$.

19256 51798 09641 13528 01314 70130 47816 43788

MATHEMATICAL PIE

No. 35 | *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England | FEBRUARY, 1962



CLOCK ARITHMETIC No. 2

Subtraction

Let us now try to do subtraction on the clock face, *i.e.*, to move the hands backwards instead of forwards.

(*a*) 5 — 2 = 3. (*b*) 2 — 3 = 5.

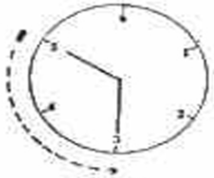


Figure 1

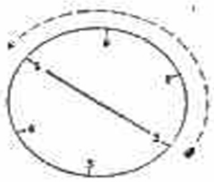


Figure 2

—	1	2	3	4	5	6
1						
2			5^b			
3						
4						
5		3^a				
6						

Figure 3 →

28035 04507 77235 54710 58595 48702 79081 43562

From the example b, you see that negative numbers are not needed in clock arithmetic. Now fill up the table in Fig. 3, the results for all possible subtraction sums in modulo 6 arithmetic. Notice that the number with which you begin is placed in the left hand column and the number being taken away appears in the top line.

From your table find the values of: (i) $4 - 2$, (ii) $2 - 4$. If this had been normal arithmetic what would the values have been?

Finally, what would you place instead of x and y if (iii) $4 - x = 5$, (iv) $4 - y = 1$?

Those of you who still have your addition table from Issue No. 34 for Modulo 6 arithmetic might care to find out how to use it for subtraction instead of the one you have just made. R.H.C.

I WANT TO BE: No. 5

A Chemical Engineer

The demand for skilled technicians has grown to such an extent that it is no longer the fashion for firms to announce their staff requirements in a two- or three-line advertisement in the "Situations Vacant" columns of a newspaper. Nowadays, some of the national dailies carry two or three pages of large advertisements offering considerable inducements to young technicians who are prepared to consider posts in various branches of industry.

Prominent among the opportunities advertised are well-paid posts in Chemical Engineering. In brief, a chemical engineer is both chemist and engineer and, as such, will be found working in practically every branch of industry. A few examples are food, petroleum, gas, coal by-products, water supply, cement, brewing, fertilisers, drugs, synthetic fibres, plastics and atomic energy.

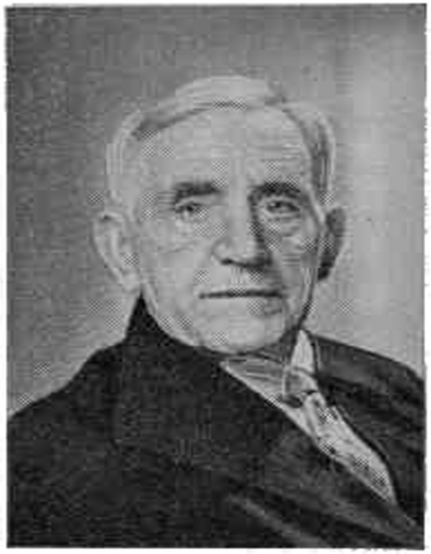
The chemical engineer's work involves a fair amount of mathematics that is full of practical interest, for example, calculation of fluid flow, heat transfer, speed of chemical reactions and strength of structural members in chemical plant. One interesting calculation concerned finding the path of a droplet of condensed liquid that had formed on the top surface of a sloping condenser tube. Other calculations concern the size of bubbles and the mechanism of their formation. Sometimes, the engineer may use high-speed photography to check his calculations. All the above combine to make chemical engineering a very interesting career.

The nature and principles of such phenomena as are embraced by the terms "total heat," "latent heat," "entropy," "order and disorder," once grasped by the young engineer, permit him to devise processes and design plant. His chemical knowledge and experience might teach him that a certain product can be made by different types of reactions; his knowledge of thermodynamics, however, would serve to show him which approach would be the most economical.

Most universities and technical colleges now have departments of Chemical Engineering; the young man interested in the subject, therefore, should find little difficulty in pursuing it further. J.F.H.

40145 17180 62464 36267 94561 27531 81340 78330

SOME SHORT CUTS



Here is a photograph of Mr. J. E. Clarkson, of Rotherham, Yorkshire. He left school when he was twelve years of age and he is now seventy-six. During all these years he has made a hobby of arithmetic. Here are three questions and the actual figures that Mr. Clarkson would write down in order to get the answers.

Can you see what he has done?

Question 1.

What is the cost of 1 ton @ $4\frac{1}{4}$d. per lb.?

$$\begin{array}{r} 17 \\ 17 \\ 5^2 \\ \hline \text{£}39\ \ 13\text{s.}\ \ 4\text{d.} \\ \hline \end{array}$$

Question 2.

What is the cost of 1 ton @ $7\frac{3}{4}$d. per ounce?

$$\begin{array}{r} 31 \\ 40 \\ \hline 1240 \\ 82^2 \\ \hline \text{£}1157\ \ 6\text{s.}\ \ 8\text{d.} \\ \hline \end{array} \qquad \begin{array}{r} 62 \\ 20^2 \\ \hline 82^2 \end{array}$$

Question 3.

What is the cost of 1 ton @ $8\frac{3}{4}$d. per dram?

$$\begin{array}{r} 35 \\ 600 \\ \hline 21000 \\ 93^1 \\ \hline \text{£}20906\ \ 13\text{s.}\ \ 4\text{d.} \\ \hline \end{array} \qquad \begin{array}{r} 70 \\ 23^1 \\ \hline 93^1 \end{array}$$

LETTERS TO THE EDITOR

19, Southwell Road East,
Rainworth,
Mansfield.

Dear Sir,

QUEER FACTS ABOUT SQUARE ROOTS.

The square root of 123,456,789 = 11,111 with a remainder of 2468. But $11{,}111^2 = 123{,}454{,}321$.

I discovered this one evening while experimenting with square roots.

Yours sincerely,

J. M. BULLEY (aged 13)
Queen Elizabeth's G.S. for Boys, Mansfield.

22, Persley Road,
Northbourne,
Bournemouth, Hants.

Dear Sir,

NUMBERS IN WORDS (Issue No. 31).

It is quite a simple fact which states "Eleven plus two minus one equals twelve."

This is true not only in figures, but also in letters, as can be seen by writing "eleventwo," striking out three letters *o*, *n*, and *e*, which gives "elevtw," which is an anagram of "twelve."

Yours faithfully,

A. MAINE.

60570 07334 92436 93113 83504 93163 12840 42512

supposed a geometrical figure to be drawn as a map, and, using the idea of latitude and longitude, measured the position of each point by a pair of numbers (x, y). Every geometrical property could then be translated into an algebraic relation between the x's and y's. The value of his idea is that "it allows one to pass continually back and forth between geometry and algebra" using each to illustrate and supplement the other. It is on Descartes' work that Newton was to base his ideas for the Calculus. R.M.S.

QUITE A TURN

The staircase at the new British Railway offices at Doncaster, shown on the front page*, would be described by most people as a "spiral" staircase, but this is a misuse of the word. A spiral is a curve in one plane. The edge of a clock spring is an everyday example. A curve such as that formed by wrapping a ribbon round a cylinder is called a *helix* and the surface formed by lines from the curve perpendicular to the axis of the cylinder is called a *helicoid*, i.e. it resembles a uniform helter skelter.

Most of the helical staircases found in old castles are built of stone steps resting on one another in the centre and their outside ends built into the wall of the staircase turret. Old staircases always turn to the right as they ascend so that the occupants could use their sword arms freely to repel unwelcome visitors.

The early staircases were narrow because they were designed for defence, but Glamis Castle in Scotland has a spacious staircase whose centre is a hollow tube which leads warm air from a furnace in the cellar to the upper rooms, and at Manderscheit in Germany there is a tower with a helical carriageway leading to the castle set on a rock above the Moselle.

Tamworth church has a most unusual staircase. The stone steps go straight across the turret and form two distinct staircases. One has an entrance inside the church and the other is reached from the street. These staircases have no landings. When they reach the flat roof of the tower they just stop. To take one step too many means a nasty fall on to the other flight.

Modern helical stairs are decorative as well as useful. The centre may be open like the staircases in the Shakespeare Memorial Theatre, or they may be built out from a slender column, like the staircase at Doncaster, or they may, like a staircase at Johannesburg have no visible means of support. C.V.G.

**The printer seems to have put this the wrong way round.*

SOLUTIONS—*Continued.*

The solution to "TRY ANGLES" will appear in issue No. 36.

GOALS, MORE OR LESS

The star of the soccer eleven had scored 10 goals.

FLOOR SQUARE

Join the mid point of each side to the mid points of the two adjacent sides.

IT'S NOT WHAT YOU DO

(*a*) *A* breaks first. (*b*) *B* breaks first.

JUNIOR CROSS FIGURE No. 31

ACROSS: (1) 99, (3) 81, (5) 63, (6) 24, (7) 40, (8) 56, (9) 68, (10) 41.
DOWN: (2) 964, (4) 140, (6) 256, (7) 464.

We are informed that "Top of the House" is not 99 as suggested in 1 across. B.A.

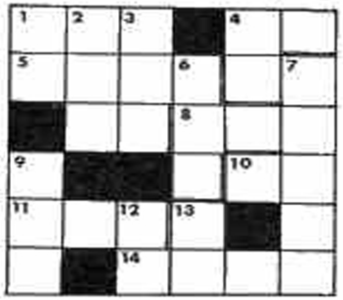
MORE GREEK

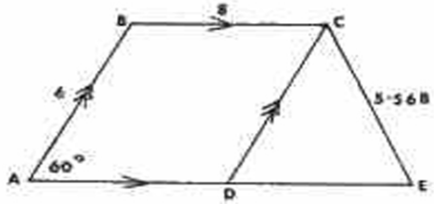
$\phi\ \beta\ \beta\ \sigma\ \beta$	=	E R A S E
$\phi\ \beta\ \mu\ \lambda\ \phi$	=	R O T O R
$\phi\ \phi\ \beta\ \pi\ \phi$	=	E R R O R
$\beta\ \sigma\ \delta\ \sigma\ \beta$	=	R A I S E
$\beta\ \mu\ \pi\ \lambda\ \phi$	=	E E R I E

A spy tried to memorise some code words. On writing them down, he got them in the wrong order as above. Find the correct order. J.F.H.

SENIOR CROSS FIGURE No. 35

Suggested by Mr. P. A. Fisher, Sir Joseph Williamson's Mathematical School, Rochester.





ACROSS:

1. $2\sqrt[3]{\tan 255°\,54'}$
4. AE in the figure.
5. AC in the figure.
8. $\sqrt{\frac{1}{2}(2\times10^3+3^4)}$
10. Maximum value of $4x^3-6x^2-9x+1$.
11. $1\div\cdot1323$ to three sig. fig.
14. The last half of the answer is one half of the first half.

DOWN:

1. d if $\left.\begin{array}{l}3c-2d=7\\4c-3d=-1\end{array}\right\}$
2. $\left[(\Sigma\,3\,!)-\Sigma\,4\right]^2$
3. $(26.66)^2$ to three sig. fig.
4. $\left[\left(5+\frac{6!}{5!}-15\right)(-3)\right]-1$
6. $3\,!\times5\,!$
7. Area of $ABCE$ in the figure.
9. $7\times4\,!+4$
12. BC^2-1
13. K if $y=K$ touches the curve $y=8x-x^2$.

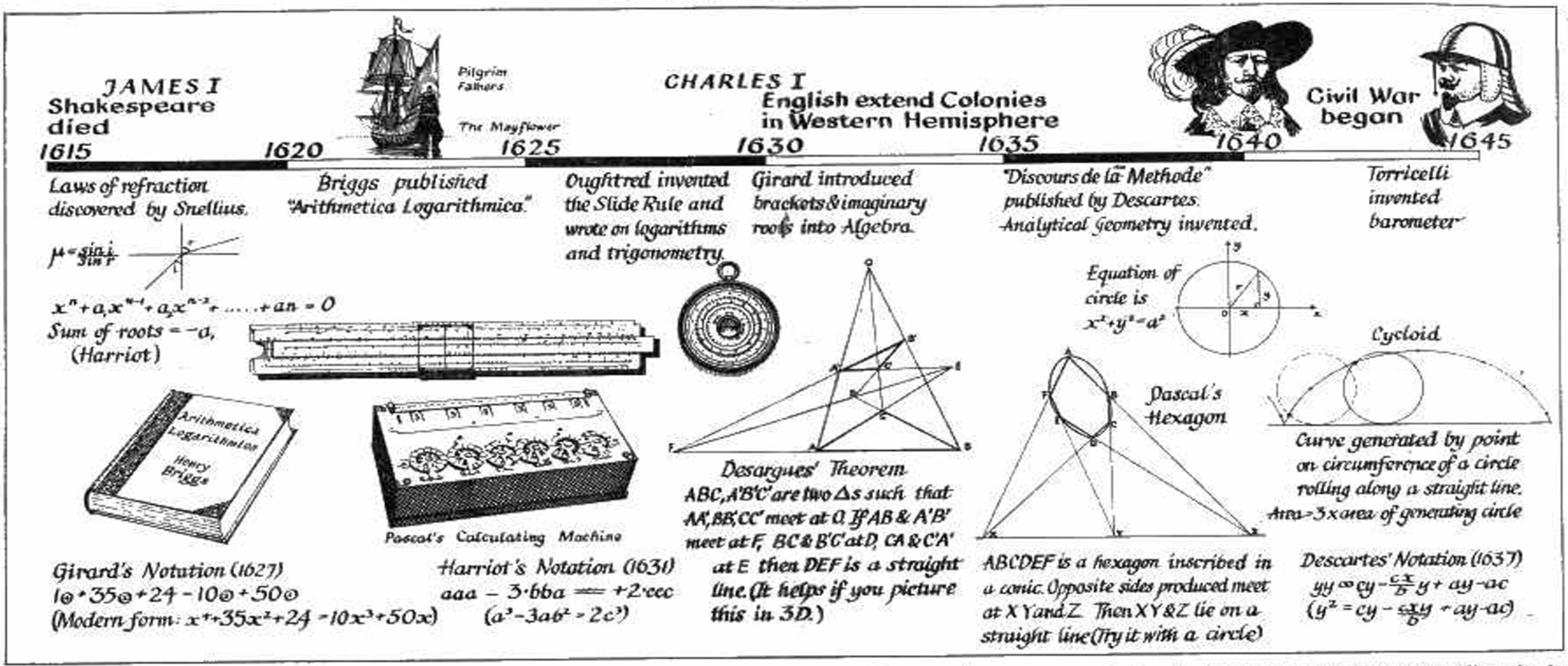
STAMP COLLECTORS' CORNER No. 21

POLAND 1959

The stamps illustrated, with portraits of Copernicus (1473–1543), Newton (1642–1727) and Einstein (1879–1955), are from a recent Polish issue. All three can be regarded as intellectual revolutionaries for the theories they put forward upset all established ideas and had profound consequences in almost every branch of science and philosophy.

The set is completed by portraits of Pasteur, Darwin and Mendel who initiated equally revolutionary theories in biological sciences. C.V.G.

TIME CHART No. 9 : 1615–1645



Time Chart No. 8 saw the start of a period of increased activity in mathematical thought. Napier's invention of logarithms, cast into a more readily usable form by Briggs, had a far reaching effect on all calculation.

Modern computers belong to one of two types, either the analogue computer which represents numbers by some physical quantity such as length, or angle, or electrical resistance (a car speedometer or petrol gauge works in this way), or the digital computer in which each digit of a number is represented by a distinct position of a wheel, as in a cycle or car milometer.

Oughtred (1574—1660) saw the possibility of adding logarithms by measuring them off along the edges of two scales which slide along each other, thus giving the *slide rule*. This standby of the engineer is a very simple analogue computer.

Pascal (1623—1662), when 18 years of age, amused himself by making a calculating machine, using gear wheels, the forerunner of the desk calculator, cash register or even electronic digital computer. As a boy he played with geometrical diagrams as other children play with toys, and at the age of 16 he produced a remarkable treatise on the conics ections (ellipse, parabola, hyperbola). It included the Pascal hexagon (above), one of the most beautiful

26063 81334 67768 79695 97030 98339 13077 10987

theorems in geometry and of such importance that over 400 colloraries have been deduced from it.

Desargues (1593—1662), engineer, artist, and mathematician developed a geometry which began as a theory of perspective. He abandoned all ideas of measurement and used only the properties of intersecting lines and planes. His work lay forgotten for almost two centries before being rediscovered to become the foundation of modern projective geometry.

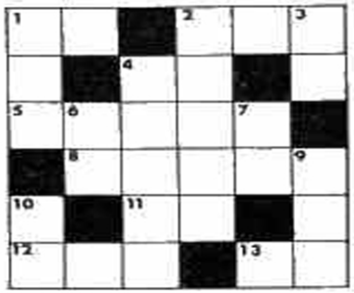
The really significant advance of the period was in algebraic notation. Successive writers had polished and transformed it from a bludgeon in to a rapier so that in 1631 Descartes was using a form not so very different from that we use today. The improved notation, as so often happened, brought new discoveries. Harriot (1560—1621), mathematician and astronomer, who surveyed and mapped New Caroline, left papers which were published 10 years after his death and shewed the relation between the roots and the coefficients of an equation, between the number of roots and the degree of an equation, and methods of transforming an equation so as to alter the roots in any desired manner.

Descartes (1596—1650) may be said to have changed the whole course of mathematical history with his invention of analytical geometry. He

04085 91337 46414 42822 77263 46594 70474 58784

JUNIOR CROSS FIGURE No. 33

Prepared by the Pre-Apprentice Course, Clowne Technical College.



CLUES ACROSS :

1. $\left[(8.485)^2 - 1\right]$.
2. $(2^3 - 2)^3$.
4. $(3.609)^3$.
5. $\sqrt{17}$ to 4 dec. places.
8. $10n^2$ when $n = 83$.
11. $\sqrt{5776}$.
12. Reciprocal of 40.
13. $(4.481)^3$.

CLUES DOWN :

1. A perfect square.
2. $\sqrt{\tfrac{3}{4}(10)^3}$.
3. $(x^3 + x)$ when x is a perfect square.
4. The cube of 35.
6. The square of x^2 when x is 2.
7. x when $(x+1)^3 = 8{,}000$.
9. The reciprocal of 50.
10. $(n^2 - 1)$ given that n is a perfect square.

NOTE BENE

A traveller at a hotel handed in a five pound note for safe keeping to the landlord. Instead of putting it away in his safe, the landlord took it to his friend the baker and paid off a bill he owed there. The baker in his turn used the same note to pay a bill of five pounds which he owed to his solicitor, and finally the solicitor took the note to the hotel and used it to pay for a dinner party he had held the previous evening. The landlord then placed the note in the safe.

Next morning the traveller came and asked for his note and, being an honest man, the landlord told the guest how he had borrowed it and then replaced it later. "Of course I don't mind," said the traveller and tore up the note, "you see it is a forgery."

Who lost on the local transactions? R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 35

MORE GREEK

The English words should have been in the following order ERROR, ERASE, EERIE, ROTOR, RAISE.

SENIOR CROSS FIGURE No. 35

ACROSS : (1) 317, (4) 13, (5) 1217, (8) 204, (10) 35, (11) 756, (14) 3618.

DOWN : (1) 31, (2) 121, (3) 711, (4) 11, (6) 720, (7) 54558, (9) 172, (12) 63, (13) 16.

JUNIOR CROSS FIGURE No. 32

ACROSS : (1) 16093, (6) 405, (8) 31, (10) 64, (11) 925, (12) 668, (13) 60, (15) 04, (16) 124, (18) 28500.

DOWN : (2) 64, (3) 009, (4) 95, (5) 63957, (7) 14841, (9) 120, (10) 660, (14) 225, (16) 18, (17) 40.

The editor apologises for two slips. 18 across should have read to the nearest 100, and clues 13 across and 5 down do not agree.

POETICAL PI

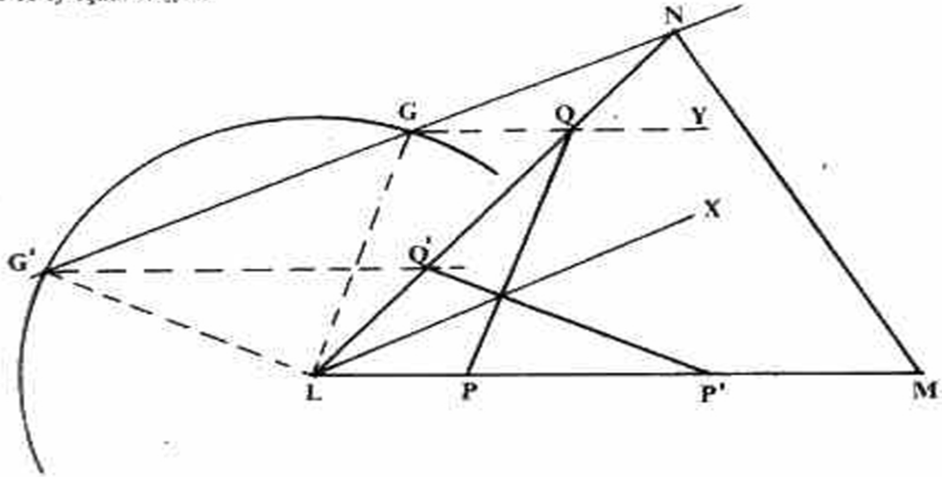
The number of letters in each word gives the sequence in the decimal equivalent of π. B.A.

MATHEMATICAL PIE

No. 36 — *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England — MAY, 1962

TRY ANGLES

The owner of a triangular field LMN has a fence of length l with which he wants to divide the field into two parts. He requires PQ to be of length l and NQ and LP to be of equal length.



Construction :—With centre L, and radius l, draw a circle. Draw in LX to bisect NLM, and construct a line through N parallel to LX to meet the circle in G and G'. GY is drawn parallel to LM to cut LN in Q. Mark off LP equal to QN. Join PQ. This is the position of the fence.

A second position of the fence can be found using G' instead of G.

Proof :— $\angle NQY = \angle NLM = 2\angle XLN = 2\angle QNG$ (parallel lines)

$\angle NQY = \angle QNG + \angle QGN$ (exterior angle of triangle)

Hence, $\angle QNG = \angle QGN$ and $GQ = QN$.

In the quadrilateral $LPQG$, $LP = GQ$ and LP is parallel to GQ; hence the figure is a parallelogram.

Hence $PQ = GQ = l$.

Write on a postcard, a short note to the Editor giving your views on the solution of the problem when (i) G and G' coincide, and (ii) the line from N neither cuts nor touches the circle. A book token will be awarded to the sender of the best postcard.

The above solution uses simple geometrical methods. It is possible to find other solutions based on trigonometrical methods and the use of conic sections other than the circle. J.F.H.

DO YOU (OR DID YOU) COLLECT CAR NUMBERS?

Whether you did or not here's an odd fact about them you can check for yourself on your bus journeys to and from School.

Choose a number from 1 to 9, say 4 for example. Now count how many registration number plates end in 4 in the next batch of 10 cars you pass. (For this purpose count DON 914 and 914 DON as both ending in 4).

Call this count the score for that batch of 10 cars and make a note of it (but *not* on the cover of your Maths. exercise book !).

Keep doing this for batches of 10 until you have 100 scores. It doesn't take very long on our crowded roads. If you've been honest with yourself and kept the scores accurately you will find that they run very close to the figures given below :

Score (*i.e.*, number of 4's per batch)	0	1	2	3	4	5 or more	Total
No. of batches with that score	37	37	18	6	2	0	100

If you compare your results with those of your friends on other bus routes you will find that they are very nearly the same. In fact if you take the average of their results and yours you will find that you get even closer to the figures given above.

Isn't it rather odd that such a "chancy" business should be capable of being predicted? Predictions like this (only rather more useful) are the everyday work of statisticians. The statistician works with Insurance Firms, in Medical and Agricultural research, in the design of Nuclear Power stations and Telephone exchanges. Perhaps you'd like to be one—ask your Mathematics teacher what it involves and how you set about it. R.M.S.

PERFECT NUMBERS

A perfect number is a number such that it is equal to the sum of all its factors, including unity but excluding itself. Thus, leaving out unity, 6 is the smallest perfect number with factors 1, 2 and 3 which, of course, add up to 6.

There are not many of these numbers, in fact, so far only twelve have been found. The next but one perfect number after 6 is 496 with factors 1, 2, 4, 8, 16, 31, 62, 124, 248.

Somewhere in between the numbers 6 and 496 there is another perfect number. It is much nearer to 6 than 496. Can you find it? J.F.H.

95625 86586 55705 52690 49652 09858 03385 07224

SEQUENCES

D / A	1	2	3	4	5
1	1	2	6	24	
2	5	9	11		60
3	10	28		8	20
4	16		27	3	5
5		126	38	0	1

Fill in the blank squares. 1D, 2D, 3D, 4D, 5D and 1A are regular sequences of numbers whose methods of formation you have to discover.

Rows 2A, 3A, 4A, 5A are obtained from the following formulae :—

2A : $n, 2n - 1, 2n + 1, 3n, 12n$.

3A : $ab, 4b - c, 2b + c, 4a, \frac{1}{2} bc$.

4A : $a^2, b^2 - 1, a + 3b - 1, \frac{1}{4}(a + b), a + 1$

5A : $x + y, xy, x - y + z, y - z + 1, z - y$

What are the numerical values of the symbols used? J.G.

THE POOR PILGRIM

(adapted from Le Facteur X)

A poor pilgrim, very pious and with very little money, entered a church where he found the three great saints ; St. Peter, St. Paul and St. John and addressed to each in turn a prayer as follows :

"Greetings, great saint, I pray that you will be pleased to double the money that I have in my purse ; I promise that to mark my gratitude, I shall give 6 francs to this Church."

His wish was granted each time, and each time he kept his promise with perfect honesty. At the end, he left the church without a sou. How much had he when he entered? J.F.H.

A SIMPLE GRAPH

Plot the graph of $y = x^3(5-x^2)$ for values of x between -2 and $+2$.

Submit solutions to the Editor stating your age. J.G.

OTHER NUMBER SYSTEMS

(a) In what number system is the identity $202 = 13 \times 13$ correct?

(b) "524 is the first perfect square larger than 441." In what number system is this statement correct? B.A.

Time Chart—Continued from page 281

this he calculated the diameter of the Earth to be 7,850 of our miles—only about 50 miles out ! He also estimated the sun's distance to be 100 million miles, which is reasonably correct. His work on prime numbers is also well known—the "Sieve" of Eratosthenes being a device for sifting out the composite numbers and leaving only the primes.

We have been able to deal with only a few of the many mathematicians of this very fruitful period—a period of creative thought which was not to be matched till Newton's day. We must postpone one of the greatest of them, Archimedes, until our next chart. He has been described as "one of the greatest mathematical physicists of all time." R.M.S.

18766 79524 06163 42522 57719 54291 62991 93064

MAGIC SQUARES No. 2

Contributed by Canon D. B. Eperson, Bishop Otter College, Chichester.

8	1	6
3	5	7
4	9	2

Fig. 1

In a magic square with 9 cells the sum of the numbers in each row, column, and diagonal is the same. In addition, the numbers in the central column, the middle row, and each diagonal, form sequences called Arithmetic progressions. In Fig. 1 these progressions are 1, 5, 9 : 3, 5, 7 : 2, 5, 8 : 4, 5, 6 : Their common differences are 4, 2, 3 and 1 respectively.

Question 1. Can you find the four progressions in Fig. 2, and their common differences ?

15	2	13
8	10	12
7	18	5

Fig. 2

Such sequences in magic squares can be used to form patterns. For instance, in Fig. 3 we have a dot marked at the centre of each cell, with lines through the four sets which correspond to the sequences mentioned above. These form a pattern similar to a Union Jack.

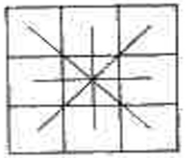


Fig. 3

Question 2. What patterns do you make by joining the dots representing the following sequences in Fig. 1?

[a] 1, 2, 3 ; 4, 5, 6 ; 7, 8, 9 ; [common difference 1]
[b] 1, 3, 5 ; 3, 5, 7 ; 5, 7, 9 ; [common difference 2]
[c] 1, 4, 7 ; 2, 5, 8 ; 3, 6, 9 ; [common difference 3]
[d] 2, 4, 6, 8. [common difference 2]

Question 3. What progressions in Figure 2, will give the same patterns as those in Questions 2[a] and [c]. What are their common differences ?

Question 4. Form patterns by joining up the dots corresponding to the numbers taken in increasing order of magnitude in Figs. 1 and 2.

7+1		7+4
	7	
7−4		7−1

Fig. 4

The two patterns are not alike, but all magic squares of 9 cells belong to one or the other of these two patterns. Test this by making your own magic squares in this way :—Choose any two numbers which are not in the ratio 1 : 2 or 2 : 1 ; then choose a third number not less than 5, which is greater than the sum of the first pair. [*e.g.*, the numbers could be 1, 4 and 7]. Put the third number in the central cell and fill the two diagonals with numbers which form A.P's whose common differences are the original pair of numbers. Then fill in the remaining cells, remembering that the rows and columns must all have the same total as the diagonals [see Fig. 4]. The join up the dots corresponding to the numbers in the cells taken in ascending order of magnitude, and you will get one of the two patterns occurring in Question 4.

		7
5		
	x	

Fig. 5

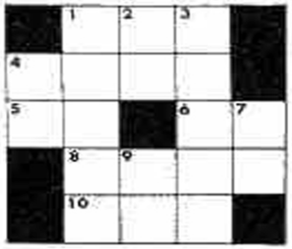
Question 5. [for those who like algebra] Can you find the value of x such that Fig. 5 can be made into a Magic square whatever number is put into the central cell, and the others filled with numbers that obey the sequence rule shown in Fig. 3?

POETICAL PI

Now I, even I, would celebrate
In rhymes inapt, the great
Immortal Syracusan, rivaled nevermore,
Who in his wondrous lore,
Passed on before,
Left men his guidance how to circles mensurate.

A. C. Orr

SENIOR CROSS FIGURE No. 36



CLUES ACROSS :

1. Value of a^2-b^2+2 when $a^3+b^3=8100$ and $a+b=30$.
4. The digits of this number are $x, 2x, x, x$; but if written $x, x, 2x, x$ it would decrease by 360.
5. Value of $s(s-a)+(s-b)(s-c)$ when $2s=a+b+c, b=4$, and $c=17$.
6. A clock is 7 minutes slow at 4 o'clock and 5 minutes fast at 5 o'clock. In how many minutes after 4 o'clock will it be correct ?
8. Value of $b^2-ac-14$ when $b+2a=122$, $b-c=52$ and $a-c=2$.
10. How many rectangles (including squares) in a rectangle 6" by 3" ruled into square inches ?

CLUES DOWN :

1. L.C.M. of 2873 and 3757.
2. Angle between the hands of a clock at 6.48 a.m.
3. Area of a trapezium, sides 1021, 25, 1007, 25.
4. Hypotenuse of a right angled triangle, perimeter 10", area 2 sq. in. (to 1 dec. place).
7. Number of diagonals in a duodecagon.
9. Numerical value of $x^2-6xy+9y^2+23x-69y+42$ when $x-3y=2$.

J.G.

FUN WITH NUMBERS No. 3

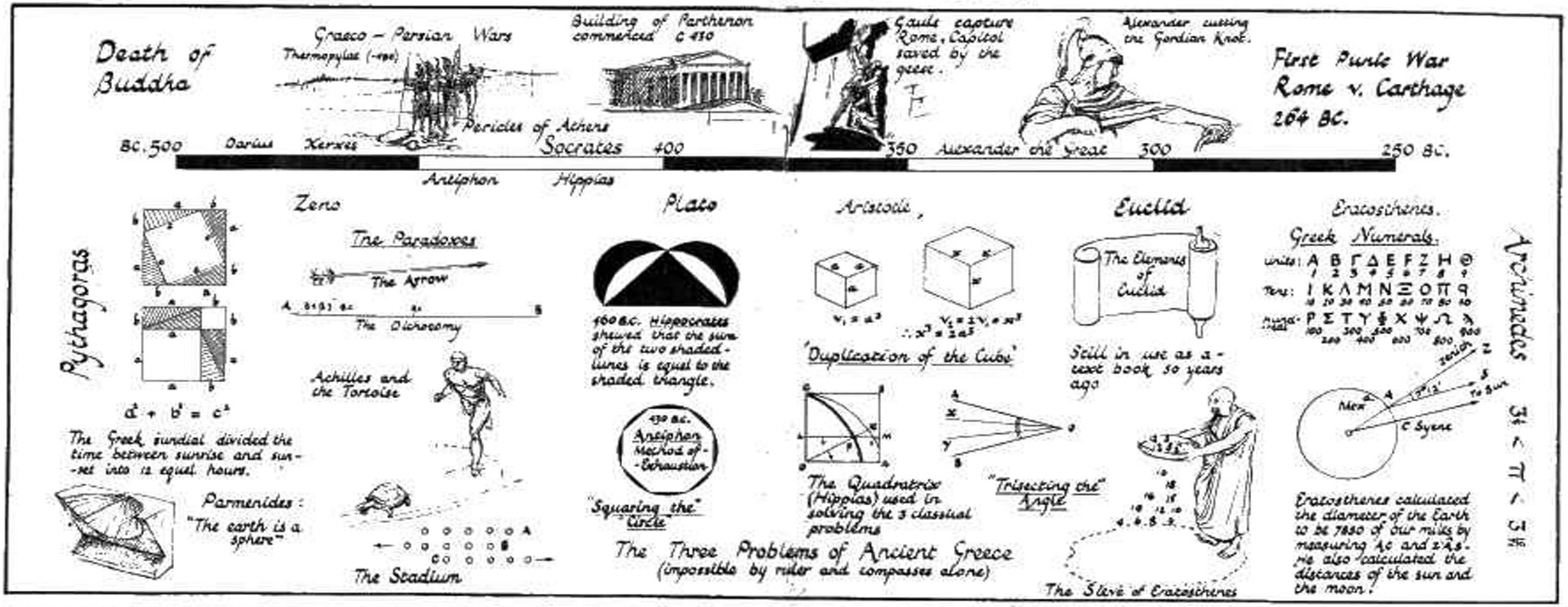
$$1+2+1 = a^2$$
$$1+2+3+2+1 = b^2$$
$$1+2+3+4+3+2+1 = c^2$$
$$1+2+3+4+5+4+3+2+1 = d^2$$

What are the values of a, b, c, d and can you write down quickly some similar problems and answers?

FUN WITH NUMBERS No. 4

$9 \times 9 + 7$ Work out these sums.
$98 \times 9 + 6$
$987 \times 9 + 5$ Can you now extend the sequence of
$9876 \times 9 + 4$ questions and answers ?

TIME CHART No. 2 : (500 B.C. — 250 B.C.)



At the end of the 6th Century B.C. we come to the death of Pythagoras and the birth of the philosopher Zeno, who is remembered chiefly for his four paradoxes, questions which challenged the ideas of some of his contemporaries who believed that space and time could be divided into an infinite number of very small parts. For example, the one about Achilles and the Tortoise :—

Achilles, who can run 10 times as fast as the tortoise, gives him 100 yards start. When he reaches the place where the tortoise was, it has gone 10 yards further on. By the time Achilles has covered this 10 yards the tortoise has gone an extra yard. When he has covered this yard the tortoise is still $\frac{1}{10}$ yard ahead and so on. How does Achilles catch the tortoise ? (Ask your teacher about the other 3 paradoxes).

The importance of this problem of infinite division is bound up with the three great problems of antiquity—Duplication of the Cube, Squaring the Circle and Trisecting the Angle. Greek arithmetic was hampered by the lack of a suitable simple number system. Various systems were in use, the simplest being the alphabetical one shewn in the chart. No wonder they avoided Arithmetic and tried to express all their ideas geometrically !

Originally the 3 problems were tackled using straight-edge and compasses only, but when this was found to be impossible, Hippocrates (c.460 B.C.) and Antiphon (c.430 B.C.) solved the Squaring of the Circle (*i.e.*, finding its area) by methods which were crude forms of our integral calculus.

Antiphon inscribed a polygon in the circle and found its area. By doubling the number of sides again and again he " exhausted " the shaded area which is the difference between the areas of polygon and circle.

The Three Problems were also tackled by the use of special curves. Deinostratus (c.350 B.C.) used the *Quadratrix* discovered by Hippias (c.425 B.C.). Neater solutions were made by using the Conchoid of Nicomedes, but this belongs to our next chart.

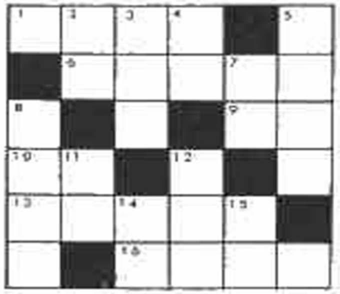
Two Greek Philosophers who, though not primarily mathematicians themselves, did much to put geometry on a sound basis, were Socrates and Plato. They insisted on accurate definitions, clear assumptions and logical proof. The Platonic school at Athens produced many brilliant men such as Aristotle (who became a tutor of Alexander the Great). Aristotle's chief interest in mathematics lay in its application to physics.

Alexander founded the city of Alexandria in Egypt, whose most famous inhabitant was Euclid, who performed the monumental task of collecting together all the mathematical knowledge of his time and arranging it in a logical sequence. The resulting " book " was still in use as recently as the beginning of our own century !

Another scholar of the University of Alexandria was Eratosthenes. He knew that the sun was at its zenith (*i.e.*, overhead) at Syrene on the Nile and measured its deviation from the zenith at Alexandria 625 miles away. From

Continued on page 283

JUNIOR CROSS FIGURE No. 34



CLUES ACROSS:

1. Palindromic number.
6. Palindromic perfect square.
9. Perfect Square.
10. A prime number.
13. Product of the first eight natural numbers.
16. Sounds like the end of a count-down.

CLUES DOWN:

2. ab when $a=2$ $b=2$.
3. Twice the square root of 6 across.
4. Unlucky for some.
5. Palindromic anagram of 1 across.
7. A perfect number.
8. Consecutive numbers.
11. Life's span.
12. Convert 1001101110 from the binary to the decimal scale.
14. Product of two primes.
15. Reverse the base of the decimal system.

Check Clue. One digit is not used and the sum of the digits in the Cross Figure is 68.

B.A.

ANOTHER SERIES

Continue the series 60, 90, 108, 120, ... and state the rule for the series.

J.G.

Time Chart—continued from page 291

Another Greek who flourished during this little renaissance of the Alexandrian School was the geometer Menelaus who wrote a treatise on the geometrical properties of spherical triangles and proved several important theorems about plane triangles, one of which appears in the chart above.

Claudius Ptolemaeus (Ptolemy) (c. 90–160 A.D.) did for astronomy what Euclid did for Geometry. He brought together in a single treatise, the "Almagest," the discoveries of his predecessors, arranging the work in such a systematic fashion as to make his work a standard reference for many centuries.

R.M.S.

SOLUTIONS TO PROBLEMS IN ISSUE No. 36

PERFECT NUMBERS

The second perfect number is 28. Its factors are 1, 2, 4, 7, 14, whose sum is 28.

SENIOR CROSS FIGURE No. 36

ACROSS: (1) 482, (4) 4844, (5) 68, (6) 35, (8) 4934, (10) 126.
DOWN: (1) 48841, (2) 84, (3) 24336, (4) 46, (7) 54, (9) 92.

FUN WITH NUMBERS No. 3

$a=2$, $b=3$, $c=4$, $d=5$.

FUN WITH NUMBERS No. 4

The answers to the sums were 88, 888, 8888, 88888.

SEQUENCES

2A. $n=5$; 3A. $a=2$, $b=5$, $c=8$; 4A. $a=4$, $b=8$; 5A. $x=9$, $y=14$, $z=15$.

THE POOR PILGRIM

The pilgrim had 5 francs 25 centimes when he entered the church.

A SIMPLE GRAPH

The result of this exercise will be announced in the next issue.

OTHER NUMBER SYSTEMS

(*a*) is true in the scale of 7. (*b*) is true in the scale of 6.

JUNIOR CROSS FIGURE No. 33

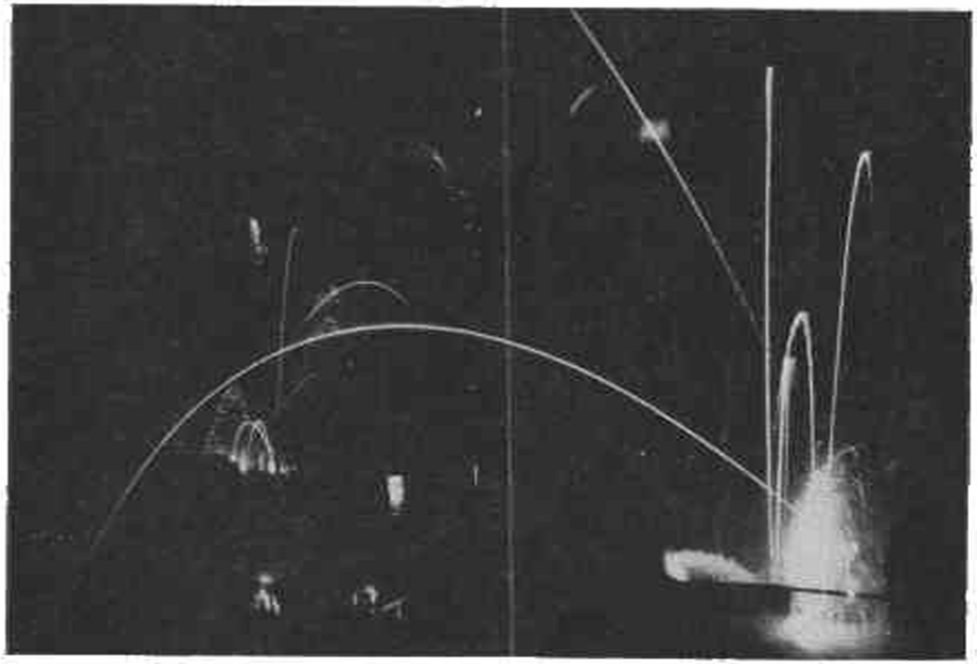
ACROSS: (1) 71, (2) 216, (4) 47, (5) 41231, (8) 68890, (11) 76, (12) 025, (13) 90.
DOWN: (1) 784, (2) 27386, (3) 68, (4) 42875, (6) 16, (7) 19, (9) 020, (10) 80.

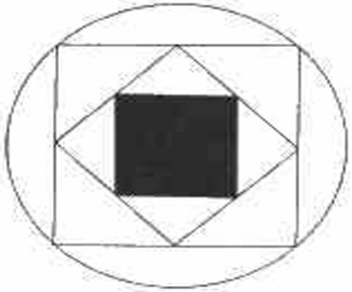
B.A.

MATHEMATICAL PIE

No. 37 — Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England — OCTOBER, 1962

FIRED!





Three squares are inscribed in a circle 34 inches in diameter as shown in the diagram. The black square represents a window which has to be filled in. Will a board 12 inches square do the trick?

DO YOU FIND MATHEMATICS DIFFICULT?

This is said for those who are dismayed at the outset of their studies, and then set out to gain the mastery over themselves, and in patience to apply themselves continuously to those studies.

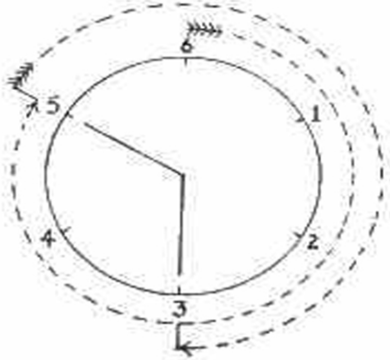
From these one sees that from these studies there result things marvellous to relate.

Leonardo da Vinci, 1452–1519. J.G.

CLOCK ARITHMETIC No. 3

Multiplication and Division

In the first article (Issue No. 34) you were shown how to make out an addition table for a circular face of scale 6 instead of a linear one in which $2+3=5$ and $5-4=3$ and from these results we constructed the addition table shown.



+	1	2	3	4	5	6
1	2	3	4	5	6	1
2	3	4	5	6	1	2
3	4	5	6	1	2	3
4	5	6	1	2	3	4
5	6	1	2	3	4	5
6	1	2	3	4	5	6

In the second article (Issue No. 35) you were shown the subtraction table and it was pointed out that the addition table above could be used to get subtraction results by working it backwards. This is similar to the use of logarithm tables to obtain anti-logarithms.

But what can we mean by multiplying numbers in modulo 6 arithmetic?

Now 3×4 means ' three fours ' *i.e.*, $4+4+4$

and 2×5 means ' two fives ' *i.e.*, $5+5$

so that every multiplication sum can always be translated into a process of repeated addition.

Thus, using the addition table above

$$\begin{array}{llll} 3\times4 = (4+4)+4 & 2\times5 = 5+5 & 4\times5 = (5+5)+(5+5) \\ \quad = 2 \quad +4 & \quad = 4 & \quad = 4 + 4 \\ \quad = 6 \text{ (see } a) & \quad \text{(see } b) & \quad = 2 \text{ (see } c) \end{array}$$

Now complete the multiplication table. Look carefully at your table and see if you can suggest a number which behaves as O does in ordinary arithmetic.

By defining division as the converse of multiplication, so that $3\times4=6$ means $6\div3=4$ and $4\times5=2$ means $2\div4=5$, we can use the complete multiplication table to do modulo 6 division. What do you make of the following divisions?

$4\div2 \qquad 3\div3 \qquad 2\div5$

The first two questions have more than one answer and the last answer need not be a fraction.

R.H.C.

x	1	2	3	4	5	6
1						
2					4^{b}	
3				6^{a}		
4					2^{c}	
5						
6						

GERONIMO'S GEOMETRY

Submitted by Marian House, Lister County Technical School.

The squaw on the hippopotamus is equal to the sons of the squaws on the other two hides.

Time Chart—continued from page 289

by which an angle could be trisected, and Diocles who invented the cissoid, which may be used in the duplication of the cube.

At about this time (150 B.C.) we get the beginnings of Trigonometry, the division of the circle into 360° and the development of a sexagesimal system of fractions (" minutes " and " seconds ") which had been suggested by the Babylonians in the first place. Hipparchos of Rhodes (c. 150 B.C.) worked out what was virtually a table of sines and developed a kind of spherical trigonometry (used in navigation). He was an excellent astronomer and left a catalogue of 850 fixed stars, which was increased to about 1,000 by Ptolemy about 300 years later, but was not materially added to until comparatively recent times.

We do not usually think of Julius Caesar as a Mathematician but he was well versed in Astronomy and planned extensive surveys of the Roman Empire. His chief claim to our interest is probably the reform of the calendar. His system of leap years was accurate enough to be only 11 days out 1600 years later and with Pope Gregory's modification about century years will need no further adjustment for the next 3,000 years.

Most of you will know Heron's formula for the area of a triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$. He was an Egyptian living in Alexandria about 50 A.D. and wrote a treatise summarising the Egyptian methods of land surveying. He invented various machines, including a simple steam turbine nearly 2,000 years before the days of the steam turbine battleship and ocean liner.

continued on page 292

THE WORLD ENCOMPASSED : No. 3

The French Mathematicians

In company with many other problems that were studied during the early years of mathematics, investigation of the size and shape of the earth was neglected for many centuries following the promising start made by the Greeks. It was not until such questions as " What is the earth made of ? " and " Is the earth a perfect sphere ? " were once more examined by mathematicians that further serious attempts were made to measure the earth and assess its shape.

In the 17th century, Newton suggested that on account of the earth's rotation it would tend to differ in shape from a true sphere and be slightly flattened at the poles. If such flattening existed, then the pole-to-pole diameter should be less than an equatorial diameter. Newton suggested that the shortening would be of the order of 1/230th part of the longer diameter.

One way of checking this possibility was by travelling along a meridian and measuring the " length of a degree " at various intervals along the meridian. Thus, supposing two places to be at sea level, one at latitude 50° and the other due north at latitude 51°, the distance between them (supposed free from mountains or hollows) would be accurately measured. Similarly, measurements further north and further south would be made, the latitudes being checked by corrected astronomical observations (Note that if the earth were a true sphere and the Pole Star were at the true astronomical north pole, the elevation of the star in degrees would be the same as the latitude at any observation point in the northern hemisphere). In regions of flattening, it would be necessary to travel further along the meridian in order to make a change of 1° in the elevation of the Pole Star than in regions of " bulge."

The French Mathematicians of the time were anxious to investigate Newton's suggestion although he warned them that the differences in the length of a degree at, say, Bayonne and Dunkirk would be so slight as to be undetectable with the instruments available. Despite the warning, Jacques Cassini made some measurements in 1718 and concluded that the length of a degree was less at a certain part of a meridian than it was at a part some distance further south. This result implied that the shape of the earth tended more to that of a lemon than to that of an orange, *i.e.*, the poles would be elongated rather than flattened.

In 1737, a team that included Pierre Maupertuis and A. C. Clairaut went to Lapland and at latitude 66° 22′ N determined the length of a degree to be 69.403 miles. Another team that went to Peru included Charles M. La Condamine, Pierre Bouguer and Godin. Their measurements, in 1744, showed that at latitude 0°, the length of a degree lay between 68.713 and 68.732 miles. Newton's conjecture was thus vindicated.

Since those early days, progress in the design, construction and operation of instruments has greatly improved the accuracy of observations. The latest determinations of the earth's shape give the polar flattening as one part in297. This is roughly equivalent to the thickness of the tissue paper compared to the diameter of orange wrapped in it, so is not as great as some folk imagine.

Let us admire the courage and pioneering spirit of these 18th century French Mathematicians and learn from their mistakes—just as they did !

J.F.H. I.L.C.

96501 17844 08745 19601 21228 59937 16231 30171

FUN WITH NUMBERS No. 5

Express each of the fractions 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8 and 1/9 by arranging the nine digits 1,2,3,4,5,6,7,8,9 using each digit once and only once.

HAPPY BIRTHDAY

A man and his grandson have the same birthday. For six consecutive birthdays the man is an integral number of times as old as his grandson. How old is each at the sixth of these birthdays ?

(*From Maths Student Journal*)

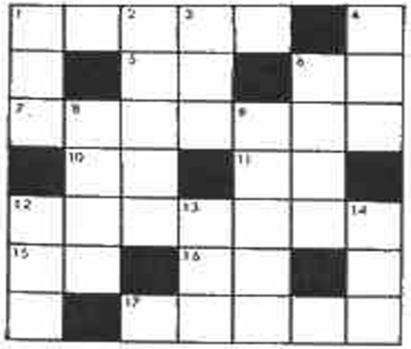
QUICK QUIZ

123456789	1
12345678	12
1234567	123
123456	1234
12345	12345
1234	123456
123	1234567
12	12345678
1	123456789

You are allowed ten seconds to decide which column of figures when added will give the larger result. Check your result after you have made your guess.

R.H.C.

SENIOR CROSS-FIGURE No. 37



$a+3b=23$, $a^2-ab+6b^2=386$, $3c\div d=cd=12$.

CLUES ACROSS :

1. $(2a+c)^3 \div 5a$.
5. $c(d^2-b^2)$.
6. Sum of the digits of 7 across.
7. Product of the first eight prime numbers.
10. Half of 5 across.
11. Reverse a.
12. A million and nine !
15. $3(a+c)$.
16. $c(a+b+c)$.
17. Leaves a remainder of 1 when divided by each of the first six prime numbers.

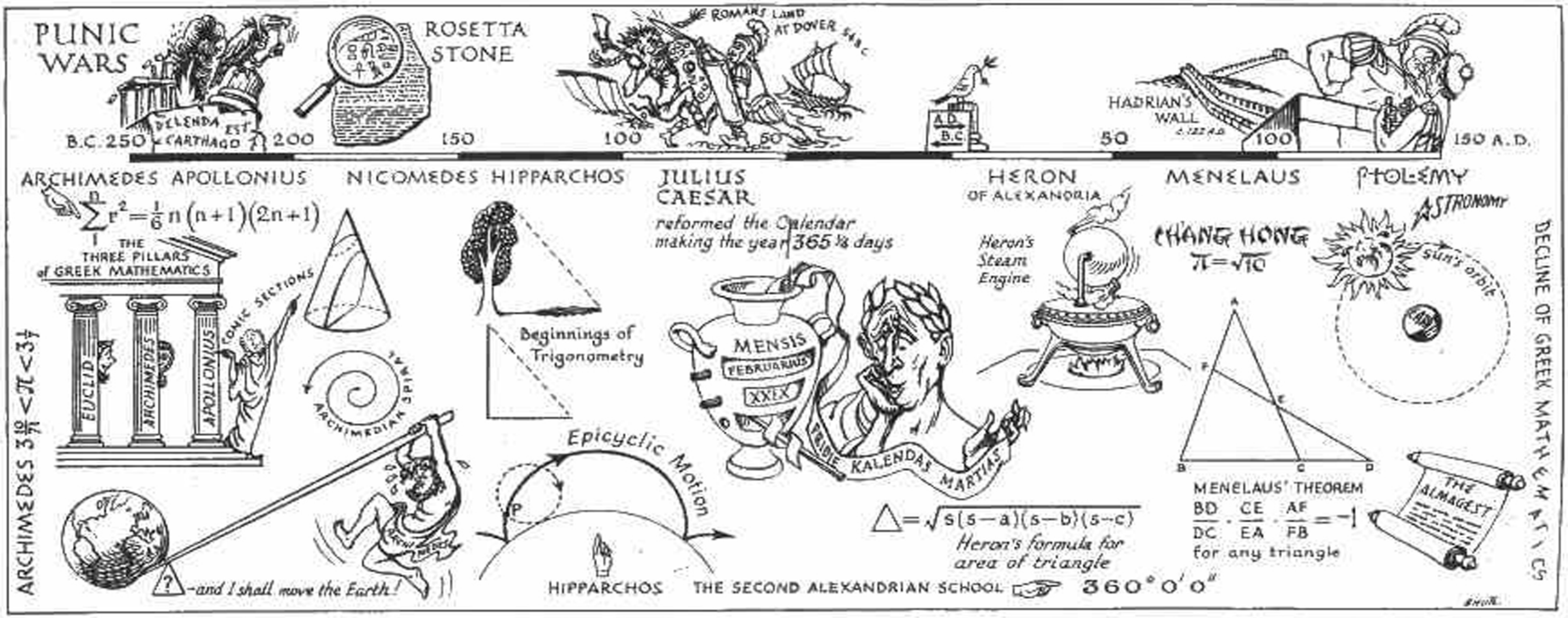
CLUES DOWN :

1. $(a-b)\ (2a+b)$.
2. $(a+d)^3+a^2b$.
3. $b(a+b+c-d)^2 \div c^3$.
4. $a(b+c-d)$.
6. LCM of 224, 616, 704.
8. $4c^2d^2\ (a-b-c-d)$.
9. $\dfrac{9!+3.6!-5!}{3!}$
12. A perfect square.
13. $abcd$.
14. ad reversed.

B.A.

29725 91677 81314 96990 09019 21169 71737 27847

TIME CHART No. 3: (250 B.C.—150 A.D.)



For a time we have a continuation of the golden age of Greek Mathematics. Archimedes of Syracuse (287—212 B.C.) was a mathematical physicist of the first order. We can get some idea of the greatness of the man if we look at a few of his contributions to Mathematics and to Science.

He calculated a value of π by considering the area of a circle to lie between the areas of the polygons inscribed and circumscribed to it. By increasing the number of sides again and again, he got closer and closer approximations to π. By taking polygons of 96 sides he found that π lies between $3\frac{1335}{9347}$ and $3\frac{1335}{9345}$. This was a remarkable achievement when one considers the clumsiness of Greek Arithmetic.

He invented a system of reckoning in octads or eighth powers of 10. By this means he extended the Greek number system as far as 10^{63} to calculate the number of grains of sand in the solar system—this involves knowledge of the laws of indices $a^m \times a^n = a^{m+n}$. He also calculated the volume of the sphere, studied the properties of several types of spirals and found that $1^2+2^2+3^2+4^2 \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$. Besides all this he put the science of Hydrostatics on a firm footing and studied the properties of pulleys and levers. He is reputed to have said " Give me a fulcrum and a place whereon to stand and I will move the Earth."

It was a great loss to mankind when an infuriated Roman soldier slew him at the capture of Syracuse because he was too absorbed in a geometrical problem to notice that the soldier had asked him a question.

The third of this great trio of Greek Mathematicians was Apollonius of Perga (c. 260—c. 170 B.C.). Like Euclid he is known chiefly for his Geometry, although he made valuable contributions to Arithmetic, notably improving the system of Archimedes by basing it on the smaller and more convenient base of 10^4, a number which under the name of *myriad* had long been in use in the East. His most important contribution to mathematics was the study of the section of a cone made by a plane. These conic sections, principally the parabola, ellipse, and hyperbola (names given to them by Appollonius and still in use today) are of great practical importance. The parabola is the path followed by a projectile if we neglect air resistance and the curvature of the earth, the approximate shape of a suspension bridge chain, and the section of a car headlamp reflector or a radio telescope bowl. Most planets and comets follow elliptical orbits round the sun and the ellipse has been used for bridge arches. The hyperbola is used extensively in the LORAN system of radar navigation. All three curves will be increasingly important in the near future in space navigation.

With the death of Apollonius we really come to the end of the Golden Age of Greek Mathematics. There were a number of minor geometers such as Nicomedes, who invented a curve known as the *conchoid* (shell-shaped)

Continued on page 291

68472 68608 49003 37702 42429 16513 00500 51683

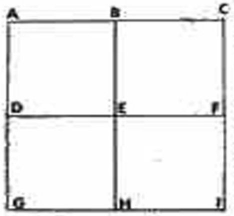
23364 35038 95170 29893 92233 45173 20138 12806

GENERALISED CUTS

If we take a line and divide it into two pieces then count up the number of different pieces or segments which could be chosen from the set of points, we find they are 01, 02, 12.

Now take a line and divide it into four pieces. Count up the number of different segments again. How many line segments can be made with the inch markings on a 12 inch ruler? Can you find a formula to give the number of different segments when there are n inches on the line?

In a similar way, a square of side two inches can have five different squares chosen from it. *ABED, BCFE, DEHG, EFIH, ACIG.* How many squares can you find in a similarly marked square of side 3 inches and how many in a foot square? Generalise the result for a square of side n inches.



Finally, the game can be extended into three dimensions. A cube of side 1 foot is divided up into inch cubes. How many different cubes can be chosen from the cubic foot and what will be the number for a cube of side n inches?

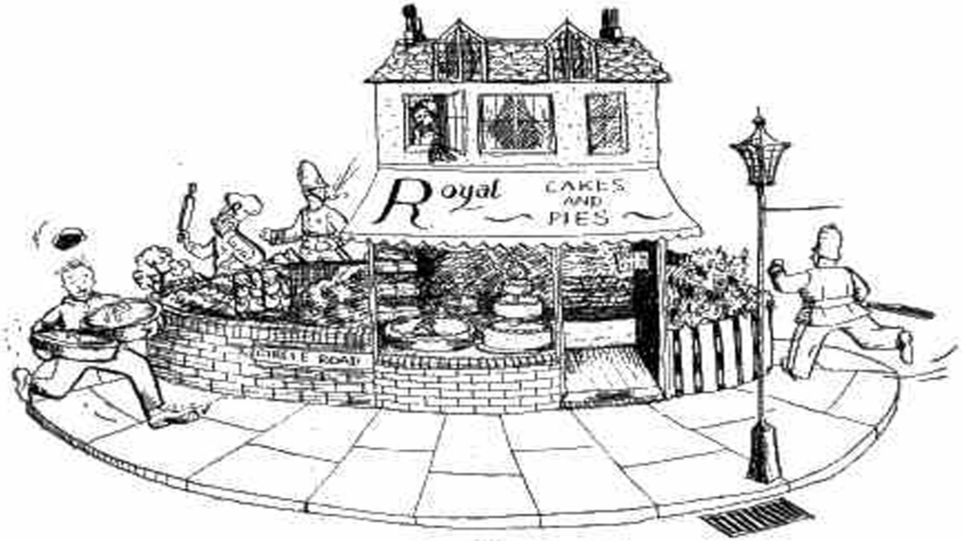
Just for fun, can you generalise your results for four dimensions? J.G.

FOOD FOR THOUGHT

$$\frac{\text{PORK}}{\text{CHOP}} = C$$

In this division sum C is greater than 2.

CIRCUMFERENCE OF A CIRCLE IS TWO PIE R



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MATHEMATICAL PIE

No. 38 *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England FEBRUARY, 1963

POINT CENTRED PATTERNS

The essential thing about a pattern is repetition of a motif. In the language of elementary geometry, a pattern is a collection of congruent figures, or more rarely similar figures, which are repeated according to some system. The particular system used determines the classification of the pattern.

The leaf pattern is an example of a point centred pattern formed by rotating the basic motif of one leaf about a fixed point through 120°. The operation of rotation through 120° which has been used to form the pattern can be applied to the pattern itself without changing its appearance.

The 6-way snowflake pattern illustrates another form of repetition. This pattern has six axes of symmetry passing through the centre point. If a mirror were placed along one of these axes, one half of the pattern would be the reflection in the mirror of the other half.

The pentacle appears at first sight to have five axes of mirror symmetry but one half of the pattern is not an exact mirror image of the other. The interlacings give the impression that the pattern has a certain depth. If we accept this and suppose that the straps really do cross, we have five axes of rotational symmetry in the plane of the paper. One half of the pattern is given by rotating the other about one of these axes as if it were a page of a book.

The fourth pattern is of a type attempted more often in Nature than in art. The pattern is formed by rotating the motif through a fixed angle and at the same time enlarging it in a fixed ratio. Such a pattern can never be completed.

LINEAR PATTERNS

To make a linear pattern, a motif is repeated by translation through a fixed distance in a fixed direction. The pattern below has repetition by translation only.

The motif repeated by translation may itself be a point centred pattern formed by the reflection or rotation of an element, so that the linear pattern may have transverse or longitudinal axes of mirror symmetry,

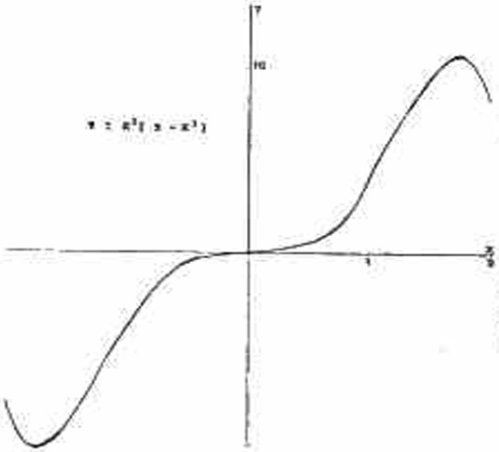
45085 04860 82503 93021 33219 71551 84306 35455

rotation through 180°, with or without symmetry.

and another type of symmetry called glide reflection, which is produced by moving the mirror reflection in an axis sideways by half the translation distance.

C.V.G.

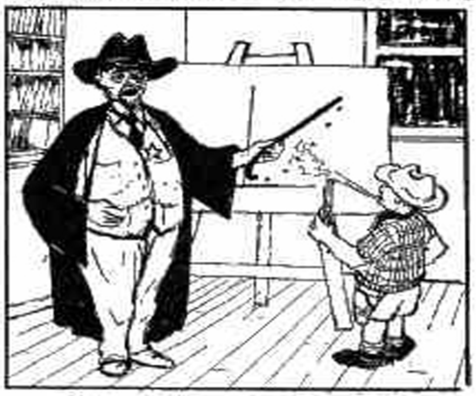
A SIMPLE GRAPH



There were several good attempts at plotting the graph of $y=x^3\,(5-x^2)$ submitted in response to the invitation in Issue No. 36. The winner of the competition was P. W. Vincent, of Dover, aged 16 years.

The main feature of the graph is that if only integral values of x are considered, the points lie on a straight line. Some of our younger readers discovered this and wrote to ask for guidance, as they recognised that the graph should be a curve. By plotting intermediate values, the shape of the graph becomes clearer and by using differential calculus the true shape of the graph can be confirmed. The greatest value in the range occurs when $x=\sqrt{3}$ and the least value of y occurs when $x=-\sqrt{3}$.

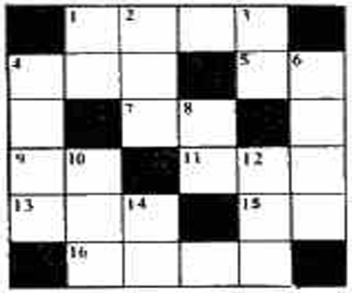
The graph shows the danger of generalising results from a limited number of observations B.A.



Now plot the point where $x=7$

00766 82829 49304 13776 55279 39751 75461 39539

JUNIOR CROSS-FIGURE No. 35



Given $a=6$, $b=7$, $c=4$, and $d=5$. Ignore decimal points.

CLUES ACROSS:

1. $2(4x+3)=19$. Find x.
4. $b\,(b-c)^4$.
5. $b\,(a+b)$.
7. $y+z-c$ (See 2 and 4 down).
9. $c(a+1)-b$.
11. $(a+d)^2$.
13. $2c^4+c$.
15. $a\,(b-d)$.
16. $10\,(a^3+b^3+c^3+d^3)$.

CLUES DOWN:

1. $a+\frac{1}{2}cd$.
2. $3y-7=20-y$. Find y.
3. $(b+2)\,(c+2)+d$.
4. $2z-100=101-2z$. Find z.
6. a^3b.
8. $bd-c$.
10. $(a+b)\,(c+d)$.
12. abd.
14. c^3.

B.A.

TREAD SOFTLY!

The famous leaning tower of Pisa has been slowly falling out of true ever since it was built 800 years ago because the foundations on one side are slowly sinking into the subsoil faster than those on the other. If we suppose that it is 180 feet high, 36 feet in diameter and has become 16 feet out of true since it was built, when is it likely to topple over? (Supposing nothing is done about it in the meantime). J.F.H.

ALL ROUND

A farmer sees a squirrel on the trunk of a tree and raises his rifle to shoot, but just then the squirrel ran around to the opposite side of the tree. The farmer kept his rifle aimed at the tree and slowly circled round it. However the squirrel kept moving ahead of him and the farmer went all round the tree without seeing the squirrel. He had gone all round the tree but had he gone all the way around the squirrel?

AM I ALL RIGHT JACK?

A bowls player carrying 3 woods came to a bridge that would only carry his weight and one wood at a time, so he decided to juggle them as he crossed so that 2 are always in the air. Smart?

SOLUTIONS TO PROBLEMS IN ISSUE No. 37

THREE SQUARES

The side of the black square is $8.5\sqrt{2}$ inches, which is just over 12 inches

HAPPY BIRTHDAY

The grandfather was 66 years and the grandson 6 years.

QUICK QUIZ

The first column of figures has the larger sum.

SENIOR CROSS FIGURE No. 37

ACROSS: (1) 74188, (5) 74, (6) 48, (7) 9699690, (10) 37, (11) 02, (12) 1362880, (15) 66, (16) 46, (17) 30031.

DOWN: (1) 779, (2) 17976, (3) 849, (4) 180, (6) 4928, (8) 6336, (9) 60860, (12) 169, (13) 240, (14) 021.

JUNIOR CROSS FIGURE No. 34

ACROSS: (1) 1221, (6) 12321, (9) 81, (10) 37, (13) 40320, (16) 3210.
DOWN: (2) 21, (3) 222, (4) 13, (5) 2112, (7) 28, (8) 2345, (11) 70, (12) 622, (14) 33, (15) 01.

ANOTHER SERIES

The series is 60, 90, 108, 120, $128\frac{4}{7}$, 135, ... The terms give the internal angles of regular polygons.

B.A.

34072 39610 36854 06643 19395 09790 19069 96395

Time has shewn that they tackled the problem from such different standpoints and used such different notations that each may be accorded a full share of the honour of the discovery.

Leibniz' interest in Mathematics really stems from his meeting with Huygens, the inventor of the pendulum clock and of the wave theory of light. Leibniz and Huygens studied the properties of the Catenary which is the curve taken up by a chain hanging freely between two points. Leibniz was also the inventor of a calculating machine of much greater potential than the simple adding machine of Pascal—indeed many modern calculating machines use a version of the Leibniz Wheel.

Newton really requires an article to himself. Like Archimedes he tried his hand at many things and illuminated everything that he touched. In two years which he spent at home because the University of Cambridge was closed during the Great Plague he laid the foundations of much of his later work. He invented the Calculus, which he called the Method of Fluxions, stated the three fundamental laws of mechanics, discovered the Universal Law of Gravitation and applied it to the Solar System, proved the general Binomial Theorem, invented numerical methods of solving equations, studied Friction, resolved white light into its component colours, and invented a telescope . . . In any Mathematical or Scientific work which stems from that period we can be sure to find the print of Newton's hand somewhere. R.M.S.

TRY ANGLES AGAIN

Dear Editor, Exmouth.

There is a misprint in the last line of the proof. It should read :— $PQ=GL=1$.

When G and G' coincide the line from N is a tangent to the circle, $\angle NGL=90°$, $\angle QGL=\angle QLG=90°-\angle LNG$, and therefore $NQ=QL$ and $LP=\frac{1}{2}NL$.

When 1 is such that the line from N neither cuts nor touches the circle, the fence will not be long enough to divide the field whilst satisfying the given requirements.

Yours faithfully,
KATHRYN SAMPSON (15 years).

Kathryn's was the best of those received after the article in Issue No. 36, and she has been sent a book token. B.A.

SITTING ON THE FENCE

Smith and Jones, two bargain hunters, were seeking estimates for the fencing in of their estates. " My property is exactly a square mile " said Smith. " Mine is exactly a mile square," said Jones.

" Then as far as I am concerned," said the contractor, " I will fence either property for £500." What did the contractor charge per mile of fencing? R.H.C.

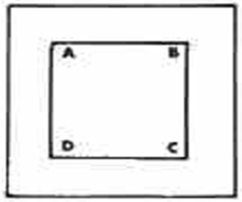
JIMMIE'S HOMEWORK BOOK

28	28	28	28	28
+39	+39	+39	+39	+39
67 ✓	65 ✓	61 ✓	63 ✓	517 ✓

Teacher gave him full marks. How was this? R.M.S.

21835 64622 01349 67151 88190 97303 81198 00497

A MONUMENTAL PROBLEM
(adapted from Le Facteur X)

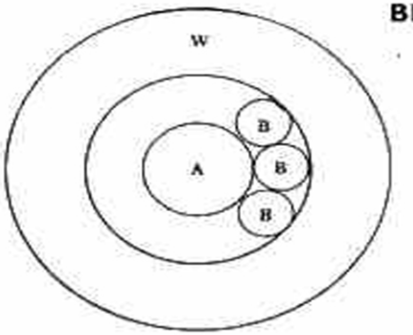


The base of a monument is a square—say $ABCD$. It is surrounded by a plinth also with a square outline. The outer edges of the plinth are distant 3 yards from the base of the monument. The area of the plinth is 120 square yards. Find the length of the side AB. J.F.H.

FUN WITH NUMBERS No. 6

Using four 1's can you express the numbers 1 to 12?
e.g., $13=11+1+1$. R.H.C.

BEARING UP



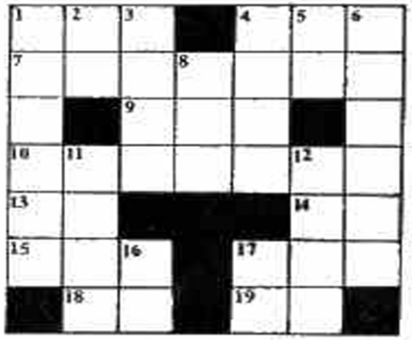
W is a wheel and A is its axle ; in between wheel and axle are ball-bearings BBB . . . . Consider two possibilities :

(1) The axle is held and the wheel is turned for one complete revolution.

(2) The wheel is held while the axle is turned for one complete revolution.

In which of these two events do the ball-bearings travel further? J.F.H.

SENIOR CROSS-FIGURE No. 38



CLUES ACROSS :

1. 55 in the scale of 6.
4. Change 1,110,001 in the scale of 2 to the scale of 10.
7. $\frac{1}{10}$ the product of the first eight primes plus $3\times7\times11\times13\times19$.
9. Change (800 — 26) from scale of 11 to scale of 10.
10. Perfect cube.
13. It has 1, 2, 3 and 4 as factors.
14. See 5 down. Shillings.
15. $13\frac{1}{2}^2 - 2\frac{1}{2}^2$.
17. Five!
18. Product of two prime numbers.
19. The sixth prime number.

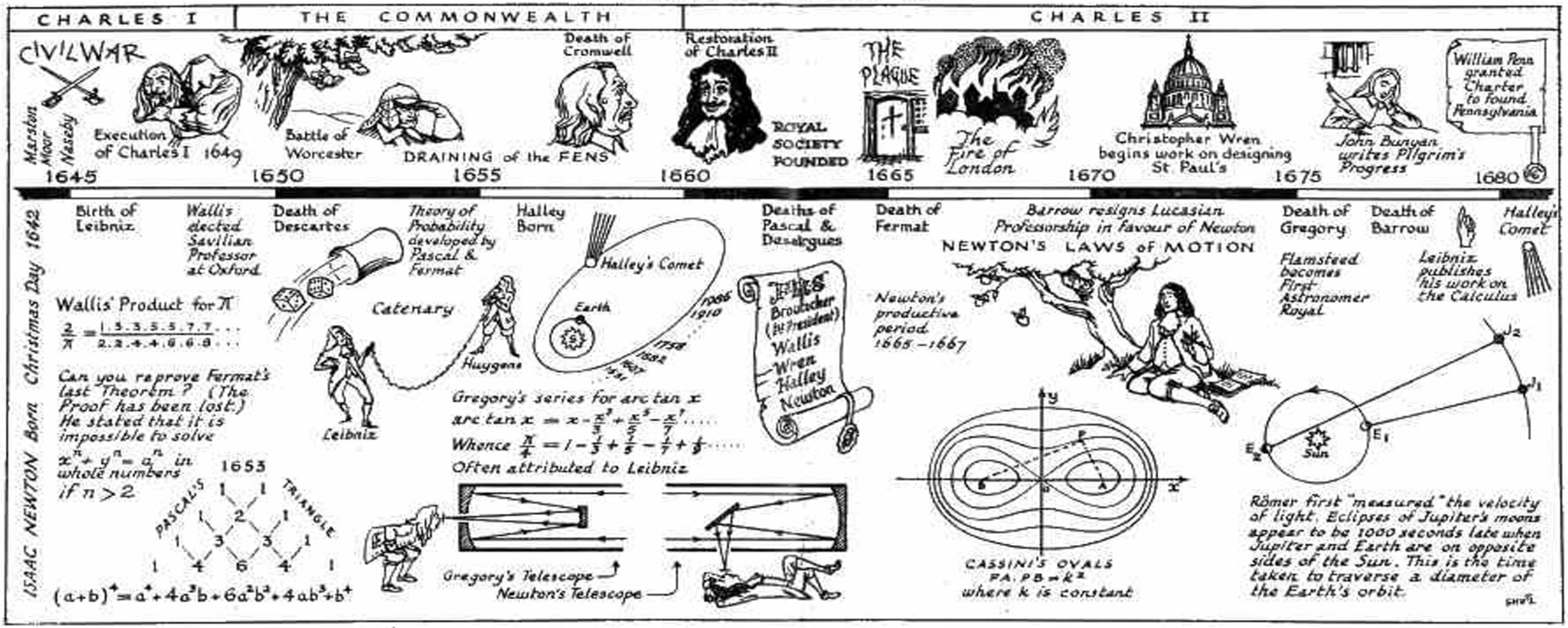
CLUES DOWN :

1. 55 in the binary scale.
2. Two more than a perfect number.
3. Perfect square.
4. Smallest number over one thousand whose digits add up to three.
5. Take a sum of money less than £12. Interchange pence and pounds and find the difference. Interchange the pence and pounds in the answer and add to the answer. Number of pounds. See also 14 Across, 17 Down.
6. 9!
8. An arithmetic progression.
11. 8 short of the number of cubic inches in a cubic foot reversed.
12. Consecutive numbers.
16. Product of two primes.
17. See 5 down. Pence.

B.A.

84683 39363 83047 46119 96653 85815 38420 56853

TIME CHART No 10



This period was one of the most productive in the whole of mathematical history. The improvements in algebraic notation which took place in the first half of the 17th century allowed ideas, which for long had been only half perceived, to be expressed with crystal clarity.

The period abounds with great names. Pascal was still at the height of his powers. In 1653 he wrote a treatise on the remarkable properties of the triangular array of numbers which goes by his name although some of its properties had been known to Omar Khayyam, the Persian Poet (c.1100) and to the Chinese two centuries later. Two of its many consequences are the Binomial theorem for expanding $(1+x)^n$, and the Theory of Probability which Pascal and Fermat (1608—1665) worked out in a now famous correspondence.

Fermat was a first rate mathematician but remained an amateur and we learn of his work mainly through his correspondence with Pascal, Descartes and others. His work on the Theory of Numbers was outstanding. This is a field of Mathematics in which the results are easy to state and to verify but surprisingly difficult to prove. *e.g.*, if p is a prime number and not a factor of a then $(a^{p-1} - 1)$ is divisible by p. (Test it yourself with $a=5$ and $p=7$ and other values). Fermat's famous Last Theorem (see chart) also belongs to the field of Theory of Numbers.

Wallis (1616–1703), chaplain to Charles II and one of the founders of the Royal Society, extended the use of indices to include negative and fractional indices as well as positive integers. He obtained the equivalent of

$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ giving the length of an element of a curve and was able to 'integrate' such curves as $y = x^n$ and $y = x^{1/n}$.

Mathematics has always been closely linked with Astronomy and we have a brilliant galaxy here including the first Astronomers Royal of Britain and France, Flamsteed and Cassini, Halley, of Comet fame and Gregory, inventor of the Gregorian telescope who also developed the series

$$\text{arc tan } x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 \ldots\ldots$$

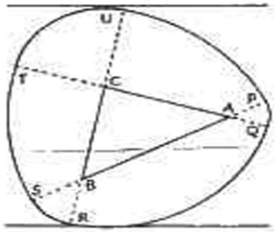
which is used in almost all modern calculations of π. He was one of the first to distinguish between convergent and divergent series and yet he was only 37 when he died. Another famous astronomer mathematician who later turned to Architecture was Christopher Wren, who rebuilt London after the Fire of 1666.

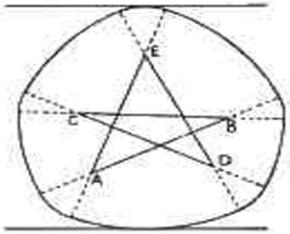
Head and shoulders above their contemporaries stand Newton (1642–1727) and Liebniz (1646–1716). There was great bitterness between them at the time, each claiming to have been the first to have invented the Calculus.

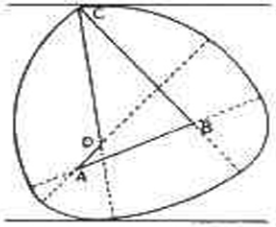
38621 86721 86725 23340 28308 71123 28278 92125 07712 62946 32295 63989 89893 58211 67456 27010

CURVES OF CONSTANT BREADTH

When the attempt was made to manufacture ball bearings by grinding between parallel plates some very queerly shaped objects were produced. There had been a confusion between a theorem and its converse. The proposition that " pairs of parallel tangents drawn to a circle of diameter d are distance d apart " is true, the converse :—" if all pairs of parallel tangents to a curve are the same distance d apart then the curve is a circle of diameter d " is not true.







A simple closed curve is formed by a continuous curve which returns to its starting point without coming to any other point on itself. Thus a circle, a convex polygon, and an ellipse are simple closed curves but a figure eight is not. The breadth of a simple closed curve is found in the following way. Draw a line in the direction in which its breadth is required, drop perpendiculars from each point on the curve to the line, and the segment of the line which contains the feet of all the perpendiculars is defined as the breadth of the curve in this direction. Most closed curves have different breadths in different directions but there are some which have the same breadth in every direction, such curves are called curves of constant breadth.

The simplest type of non-circular curve of constant breadth is formed by drawing on an equilateral triangle, three arcs whose radii are equal to the length of the side of the triangle and whose centres are the three vertices, each arc lying between two vertices. Can any regular figure be treated in this way to give a curve of constant breadth?

The next simplest type of non-circular curve of constant breadth is formed by six circular arcs whose centres are the vertices of a triangle. ABC is a triangle (see first figure). With centre A (it is advisable to start at the vertex opposite the shortest side) and with any radius describe an arc cutting CA at Q and BA produced in P. Then with centre C and radius CU describe an arc cutting AC produced at T, with centre A and radius AT describe an arc to cut AB produced in S, with centre B and radius BS describe an arc to cut CB in R. It is now easy to prove that the arc with centre C and radius CR passes through Q, so closing the curve, and also that $AP+AS=BR+BU=CQ+CT$. Two parallel tangents to the curve must touch arcs with the same centre, therefore the perpendicular distance between tangents is always equal to PS.

The next two figures are based on a pentacle and a re-entrant quadrilateral respectively. The only limitation on the polygon used is that the sum of the acute and obtuse angles plus the supplements of any reflex angles must be two right angles. (*to be concluded*). C.V.G. B.A.

44604 77464 91599 50549 73742 56269 01049 03778

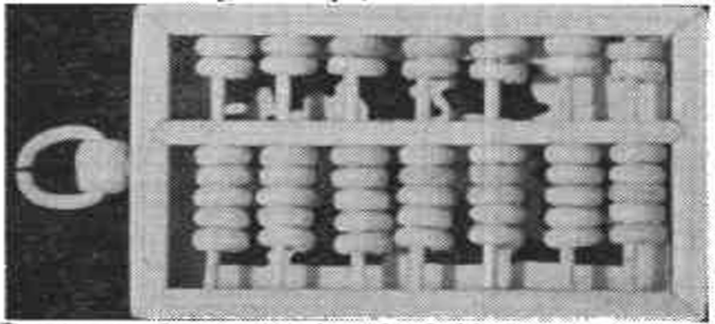
MATHEMATICAL PIE

No. 39 — *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England — MAY, 1963

MATHEMATICAL INSTRUMENTS No. 8

The Abacus

The little ivory Chinese abacus, reproduced full size, must have needed dainty fingers. Those in common use in China are rather larger, about the size of a foolscap envelope, and made of wood and wire. Some kind of



abacus, or counting frame, has been used in almost every part of the world. The Romans used marbles on a grooved board. Our word calculation is derived from the Latin word *calculus* which means a small pebble. In the Middle Ages in Europe, an abacus was a board ruled in squares and the counters were like the "men" used in the game of draughts. The Chinese abacus, the Japanese abacus and the Bulgarian abacus, used in Balkan countries until quite recently, have beads on wires. The five beads on the right hand wire stand for units. The two beads above are fives. On the next wire the five beads represent tens and the two beads above represent fifties. The beads count when they are pushed towards the centre bar. Each wire of a Japanese abacus has only four beads below the bar and one above.

Addition, subtraction, and multiplication or division by two are easy. To divide a number by two start at the right hand side. To double a number start on the left hand side. For multiplication the method of duplication or " Russian multiplication " is used. Only very small multiplications could be performed on the little ivory abacus.

Using figures instead of beads this is how multiplication of 23 by 19 is done

19	23	23
9	46	69
4	92	69
2	184	69
1	368	437

The three columns represent different parts of the working on the abacus. The answer is found in the last column.

The numbers 19 and 23 are entered on the abacus. 19 is odd therefore enter 23 in the last column.

56803 44903 98205 95510 02263 53536 19204 19947

Line 2. Halve 19 and double 23, omitting remainders. 9 is odd therefore add 46 to the last column.

Line 3. Halve 9 and double 46

Line 4. Halve 4 and double 92

Line 5. Halve 2 and double 184. 1 is odd therefore add 368 to the last number in the last column.

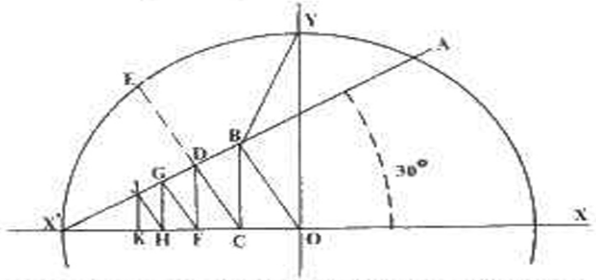
The number in the last column is now 23×19. Try it using match sticks.

Division is a different matter. When Pope Sylvester published a treatise explaining how it could be done many people thought that he must be in league with the Devil. C.V.G.

RULER AND COMPASSES ONLY

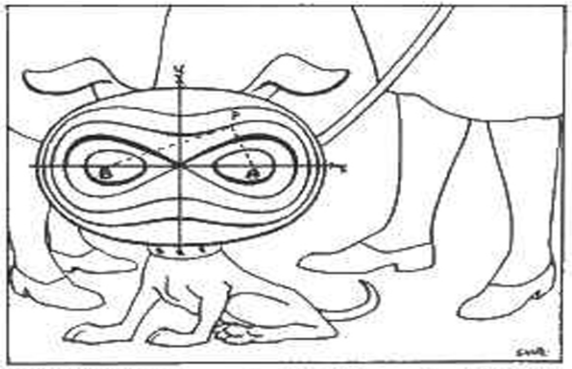
Regular polygons with six and eight sides are easy to construct by dividing up a circle. With a prime number such as 5, 7, 11 or 13 representing the number of sides, the task of drawing a *regular* polygon is not easy. The following method, however, gives a very close approximation to a truly regular polygon.

With centre O, describe a circle. Draw two diameters XOX' and YOY' at right angles to each other. From X' draw a line making an angle of 30° with $X'OX$; let this cut the circle at A. Draw OB perpendicular to $X'A$. Join YB; with compasses set to this distance, the circle may be divided into ten. Joining every other point on such a division gives a pentagon. In the same way use $X'B$ for a 7-sided polygon (heptagon). You can try some others as follows: DE for 13 sides; DF for 19 sides; $X'G$ for 17 sides; $X'J$ for 23 sides; and $X'K$ for 26 sides. J.F.H.



AN ODD RESULT

Add 1,000 to a certain whole number and the result is actually more than if the original number were multiplied by 1,000. What is the number?



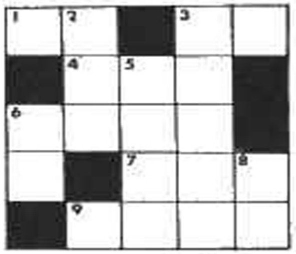
'He's looked like that ever since I fed him on Cassini's Ovals.'

45538 59381 02343 95544 95977 83779 02374 21617

JUNIOR CROSS-FIGURE No. 36

Submitted by R. Anne Horton, Girls Grammar School, Maidstone.

Correct each answer to the appropriate number of significant figures.



CLUES ACROSS:

1. Last year, John was half Anne's age. Now he is 13, how old is Anne?
3. Prime number, large as possible.
4. The water is just about to boil.
6. Earth's mean radius in miles.
7. Reciprocal of 1.1.
9. Log 2.048.

CLUES DOWN:

2. $(IXIIX)^2$.
3. Express the decimal number 25 in the binary scale.
5. tan 9° 36′.
6. $x - y = 15$, $2y + 3x = 9$. Find y.
8. $x + 114$ (See 6 down).

SOLUTIONS TO PROBLEMS IN ISSUE No. 38

A MONUMENTAL PROBLEM

AB was seven yards.

FUN WITH NUMBERS—No. 6

$1 = \frac{1+1}{1+1}$, $2 = \frac{1}{1} + \frac{1}{1}$, $3 = \frac{1}{1} + 1 + 1$, $4 = 1+1+1+1$, $5 = \frac{1}{.1} \div (1+1)$,

$6 = (1+1)\sqrt{\frac{1}{.1}}$, $7 = \frac{1}{.1} - 1 - 1$, $8 = \frac{1}{.1} - 1 - 1$, $9 = \frac{1}{.1} - \frac{1}{1}$,

$10 = \frac{1}{.1} + \frac{1}{1}$, $11 = 11 \times \frac{1}{1}$, $12 = 11 + \frac{1}{1}$

BEARING UP

Event (2) causes the ball-bearings to move further.

SENIOR CROSS FIGURE No. 38

ACROSS: (1) 131, (4) 113, (7) 1027026, (9) 940, (10) 1061208, (13) 12, (14) 18, (15) 176, (17) 120, (18) 15, (19) 13.

DOWN: (1) 110111, (2) 30, (3) 1296, (4) 1002, (5) 12, (6) 362880, (8) 741, (11) 0271, (12) 0123, (16) 65, (17) 11.

SITTING ON THE FENCE

The cost per mile is £125. Each property must be a mile square as this is the only rectangle with the same perimeter as the first property.

JIMMIE'S HOMEWORK BOOK

Problems are, tens and units, shillings and pence, pounds and ounces, stones and pounds, Pounds and shillings.

JUNIOR CROSS FIGURE No. 35

ACROSS: (1) 1625, (4) 567, (5) 91, (7) 53, (9) 21, (11) 121, (13) 516, (15) 12, (16) 7480.

DOWN: (1) 16, (2) 675, (3) 59, (4) 5025, (6) 1512, (8) 31, (10) 117, (12) 210, (14) 64.

TREAD SOFTLY

The top has moved 1 foot every 50 years out of true. Hence the next 20 feet will take another 1,000 years.

ALL ROUND

The farmer has not moved around the squirrel.

AM I ALL RIGHT JACK?

The bridge would collapse as the force required to maintain the two bowls in the air is greater than the weight of one wood.

FOOD FOR THOUGHT

$9867 \div 3289 = 3$.

CIRCUMFERENCE OF A CIRCLE IS TWO PIE R

Will the girl who submitted this cartoon please write to the editor as we have lost her address? B.A.

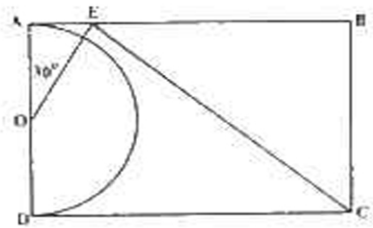
Continued from page 305

Crystallographers are concerned with patterns in three dimensions in which the motif is the group of atoms forming a molecule of a crystaline substances. There are exactly 230 possible patterns in three dimensions.

The analysis of point patterns, such as might be used for a medallion, of linear patterns, and of plane patterns was carried out by da Vinci in the 15th century. The analysis of three dimensional patterns was made by a number of crystallographers in the 19th century. C.V.G.

78111 96358 33005 94087 30681 21602 87649 62867

APPROXIMATE CIRCLE SQUARING



Draw a rectangle *ABCD* where the ratio *AB* to *BC* is as 3 : 2. Describe a semicircle with *AD* as diameter. From *O*, the centre of *AD*, draw a radius making an angle of 30° with *OA*. Let this radius meet *AB* at *E*; join *EC*. Show that *EC* is very nearly equal to the length of the semicircle *AED*.

J.F.H.

AN ODD FARMER'S PROBLEM

Is it possible to put nine pigs in four pens so that there is an odd number of pigs inside each of the four pens?

THOSE CAR NUMBERS AGAIN!

In Issue 36, we asked you to carry out an experiment by counting the number of cars which had 4 (say) as the last digit on their number plate in each batch of 10 cars which passed you.

Now two members of your Editorial Board have been indulging in a friendly wrangle about the figures we published in that issue. The point at stake is that there are two ways of calculating the figures according to which of two distributions we assume to fit the case. (For those of you who may have heard of them they are the Poisson and Binomial Distributions). Our two friends agree to differ about which is correct.

Here are the two sets of figures.

Score (*i.e.*, No. of 4's in batch of 10)		0	1	2	3	4 or more
According to Poisson	..No. of times	37	37	18	6	2
According to Binomial	..in 100 ..batches	35	39	19	6	1

Before our two friends resort to loaded Slide Rules at ten paces we want you to carry out an experiment to help them. Will you carry out a test on at least 100 batches of 10 cars (the more the merrier) and send the results of your test on a post-card to the Editor? Mark your P/C clearly CAR NOS. in the bottom left hand corner.

R.N.S.

STAMP COLLECTORS' CORNER No. 22

SIMON STEVIN (1548–1620), of Bruges, gave the first systematic treatment of decimal fractions and campaigned unsuccessfully for the introduction of a decimal system of weights and measures. In applied mathematics he studied problems of equilibrium and solved problems on bodies resting on inclined planes.

C.V.G.

Belgium 1942,
50c – 10c *fawn*.

HAPPY BIRTHDAY

A man was born in the nineteenth century. He was x years old in the year x^2. Find his age in 1875.

PAINTER'S PUZZLE

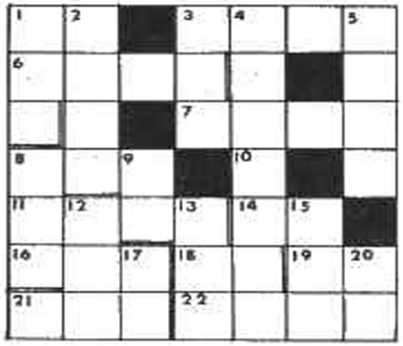
A paint tin weighs 5 lb. when half full and 4 lb. when it is one third full of paint. Find the weight of a full tin of paint.

DESERT VICTORY

An explorer wishes to cross a barren stretch of land which will take 6 days to cross. However he can only carry 4 days' supplies. He hires native bearers who can each carry 4 days' supplies. What is the smallest number of bearers that can make the trip possible?

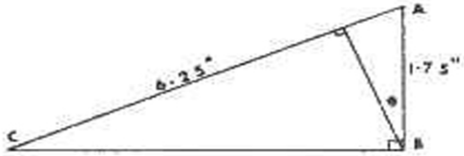
SENIOR CROSS-FIGURE No. 39

Submitted by J. G. B. Byatt-Smith, King Edward VI G.S., Totnes.



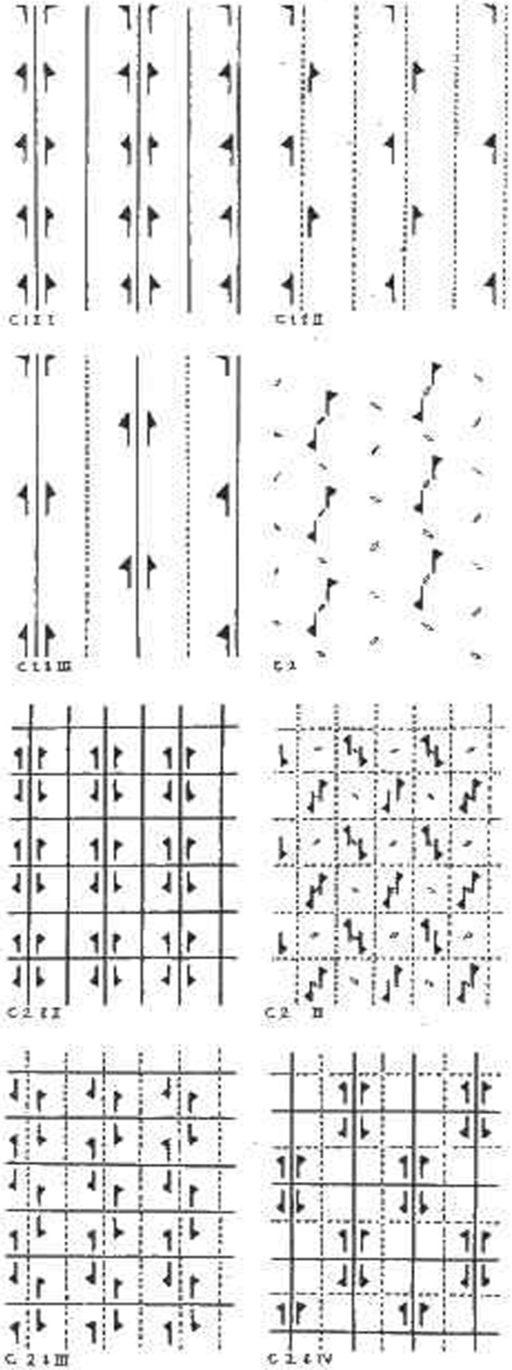
CLUES ACROSS:

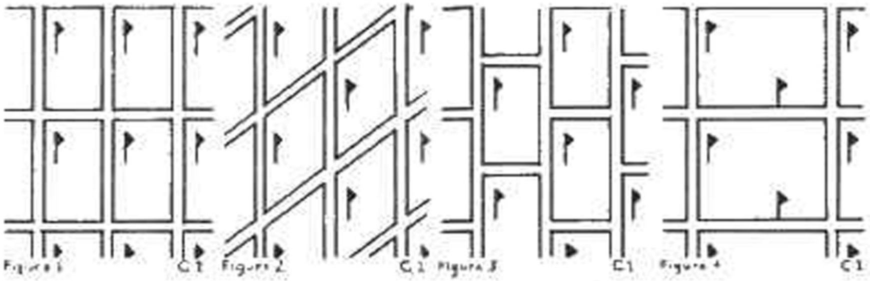
1. Minimum value of $11\left(x + \frac{1}{16x}\right)$
3. Sin 2θ, see figure.
6. Sum of the first 21 terms in the progression 25, 60, 95,
7. $3 \cdot 6! + 6$.
8. Product of two perfect numbers minus ten times the smaller.
11. Number of feet in a mile.
14. The square of the sum of the digits gives the number reversed.
16. A prime number whose digits are each perfect squares.
18. $25\frac{1}{4}$ miles per hour in knots.
19. $355\left[1 - \frac{1}{1+}\ \frac{1}{3+}\ \frac{2}{3-}\ \frac{1}{3+}\ \frac{2}{3}\right]$ continued fraction
21. An obtuse angle whose sine is one half.
22. Sum of the squares of the roots of $x^2 + 4x - 621 = 0$.



CLUES DOWN:

1. $2\ BC^2\ (AB + AC)$, see figure.
2. Sum of roots times the product of the roots of $x^2 - 13x + 450 = 0$.
3. Half the sum of the 4th, 6th, and 10th terms of 2, 4, 8, 16, ——.
4. Square of a prime number. The first two, and also the last two reversed, are perfect squares.
5. One twelfth of the coefficient of x^7 in $(1+x)^{20}$.
8. Area of the ellipse $x^2 + 100y^2 = 49$.
9. $a^3\ (3a+5)$ when a is the first prime number.
10. 17 down written to the base 4.
12. $x(x-1)(x+1) - 4$ when $x = 6$.
13. One tenth of the product of the partial fractions of $\frac{-40}{21x^2 - 26x - 11}$ when $x = 3$.
15. xyz when $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 64$ and $x^2 = \frac{4900 - y^2z^2}{y^2 + z^2}$
17. $\frac{1}{4}\pi$ radians in degrees.
20. Area between the graphs $y = 12x^2 - 12x + 1$, $y = 0$, $x = 1$, and $x = 3$.





Every pattern has a motif, which may be a group of flowers or figures, a scene or just a geometrical shape, and it has a system for the repetition of the motif. In the two patterns above the motif is a simple shape which is repeated by translation in two directions. Joining corresponding points of four adjacent repeats gives a parallelogram (or rectangle or square), called a unit cell, which contains the motif. These parallelograms together form the lattice of the pattern. Geometrically, rectangles and squares are included in parallelograms, therefore simple one-way patterns which are produced by translation alone are classed together as type C1. C is the symbol for a plane pattern, the suffix 1 indicates a one way pattern without axes of symmetry. Designers of fabrics and wallpaper usually prefer to deal with rectangles. Instead of the oblique grid of figure 2, they would generally consider this pattern as based on half drop rectangles. Metallurgists and crystallographers, who are interested in patterns of atoms and molecules, also prefer to deal with rectangular lattices whenever possible and might divide the pattern of figure 2 into double cells.

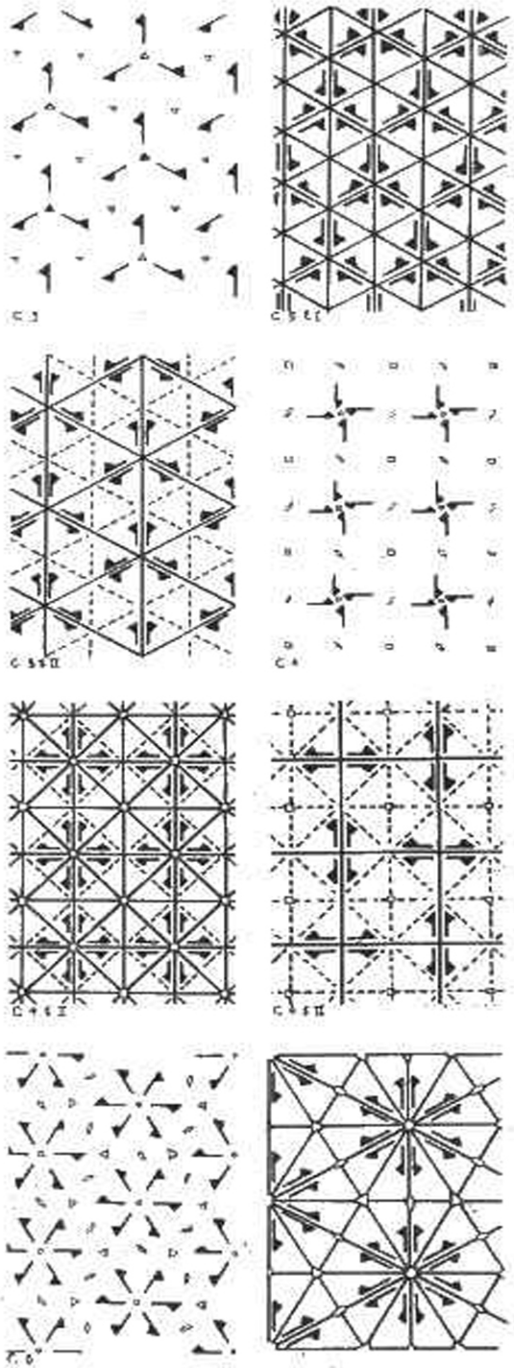
Every pattern must repeat by translation. A pattern can also repeat in three other ways :—(1) by reflection or reversal, (2) by a combination of reflection and translation called glide reflection, (3) by rotation through 180°, 120°, 90°, 60°. The symbols for an axis of reflection or symmetry, for a glide axis and for the four types of rotation are shown below.

If we add axes of symmetry to the basic one-way pattern we form three new types. (To show that there are axes of symmetry we add a suffix *l* to the type name). C1/I with two sets of axes of symmetry, C1/II with two sets of glide axes and C1/III with axes of symmetry and glide axes. By making a tracing of the simple pattern of figure 1, reversing the tracing and placing it over the figure patterns of these three types are produced.

Rotating a simple one-way pattern through 180° produces a simple two way pattern of type C2. In the figure the centres of rotation are marked by the symbol for 180° rotation. It is interesting to note that there are four centres of rotation in each unit cell. Adding reflection to the 180° rotation produces four more types :—C2/I with two sets of axes of symmetry at right angles, C2/II with two sets of glide axes at right angles, C2/III with axes of symmetry in one direction and perpendicular glide axes and type C2/IV with axes of symmetry and glide axes in each direction.

Axes of symmetry at 120°, 90°, and 60° produce patterns based on 60° rhombi and square lattices. The names of the various types are C3, C3/I, C3/II ; C4, C4/I, C4/II ; C6, C6/. This brings the number of types of plane patterns to seventeen and no more are possible.

Continued on page 307



TIME CHART—Concluded from page 313

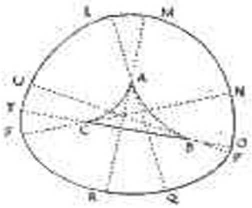
The scene now shifts to Baghdad to the Courts of the great Caliph Hârûn al Raschid (of 1,001 Nights fame) 786–809, and of his son Al-Mâmûn (809–833). Here mathematics was so encouraged that Baghdad may be likened to a second Alexandria. Mathematics came to Baghdad with the works of Brahmagupta in 766, but it was 20 years later when mathematical learning really began to flourish. The works of Euclid and Ptolemy were translated into Arabic and it is recorded that two geodetic surveys were carried out in Mesopotamia with the idea of calculating the meridian.

The greatest mathematician at this court was Mohammed ibn Mûsâ al-Khowarismi (d. c.840). He is best known for having written the first work bearing the name Algebra—" ilm *al-jabr* w'al mugabala "—" the science of reduction and cancellation "—this work later being translated into Latin by either Robert of Chester or Abelard of Bath under the title of " Algoritmi de numerico ludorum " whence our term " algorithum " for a device for carrying out a particular calculation.

The world owes a lot to the Arab scholars of Baghdad for preserving or passing on the classics of Greek and Hindu mathematics for much of it reached Europe via Baghdad. Little original thought was done in Europe at this period although a few great teachers such as Boethius (c.500), the Venerable Bede (c.700) and Alcuin of York (c.775) had their own followings. Much of the " mathematics " of the time consisted of propounding puzzle problems which, though interesting, did little to advance the main stream of mathematics.

R.M.S.

CURVES OF CONSTANT BREADTH *(Concluded from last issue)*



Curves of constant breadth need not be composed of circular arcs. The figure on the right is an example in which the basic figure is a ' triangle ' with two curved sides. The tangents to these curves at the vertices are shown as dotted lines. An arc with centre A cuts the tangents to BA and CA in L and M. Now imagine a line, which initially coincides with the tangent AM, rolling round the curve AC so that the point which was at M traces out the curve MN. The easiest way to do this is to use a block of wood shaped to fit the curve and unwrap from this block a string to which a pencil is fastened. A curve constructed in this way is called an involute. NO is an arc centre C, OP an arc centre B and PQ an involute of BA. Continuing in this way the curve is completed. If the sides of the basic figure had been tangential the closed curve would have been formed entirely of involutes.

The four curves described are composite curves made up of arcs of several simple curves. Non-composite curves of constant breadth other than circles, have rather complicated equations. The simplest is:

$$x=8\cos\theta+2\cos 2\theta-\cos 4\theta,\quad y=8\sin\theta-2\sin 2\theta-\sin 4\theta.$$

To plot the curve complete the table

θ	0°	10°	20°	. . . . . .	360°
x	9	8.99	8.88		9
y	0	0.06	0.54		0

and plot the values obtained for x and y.

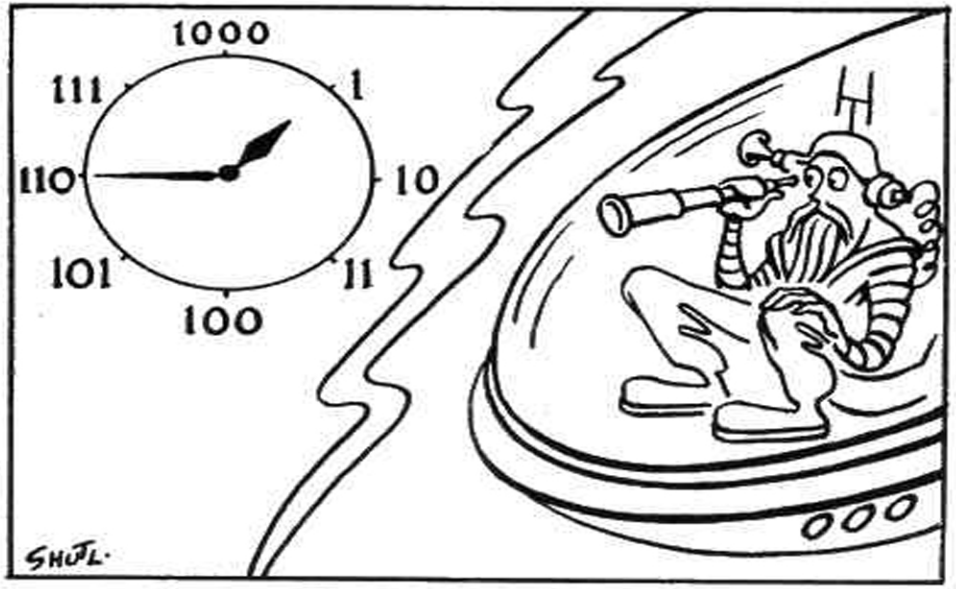
In 1917, an American took out patents for a tool which drilled a square hole. The basis of the drill was a curve of constant breadth.

C.V.G., B.A.

87925 30343 51398 44253 22341 57623 36106 42506

MATHEMATICAL PIE

No. 40 *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England OCTOBER, 1963



THE VENUS CLOCK

Professor Gottnowitz, just arrived from Earth, was circling the planet Venus looking for a suitable landing place. Through his telescope, he saw a tall tower with the rather strange sort of clock face shown in our illustration.

He found by observation that the planet's day was about 24 hours just like our own; he further noticed that both hands of the clock were in the position corresponding to Earth midnight when it was midnight at that particular part of the planet. The short hand apparently made one round of the dial in the Venus day, while the long hand made eight rounds.

When the professor first observed the clock, the hands were in the position shown; this we may call Venus time. Can you say what would have been the corresponding Earth time?

J.F.H.

SALUTE TO GHANA

Submitted by Mrs. I. E. Yelland, Achimoto School, Ghana.

On graph paper, place your axes so that x ranges from -3 to $+3$, and y from -2 to $+4$.

Now draw the following lines:

(i) $y=\frac{2}{3}x$ (ii) $y=-\frac{2}{3}x$ (iii) $y=2$ (iv) $y=4-3x$ (v) $y=4+3x$.

Take the same scale on the two axes.

19868 35938 14657 41268 04925 64879 85561 45372

A SLICE OF π

From time to time we get enquiries about the row of figures which forms a border to the bottom of each page of PIE. They are, in fact, part of an approximation to the value of π which most of you first meet as the ratio of the circumference to the diameter of a circle. This, however, is one of the least of its applications in Mathematics for it has a habit of popping up in the most unexpected places.

In a previous article on this topic (Issue 26, Feb., 1959—if your School has a Library file of PIE) we likened π to a spoonful of treacle with a thin streamer dangling from it. Since we can't wait all day for it to stop running we give the spoon a twist to cut it off short, thus getting an approximation to the true value.

Most people know the rough and ready value $\frac{22}{7}$ or $3\frac{1}{7}$. A better one is $3\frac{10}{71}$ and Archimedes, about 200 years B.C. knew that the true value lay somewhere between these two values which was a rather remarkable achievement in view of the clumsiness of Greek Arithmetic of that time. Mathematicians have, since then, tried to get better and better approximations to π, though it has long since been proved that it is impossible to get an exact value, π being a non-terminating and non-repetitive decimal.

About 100 years ago π was calculated to 707 places by William Shanks, taking 20 *years* to do so. Unfortunately he made an error after 530 places, but even this was no mean feat without the aid of calculating machines. It was the development of the electric computer after the last war which started a new chapter in the calculation of π. In 1949 the ENIAC computer calculated π to 2035 places in 80 *hours* machine time.

PIE began printing these figures along the bottom of the page beginning with Issue No. 18 (May 1956). Since 320 figures are printed per issue the ENIAC determination would have been used up by the end of Issue 24 (May 1958). Before this happened Mr. G. E. Felton, in 1958, using a Ferranti Pegasus computer took the calculation to 10,021 places in 33 *hours* machine time. He used the formula (attributed to Klingenstierna)

$$\frac{\pi}{4} = 8 \arctan \frac{1}{10} - \arctan \frac{1}{239} - 4 \arctan \frac{1}{515}$$

Each of the arctangents was evaluated using the formula devised by James Gregory (of telescope fame) :—

$$\arctan x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \ldots\ldots$$

And now we hear of a new calculation in America by Dr. Daniel Shanks and Dr. John W. Wrench Jr who, in 1961, have calculated π to 100,265 decimal places. The computation was done on an IBM 7090 installation in New York and took 8 hours 43 minutes time compared with the 20 years of Dr. Shanks' namesake. After the first calculation, a check run using a different formula confirmed the accuracy of the first as far as 70,695 decimal places and revealed a machine error which affected the rest. The calculation and checks have now been repeated to give results which agree perfectly to 100,265 decimal places.

The first calculation used Störmer's formula (1896) :—

$$\frac{\pi}{4} = 6 \arctan \left(\frac{1}{8}\right) + 2 \arctan \left(\frac{1}{57}\right) + \arctan \left(\frac{1}{239}\right)$$

and the check calculation used that of Gauss (c.1800) :—

$$\frac{\pi}{4} = 12 \arctan \left(\frac{1}{18}\right) + 8 \arctan \left(\frac{1}{57}\right) - 5 \arctan \left(\frac{1}{239}\right).$$

JUNIOR ALGEBRAIC CROSS DIAGRAM

1 8a	2	3	■
4b			■
■	5		6
■	7		

Ignore signs in the answers. One term is placed in each square. See 1 down.

CLUES ACROSS :

1. $2(6a - 5b + 4c) - 2(2a - 2b - c)$
4. $4b(1 + a)^2$.
5. $(a + b + c)^2 - 2ab - 2bc - 2ca$.
7. This is zero when $3bx = a(a - 4b)$.

CLUES DOWN :

1. Numerator of $\frac{b+8}{b} - \frac{a-4}{a}$
2. $(a + 5b)(a + 3b) - 3b(5b - x + 2)$.
3. Value of x when $\frac{1}{2}(x - b^2 - a^2) = 5c + 2a^2b$.
6. Area remaining when a rectangle, sides $2a$ and $2b$, is removed from a square of side c. J.G.

SCENE AT WIMBLEDON

In the final of the men's doubles it was observed that at one stage any of the six lines joining any two of the players was at right angles to the line joining the other two. Draw the diagram showing their positions and the position of the net. J.G.

SPECIALLY FOR RED INDIANS

There seem to be quite a large number of hairs in the average scalp, yet it has been stated that in London there must be more than one person with the same number of hairs per head. Suggest a method of testing this statement and make a rough estimate of the probable number if the statement were to be true.

(Suppose London has 10,000,000 inhabitants). J.F.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 39

In the Solutions : $7 = \frac{1}{.i} - 1 - 1$.

AN ODD RESULT

The number is 1. If it were not an integer, it could be any number between 1 and 1,001.

HAPPY BIRTHDAY

The man was 69 years of age. $43^2 = 1849$.

PAINTER'S PUZZLE

A full tin weighs 8 lb. 6 lb. of paint and 2 lb. for the tin.

DESERT VICTORY

The smallest number of bearers is 2.

SENIOR CROSS FIGURE No. 39

The clue to 12 down should have read : $x(x - 1)(x + 1) + 5$ when $x = 6$.

ACROSS : (1) 55, (3) 5376, (6) 7875, (7) 2166, (8) 108, (11) 5280, (14) 18, (16) 419, (18) 22, (19) 75, (21) 150, (22) 1258.

DOWN : (1) 576, (2) 5850, (3) 552, (4) 361, (5) 6464, (8) 154, (9) 88, (10) 1122, (12) 215, (13) 021, (15) 875, (17) 90, (20) 58.

AN ODD FARMER'S PROBLEM

Each of three pens contain 3 pigs and the fourth pen encloses the other three.

THOSE CAR NUMBERS AGAIN !

The editor thanks readers for their cards which are being analysed.

JUNIOR CROSS FIGURE No. 36

ACROSS : (1) 25, (3) 19, (4) 211, (6) 3960, (7) 909, (9) 3113.

DOWN : (2) 529, (3) 11001, (5) 1691, (6) 36, (8) 93.

CURVES OF CONSTANT BREADTH

In the fourth paragraph, line 5, after "P" insert "With centre B, radius BP describe an arc to cut BC produced in V." B.A.

MATH MAGIC

A Topological Twist

Take your handkerchief (preferably clean and hereinafter known as hankie for short, although a long one is better) and roll it to form a narrow bandage. Get one of your friends to hold out the first finger of his right hand. You wrap the hankie round his finger, tie in the first finger of the other hand, remove that finger and apparently pull the hankie straight through the original right forefinger.

Although the hankie appears to be tightly wrapped round both fingers, it is done in such a way as to leave the right finger *outside* the closed curve formed by the hankie. This closed curve can be made to a circular form once the left forefinger has been pulled out.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

The tying should be done in the following way :—

1. Hang the hankie over the right forefinger, Fig. 1, and cross it under the finger, Fig. 2, with the end *A* towards you at the crossing. *It is essential to keep the end* A *towards you every time the ends cross.*
2. Cross the ends above the finger, Fig. 3.
3. Get your friend to put his left forefinger on top of the crossing, Fig. 4.
4. Cross the ends above the left finger, Fig. 5. *N.B.—Take care.*
5. Cross the ends below the fingers, Fig. 6.
6. Bring the ends up and hold them together. Your friend's fingers now seem to be tied tightly together and the hankie cannot be pulled off. (It is not at this moment a simple curve, Fig. 7).
7. Grasp the tip of the right forefinger (lower one). Tell your friend to remove his other finger. (The curve has now become a simple one). By pulling the hankie upwards it will come away, apparently cutting the finger you hold.

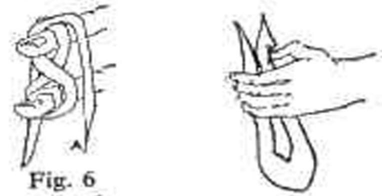


Fig. 6

Fig. 7

Fig. 8

This trick depends on being able, by removing the finger, to turn one curve with certain properties into another curve with different ones.

This study of the ways in which one curve may be 'deformed' into another belongs to an important branch of mathematics known as Topology. It is sometimes known as 'Rubber Sheet Geometry' and is a fascinating subject. It is of considerable practical importance, for instance in the design of knitting machines and looms and in map colouring. R.M.S.

75715 55278 19653 62132 39264 06160 13635 81559

PIE, therefore, will have no fear of running short of figures—this new calculation ought to last us until about the October Issue in the year 2060 at the present rate ! [For good measure, Shanks and Wrench have also calculated e to 100,265 decimal places and we can always fall back on this if we run out of π].

Note to non-Sixth Formers : e is another important universal constant like π and may be determined from the formula

$$e = 1 + \tfrac{1}{1!} + \tfrac{1}{2!} + \tfrac{1}{3!} + \tfrac{1}{4!} + \tfrac{1}{5!} + \dots\dots\dots$$

where $n! = n(n-1)\ (n-2)\ (n-3) \dots\dots 4 \cdot 3 \cdot 2 \cdot 1.$ R.M.S.

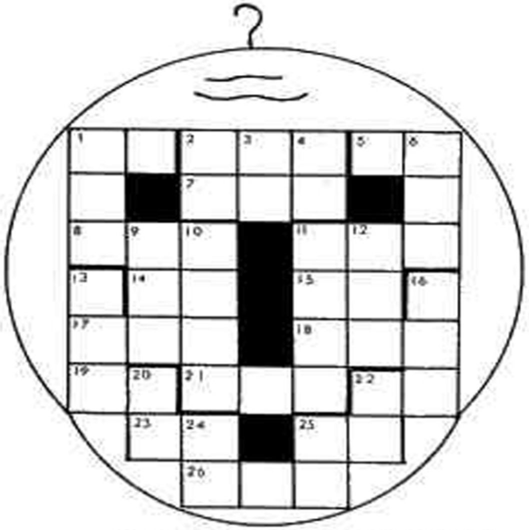
SIMPLE SOLITAIRE

Take a piece of pegboard with eleven holes along its length, and ten pegs, five of each of two colours. Place one set of five in the five holes at one end of the pegboard and the other set at the other end of the pegboard. The object of the game is to interchange the two sets of pegs by moving one at a time forward or by jumping over a peg of the other colour into an empty hole. The pegs must not be moved backwards.

What is the smallest number of moves to complete the game? B.A.

SENIOR CROSS FIGURE No. 40

Are you hard-boiled? This is for egg heads only.



CLUES ACROSS :

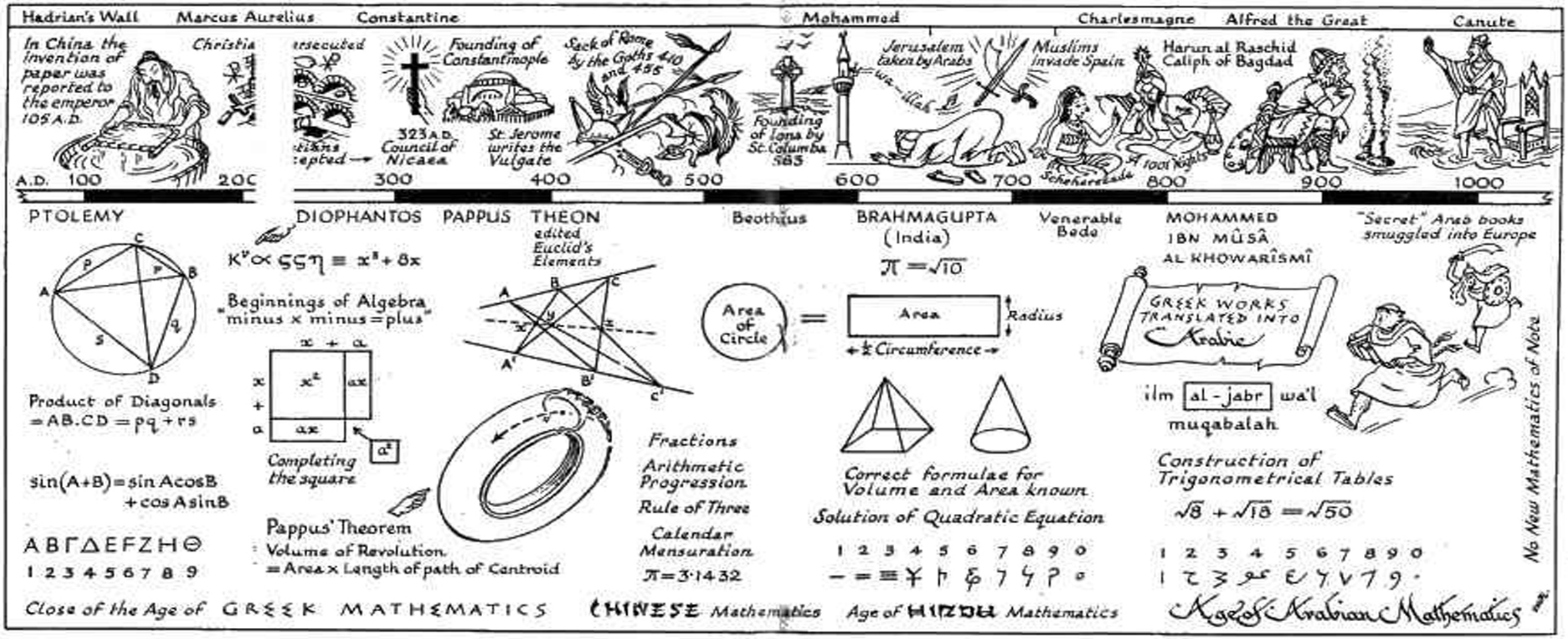
1. Ten times the age of the Egg (always given in months).
2. Number of good Eggs in 1 gross. (The digits of 13 down are no longer palindromic).
5. Half of 11 across, also HEN × age of the Egg.
7. Cube of the age of the Egg.
8. Three consecutive digits.
11. See 5 across.
14. 5 across + 2 down.
15. 2 HEN + one factor of 2 across. (See 22 down).
17. 7 across + 1.
18. 22 across reversed and squared.
19. Quarter of 1 across.
21. (Reversed) Cube of the other factor of 1 across . (See 2 down).
22. (HEN — 2)^2.
23. A multiple of 15 across + 1 is PRIME.
25. Twice 2 down.
26. 1 down divided by the age of the Egg.

CLUES DOWN :

1. 5 across has swallowed a DUCK.
2. One factor of 1 across.
3. HEN (Sorry?).
4. Twice 1 more than 2 down.
6. Sum of its digits is the square root of 22 across.
9. Internal Pressure of the Egg in mm. of mercury.
10. Surface area of Egg in sq. mm., also hat size of Egg Head (if you shift the decimal point).
11. HEN as seen by Picasso?
12. Time for which the Egg was boiled in seconds.
13. A square.
16. Square of the sum of the digits of 8 across.
20. Half of 2 across.
22. Age of the Egg × the other factor of 2 across. (See 15 across).
24. Its that HEN again.
25. 2 down reversed. R.M.S.

56387 29317 48723 32083 76011 23029 91136 79386

TIME CHART No. 4. AD 150 — 1000



This was not one of the great ages for epoch making discoveries in Mathematics—rather it was a period of consolidation. It was like that stage in the building of a house when the shell is complete and from outside nothing much seems to be happening though inside one finds a carpenter at work on a doorway here, an electrician fitting a socket there and a plumber at work up in the loft. Many different civilisations contributed to this consolidation, notably the Chinese, Hindus and Arabs.

The last two names of any note in Greek Mathematics were Diophantos (c.260) and Pappus (c.390). Diophantos wrote on the theory of Numbers, was able to solve linear, quadratic and some cubic equations. He is best known for his work on indeterminate equations, *i.e.*, equations with too many unknowns, so that it is possible to obtain a whole set of solutions (*e.g.*, $Ax - By = C$)—this type of equation is known as a Diophantine Equation to this day. These advances were due partly to an improved algebraic symbolism far beyond anything in use before his time.

Pappus was a geometer of note, writing about many subjects such as spirals and the set of figures having a given perimeter. He is chiefly remembered for his generalisation of the Theorem of Pythagoras and for the theorem usually attributed to Guldin, on the calculation of the volume generated by rotating any plane area about an axis in its plane.

We now move to China where the interest centres on Mensuration and Astronomy. The Chinese were able to handle fractions, knew of Arithmetic Progressions and indeterminate linear equations and produced a whole rash of "values" for π.

From China to India, where we find an interest in number and in calculation completely missing from Greek Mathematics. This was no doubt because the Hindu system of numerals was like our own, based on place value and it is certain that in the latter years of the period of this chart the Hindus were using zero for an empty column.

They were great astronomers and published astronomical and trigonometrical tables more accurate than any before, while *Vaharamihara* (505) left papers which included the calculation necessary for finding the position of a planet.

The most prominent of the Hindu Mathematicians was Brahmagupta (c.630) who could work with integers and fractions, could sum progressions, and knew Simple Proportion and Simple Interest. He took π as $\sqrt{10}$ and gave the area of a quadrilateral as $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semiperimeter, *i.e.*, $2s = a+b+c+d$, but this formula happens to be true only for cyclic quadrilaterals. He understood the use of negative numbers and gave a formula for the solution of the quadratic equation.

Continued on page 315

27089 43879 93620 16295 15413 37142 48928 30722

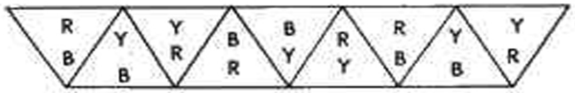
01269 01475 46684 76535 76164 77379 46752 00490

FLEXAGONS

(Submitted by Miss S. Wallis, Newcastle upon Tyne).

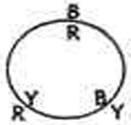
Flexagons are figures made by folding a strip of equilateral triangles into a hexagon and joining the ends in such a way that if the triangles are coloured in a certain manner, the model can be continuously folded and unfolded to exhibit these different colours in turn.

The simplest flexagon is constructed from nine triangles giving eighteen faces, six of which are coloured green, six red and six yellow. A colouring chart for the strip is shown.

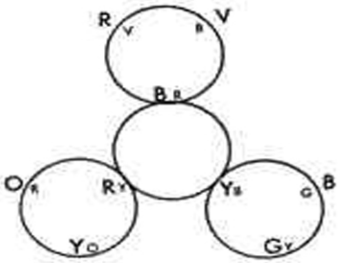


Y = yellow B = blue R = red Front face colour at the top

The strip is folded by giving it three half twists, hiding all faces of one of the colours and the two ends are stuck together. A plan to show the colour changes when the flexagon is operated is given, and as each operation can be reversed, the circular plan can be followed in either direction.



The second flexagon is made from eighteen triangles, coloured with the three primary colours, red, yellow and blue, and in addition three secondary colours, green, violet and orange. When the strip has been coloured, it is given nine half twists to hide all the secondary colours so that it then looks like the first strip shown above, and is completed in the same way. Here the plan of operation has a primary cycle and three secondary cycles branching from it, each containing two of the primary colours with their associated secondary colour. The directions of operation are again shown and it is seen that one cannot from one secondary produce another without first turning back to the central primary cycle.



The third flexagon is a development of the second, as the second is of the first. It consists of 36 triangles, but instead of introducing further colours, the extra triangles have been coloured to produce hexagons of two of the already existing colours in an alternating pattern.

This family of flexagons can be extended, theoretically at least, the difficulty in actual construction and manipulation increases with the size, thin cardboard must be used and a sufficient gap left between each triangle to enable the flexagon to fold.

Note :—there are other types of flexagons which are constructed from triangles not arranged in a straight row so that the plan may take different forms, some of which will be shown in a future issue.

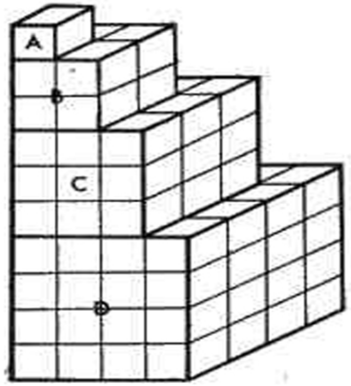
26749 16315 70591 96749 53740 06082 67999 43265

MATHEMATICAL PIE

No. 41 *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England FEBRUARY, 1964

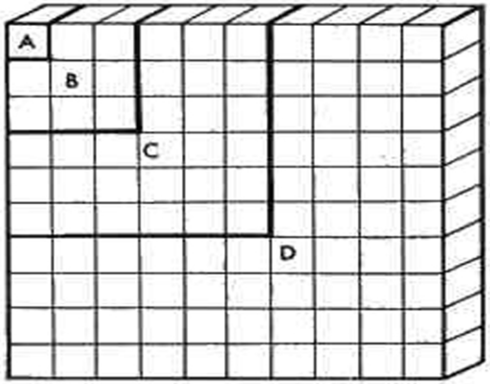
CUBES AND SQUARES

If you have patience and a plentiful supply of sugar cubes, it is easy to build up the structure shown in Figure 1. It consists as you can see of a succession of cubes, the top one having one unit side, the next one two units, the succeeding one three units and the lowest one four units in each of its sides, or edges.



The number of unit cubes (lumps of sugar) contained in each of the successive cubes increases rapidly. In the bottom one these are $4^3 = 64$ units. If the structure were continued until a cube of side 10 units formed the 'foundation' cube, this one would contain 1,000 unit cubes; the number in the whole structure would then be very large, more than 3,000. Figure 1, which reminds us perhaps of some modern architecture, is now to be demolished, but not by an earthquake. It is rearranged systematically, to form a kind of pavement, one unit thick on the ground. Figure 2.

The first cube (1 unit edge) is placed in the position A. The 8 elements of the next cube are arranged round A and will form a square B of 3 units side, for $1 + 8 = 9 = 3^2$. The single cubes of the next cube (27 of them) are placed as at C, and you will see from Figure 2 that they can be laid to complete another square of side 6, for $1 + 8 + 27 = 36 = 6^2$. Similarly the last cube, containing 64 units, placed at D will increase the existing square to another with side 10, for $1 + 8 + 27 + 64 = 100 = 10^2$.



Now when we rearrange the cubes in each instance, they form a square. Thus $1^3 + 2^3 = (1 + 2)^2$; $1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$; $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$. It appears that this is a property of numbers. You can test it quite easily. You will find that $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 225$.

Assuming the rule suggested by the foregoing, what is the sum of the cubes of the numbers from 1 to 10? Generalise this result for the numbers from 1 to n.

J.G.

39049 75008 65627 10953 59194 65897 51413 10348

THE HISTORY OF NUMERALS

Primitive people very rarely needed to use large numbers ; an analysis of 30 different tribes of Australian Bushmen shews that none of them went beyond 4, and their counting was of the form 1, 2, many ; 1, 2, 3, heaps ; or 1, 2, 3, 4, lots ; so that their methods of writing them would have been very similar to our illustrations in Issue No. 27 and Issue No. 30.

AFRICAN PIGMIES

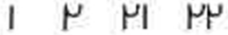
A, OA, UA, OA-OA, OA-OA-A, OA-OA-OA

A TRIBE IN THE TORRES STRAITS

URAPUN, OKOSA, OKOSA-URAPUN, OKOSA-OKOSA, OKOSA-OKOSA-URAPUN, OKOSA-OKOSA-OKOSA, RAS

Fig. 1

1 2 3 4 **ANCIENT SYRIAC**



HINDU PROBABLY 1ST CENTURY B.C.

1 2 3 4 5 6 8

I II III X IX IIX XX

Fig. 2

WHAT WE SOMETIMES USE NOW

1 2 3 4 5 6 7 8

| || ||| |||| ~~||||~~ ~~||||~~| ~~||||~~|| ~~||||~~|||

As races developed their mathematical ideas improvements could not take place without improvements in their written number systems. Such improvements were first reflected in the spoken word as shown in Figure 1 and it will be seen that they are counting in groups of two or three rather as we count in groups of 10.

This counting in groups formed a basis for writing the abstract numerals and some of the earliest examples are shown in Figure 2.

R.H.C.

ENGLISH BY ARITHMETIC

Ever thought of assembling a dictionary ?—the easiest way is to call in a computer. But computers, you will say, use numbers, not words—and binary numbers at that. All right then, all we have to do is translate the words which are to go into the dictionary into numbers, and the computer will do the rest, *viz.* : put them into alphabetical order. Actually, we can do this in two stages, first translate words into ordinary (decimal) numbers and then we (or the teleprinter) can easily translate from there into binary numbers.

Suppose we give the letters *A* to *Z* a number apiece starting from 01 and going up to 26.

Then, for *A* we read 01,
for *B* we read 02,

and so on up to *Z* which is 26.

Then the word Art would be written 01 18 20.
01 for *A*, 18 for *R* and 20 for *T*
(translate Maths and 160905).

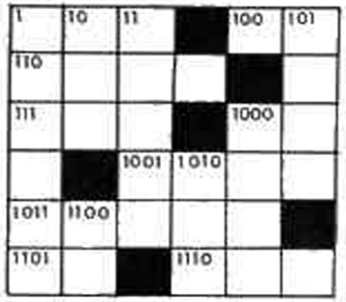
Unfortunately, words may be of different lengths—2, 3, 4, 5, 6 or more letters, and these would lead to 4, 6, 8, 10 and 12 digit numbers which would be difficult to compare.

However, there is an easy way round this. All we have to do is put a decimal point at the beginning of each word. Thus, Art now becomes .011820.

22769 30624 74353 63256 91607 81547 81811 52843

BINARY CROSS FIGURE

(*Submitted by R. G. Everett, Lincoln*).



CLUES ACROSS :

1\. $1011011 \div 1101$.
100\. $\sqrt{1001}$.
110\. $11\,! + 11$.
111\. $100011 \div 111$.
1000\. $110010 \div 11001$.
1001\. $(10 \times 11\,!) + 1$.
1011\. $\dfrac{101\,! \div 101}{110}$
1101\. $\sqrt{\dfrac{100\,!}{11\,!}}$
1110\. $\sqrt{11001} - 1$.

CLUES DOWN :

1\. £1 — 10011s. — 100d. $\div$ 1000. Answer in pence.
10\. Maximum value of $(10 - x)(10 + x)$.
11\. $\dfrac{1{,}000{,}000 - 1}{11} + 1$.
101\. $\sqrt{10101001}$
1000\. (H.C.F. of 110, 1010, 1100) $\times$ 101
1010\. $\sqrt[11]{1111101}$
1100\. $\sqrt[110]{\dfrac{101\,!}{10} + 100}$

The clues are given in the binary notation. Answers to be given in the binary notation.

SOLUTIONS TO PROBLEMS IN ISSUE No. 40

THE VENUS CLOCK

The time shown on the clock was 2.15 a.m.

SALUTE TO GHANA

The graph produced the Ghana star.

SIMPLE SOLITAIRE

Label the pegs B1, B2, B3, B4, B5, W1, W2, W3, W4, W5 from the centre. Move B1, jump over B1 with W1 and advance B2 ; jump over B2 with W1, jump over B1 with W2 and advance W3, and so on. The number of moves required is 35. If n pegs of each colour and $(2n+1)$ holes are used, the number of moves is $n(n+2)$.

SENIOR CROSS FIGURE No. 40

CLUES ACROSS : (1) 60, (2) 112, (5) 66, (7) 216, (8) 678, (11) 132, (14) 78, (15) 30 (17) 217, (18) 224, (19) 15, (21) 521, (22) 81, (23) 61, (25) 24, (26) 101.

CLUES DOWN : (1) 606, (2) 12, (3) 13, (4) 26, (6) 612, (9) 771, (10) 8875, (11) 1321, (12) 302, (13) 121, (16) 441, (20) 56, (22) 84, (24) 11, (25) 21.

JUNIOR ALGEBRAIC CROSS DIAGRAM

CLUES ACROSS : (1) $8a$, $6b$, $10c$, (4) $4b$, $8ab$, $4a^2b$, (5) a^2, b^2, c^2, (7) $3bx$, a^2, $4ab$.
CLUES DOWN : (1) $8a$, $4b$, (2) $6b$, $8ab$, a^2, $3bx$, (3) $10c$, $4a^2b$, b^2, a^2, (6) c^2, $4ab$.

SCENE AT WIMBLEDON

Three of the men form the vertices of a triangle and the fourth must stand at the point of intersection of the altitudes of the triangle ; the net may be drawn anywhere so as to pair off the players.

SPECIALLY FOR RED INDIANS

To test the statement given, find the maximum number of hairs on a person's head. This will be less than 10,000,000 ; Suppose it is 1,000,000. Take 1,000,001 people in London, then these may have a different number of hairs on their heads from 0 to 1,000,000. The next person considered must also have 0 to 1,000,000 hairs on his head and therefore the same as one of the first million and one considered.

To find how many hairs a person has on his head, take a unit square of scalp, count the number of hairs and multiply this by the number of square units of scalp.

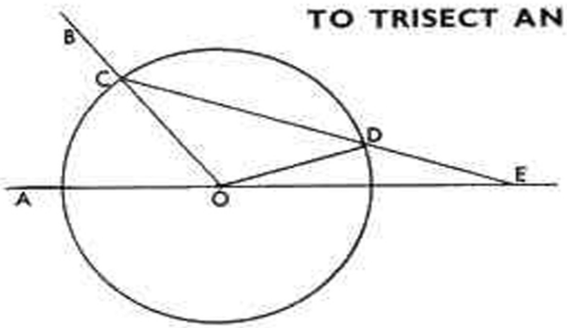
B.A.

68170 93804 47058 44802 93734 04320 85948 11872

Philosophy (Science) is written in that great book, ... (the Universe); but we cannot understand it if we do not first learn the language, and comprehend the characters in which it is written. It is written in the mathematical language and its characters are triangles, circles, and other geometrical figures.

Galileo Galilei 1610. J.G.

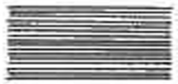
TO TRISECT AN ANGLE



To trisect the angle AOB, draw a circle, centre O to cut OB at C. Produce AO. From C, draw a line CDE so that DE = the radius of the circle. Then $\angle DOE = \frac{1}{3} \angle AOB$.

J.F.H.

A CARD TRICK



A pack of cards stacked as a rectangle is displaced as shown.

Therefore the area of a parallelogram is the product of the base and the height. J.G.

OBSTRUCTION

In the course of making a relief map, a student got as far as shown in the Figure. It represents a hill on a plain.

The line AB represents a map coordinate and the student discovered, after he had stuck the model hill in place, that he should have continued the line AB on the other side of the hill. The only ruler that he had was a single-sided straight-edge and, of course, he could not lay this over the hill. Find a method of drawing the required line A^1B^1 without crossing the hill. J.F.H.

DARTS

Two men play darts, one to score 301 all in doubles, and the other to score 1,001 under normal rules. Who has the better chance of winning?

In which scale of notation would it be worthwhile taking a bet? R.H.C.

TV TIMES

In a current advertisement for Pye Electronics, the following statement appears: $\pi \times 625 = 1964$.

How nearly true is this statement? R.H.C.

67286 80336 99136 76363 17847 04533 36146 92473

Now let us see what happens if we translate the, their, there, and put them in alphabetical order, we have

the	.200805.
their	.2008050918
there	.2008051805

Thus, if the words are in alphabetical order, the corresponding numbers are in increasing order of magnitude — and the fact that some words are longer than others doesn't matter any more to the computer. If the computer is asked to put two words in alphabetical order it simply subtracts the first from the second. If the answer is positive then the words are not in alphabetical order; if it is negative then they are.

In this way it can order all the words we feed into it — and out come the words for a dictionary, in alphabetical order. K.A.

ALGEBRA CORNER

1. Verify that the difference between the cost of a lb. b oz. of meat at x shillings y pence per lb. and x lb. y oz. at a shillings b pence per lb. is $ay - bx$ farthings.

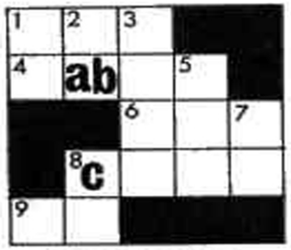
2. Invent a simple algebraic expression which is such that when it is divided by a, the remainder is b, and when it is divided by b, the remainder is c.

3. Simplify $\dfrac{0.a + 0.b + 0.c}{0.abc}$ and $\dfrac{0.a + 0.b + 0.c}{0.abc}$ J.G.

FIVERS

A man dies leaving £11,617 to be divided amongst his relatives. He stipulated that the money was to be distributed in single pounds or whole numbers of pounds which are powers of five. He further stipulated that the numbers of pounds was not to be given to more than four persons. How same sum of money was not to be given to more than four persons. How many relatives had he and how much did each receive? R.M.S.

SENIOR ALGEBRAIC CROSS DIAGRAM



CLUES ACROSS:

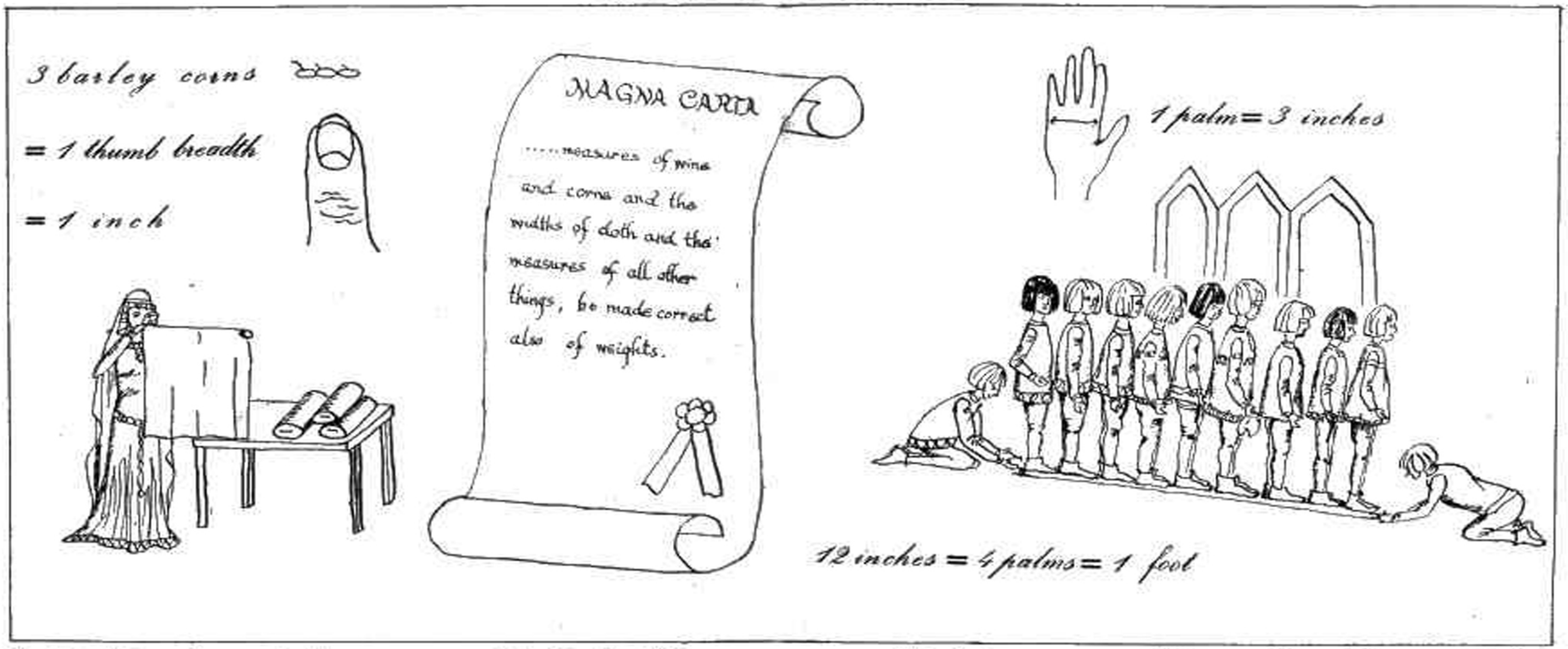
1. $\dfrac{(ax+b)^2}{a} - \dfrac{b^2 - ac}{a}$
4. $(ax - 3c)(a - c) + ac(x+3) + a(a^2+b)$.
6. What must be taken from $4c^3 + 2b^3$ to give $b^3 + c^3 - a^2$.
8. $\dfrac{c(c^4 - 1)}{(c - 1)}$
9. The square root of $b^4 - 8acb^2 + 16a^2c^2$.

CLUES DOWN:

1. Divide $ax^3 - a^3x$ by $x - a$.
2. The numerator of $\dfrac{x}{x-b} - \dfrac{x+a}{x+a+b}$ when written as a single fraction.
3. Expand $c(1+c)^3$.
5. $(a+b+c)(a^2+b^2+c^2 - ab - bc - ca) + 3abc$.
7. $ac\left(\dfrac{a}{c} + \dfrac{c}{a}\right)$
8. x when $\dfrac{x}{c} + 4a = 1$.

Ignore signs and place one term of each answer in each box. J.G.

66795 70611 08615 33150 44521 27473 92454 49454



The first of this series on units of measurement, published in May 1958, described how the early civilisations in the Mediterranean countries introduced units of length for the measures that were required and how some of these units were eventually standardised. Such units as the cubit, span, etc. continued in use for many hundreds of years and were introduced to new countries as culture spread throughout Europe.

However, although the Egyptians had realised the need for standard units of measure as early as the time of the building of the pyramids, the measures used in Europe in, for example, Anglo-Saxon times were as unreliable as any of the early cubits had been. As new units were introduced so was the pattern of history repeated. The yards, feet and inches we use today came from different body measurements just as did their predecessors the cubit, span, palm, etc. and consequently depended on the size of the person making the measurement.

The inch was measured in either of two ways. It could be the length of three full grown barley corns placed end to end or the breadth of the thumb nail measured at the base of the nail. If you make this last measurement on yourselves and compare with your friends you will see how much the different "inches" can vary. Another measure you can check is the palm, which was given as three inches. This is simply the breadth of your hand measured across the palm.

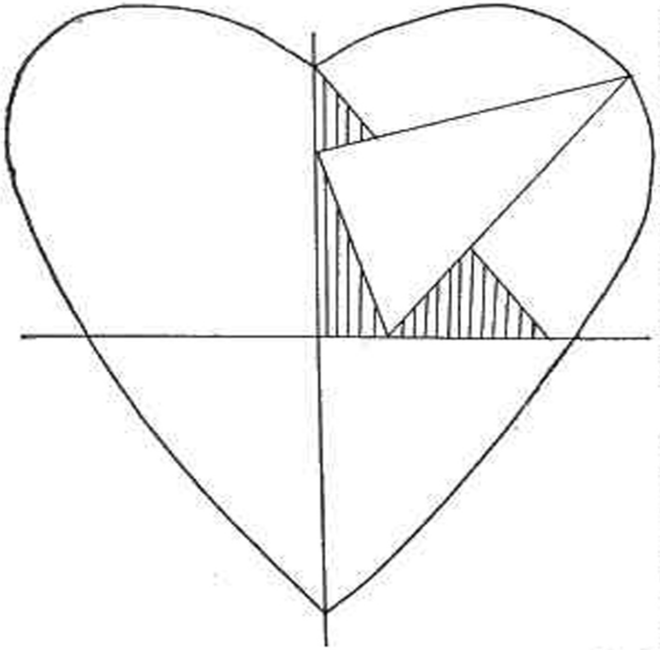
The foot as a measure probably spread through Europe via the Romans who had originally adopted it from the Greeks. This again obviously varied very much from person to person and was eventually linked with the existing measures in England by Henry I. The story is that a number of men were lined up so that each man's toes touched the heels of the man in front and from the total length was calculated the average foot length. As this came to be twelve inches they either had large feet or small inches !

The yard was also in use in the time of Henry I and was most used for the measurement of cloth. You have probably seen cloth measured approximately in just the way it was measured by the merchants of the 11th century by using the distance between the tip of the nose and the end of the outstretched arm. Naturally such a rough method of measurement led to a certain amount of cheating on the part of the merchants and this was one of the reasons for the clause in Magna Carta stating that measures must be made correct.

However, it was not until the reign of Edward III that standard measures were created by law, which stated ". . . . we will, and ordain, that one weight, one measure and one yard be used throughout all the land."

Today one can still see, in the Standards Office, the standard yard instituted by Henry VII. This is an eight sided brass bar, one side of which is divided into three feet with one of them marked in inches ; and another side marked to show $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ of a yard. I.L.C

WITH LOVE FROM ME TO YOU



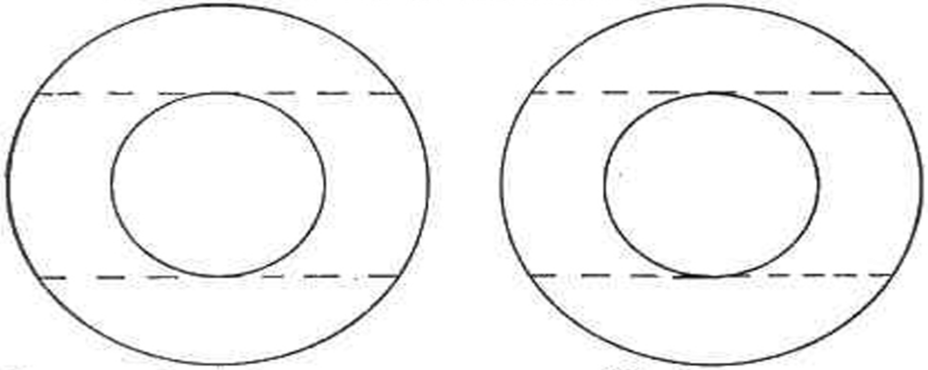
When anything moves, any point on it traces out a path which may be called a graph or a locus. It is usually invisible but can be represented in a diagram. A triangle is often connected with love and the traditional symbol of love is traced out when a right-angled triangle moves in a certain way.

On a fairly large sheet of paper, draw axes at right angles to each other. Cut out a right-angled triangle from cardboard—a 3, 4, 5 triangle is a suitable shape—lay it with the right angled vertex and the vertex where the shorter side meets the hypotenuse on the two axes and trace out the locus of the third vertex.

Every curve can be represented by an equation and the heart-shaped locus of the eternal triangle can be written $y=\frac{ax}{b}\pm\sqrt{(b^2-x^2)}$, where a is the length of the shorter side and b is the length of the longer side of the original triangle.

J.G.

POINTS OF VIEW (*Adapted from Le Facteur X*)



Given *Front view* *Side view*

Required :—the plan view of the body.

J.F.H.

99771 52515 64682 33580 31653 63270 64184 48135

MATHEMATICAL PIE

No. 42 *Editorial Address :* **100, Burman Rd., Shirley, Solihull, Warwicks, England** **MAY, 1964**

THE WANKEL ENGINE

If a circle of radius a rolls without slipping round a fixed circle of radius $2a$, a point on the rim of the rolling circle follows a curve called an epicycloid. Exactly the same curve can be generated by rolling a hollow circle of radius $3a$ round a fixed circle of radius $2a$, like a large ring rolling round a slender finger. This alternative construction is shown by the dotted lines in Fig. 1.

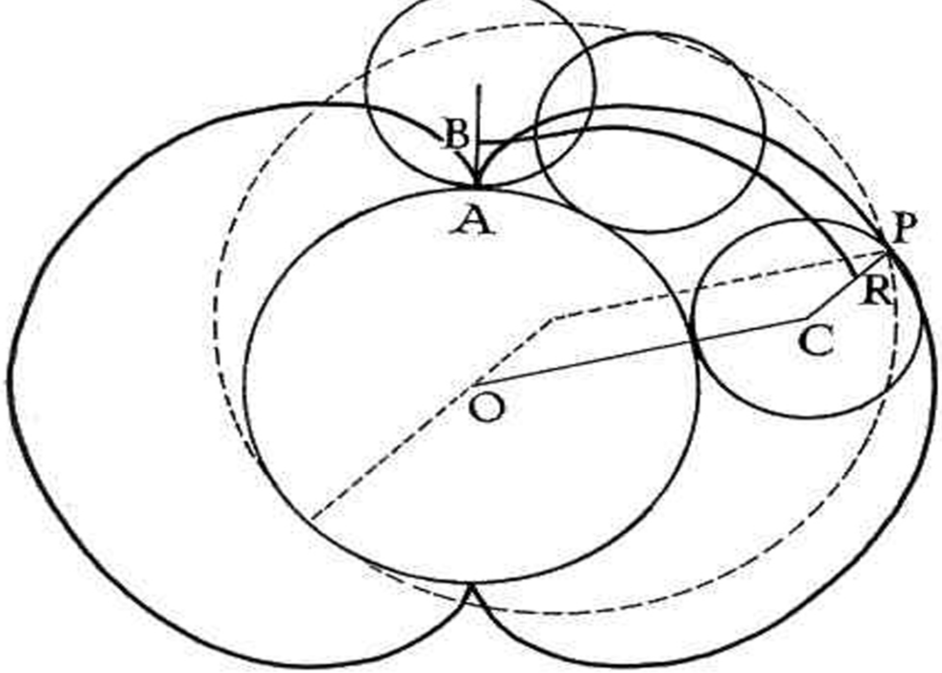


Figure 1

Any student of *A* level mathematics can prove the peculiar property of this particular curve which is that an equilateral triangle of side $3\sqrt{3}a$ can be rotated with all three vertices on the curve whilst its centroid moves round a circle of radius a. Although the triangle is inscribed in the curve, we cannot say that it moves round inside the curve because its sides cut across the cusps (pointed parts) of the curve, as in Fig. 2.

If we look again at the rolling circle and consider a point which is not on the rim, but at some smaller distance—say b—from its centre, we find

05527 23231 43898 04477 85279 06769 03700 23206

that it moves round a curve which has two 'dimples' instead of two cusps. If the point is sufficiently near the centre of the circle the dimples become very small and we obtain a curve—an epitrochoid—inside which the triangle can turn without its sides cutting across the curve.

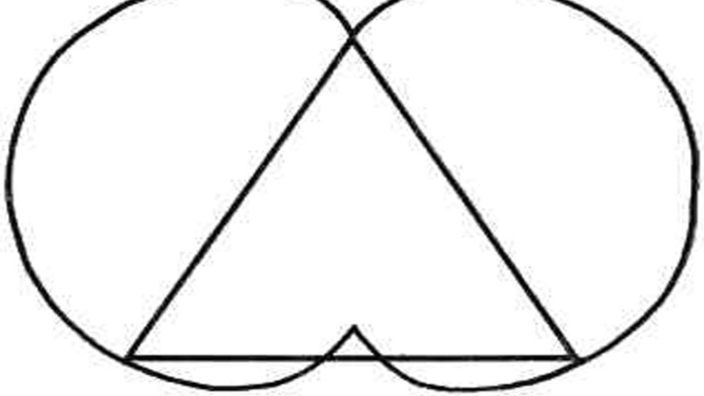


Figure 2

A chamber with a cross-section of this shape and a rotor having a cross-section that is triangular, with curved sides, Fig. 3, are used in the revolutionary Wankel engine which has been successful in experimental cars and may soon be in full scale production.

The three spaces between the rotor and the chamber take the place of the cylinders of the piston engine. In the piston engine the four 'strokes' of the cycle take place in each cylinder in turn. Air-petrol mixture is drawn in, induction, through the inlet valve, compressed, ignited by the spark and expanded, then exhausted, but in the Wankel engine these four strokes take place in different parts of the chamber.

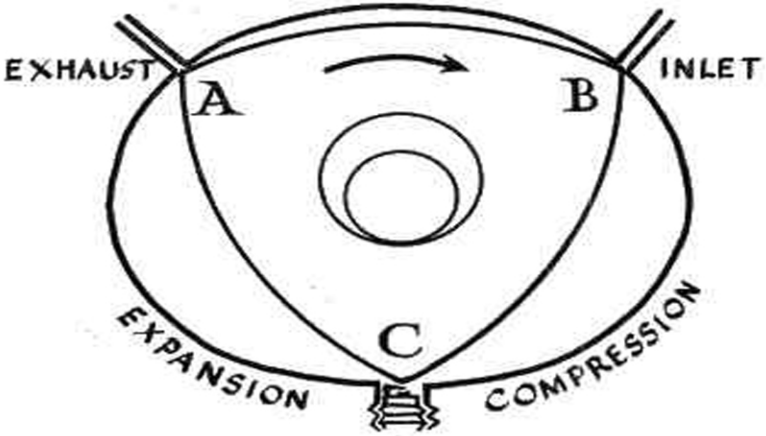
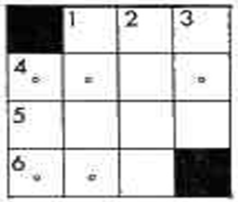


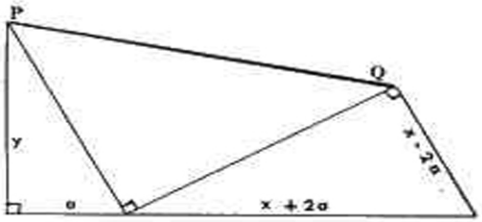
Figure 3

Trace the outline of the rotor and place your tracing over the chamber in Fig. 3.

In the diagram the space on AB is at the start of the cycle. As the rotor turns the space increases and air-petrol mixture is drawn through the inlet port. When AB is vertical A passes over the inlet, which is then connected to the next space, and compression begins. When AB is horizontal at the bottom of the chamber, the mixture is fully compressed, then ignited by the spark, it expands and drives the rotor until B passes over the exhaust port. As AB turns into the horizontal the space decreases, the spent gas is expelled through the exhaust port and the cycle is completed. Each space passes through the complete

JUNIOR CROSS DIAGRAM No. 2





HINT: Terms are usually in alphabetical order. Small circles indicate ± squared terms.

CLUES ACROSS:

1. $(c-d)-4(a+b)-2(a+b)-(c-5d)$.
4. $(y+a-b)(y-a+b)$.
5. Area of a rectangle; adjacent sides $2(a+b)$ and $2(x+y)$.
6. $(a-bc)^2-b^2(c-1)(c+1)$.

CLUES DOWN:

1. Numerator when $\dfrac{b^2-a^2}{a}+4y-6$ is written as a single fraction.
2. Multiply the difference of $(a+2x)$ and $(ac+3)$ by $2b$ and rearrange.
3. Divide $6dx^2-x(b^2x-4bxy)$ by $(-x^2)$.
4. PQ^2 in the figure. J.G.

"Who's the Square on the Hypotenuse?"

DROP A BRICK?

(*Adapted from Le Facteur X*)

Johnny was playing with his model boat in the bath. As cargo for the boat, he had a half-brick. After a time, he took the piece of brick out of the boat and put it at the bottom of the bath. Does the level of the water in the bath rise, fall, or remain the same? J.F.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 41

FIVERS

11,617 in the decimal scale equals 332,132 in the scale of five. Hence two received £1 each, three £5, one £25, two £125, three £625, and three £3,125. Number of relatives is 14.

SENIOR ALGEBRAIC CROSS DIAGRAM

CLUES ACROSS: (1) ax^2, $2bx$, c; (4) a^2x, ab, $3c^2$, a^2; (6) $3c^3$, b^3, a^3; (8) c, c^4, c^3, c^2; (9) b, $4ac$.

CLUES DOWN: (1) ax^2, a^2x; (2) $2bx$, ab; (3) c, 3^2c, $3c^3$, c^4; (5) a^3, b^3, c^3; (7) a^2, c^2; (8) c, $4ac$.

OBSTRUCTION

See next issue.

DARTS

The first man cannot win as 301 is an odd number. He could win if the scale were to an odd base *e.g.*, 5.

T.V. TIMES

$\pi \times 625$ is more accurately 1963.5.

BINARY CROSS FIGURE

CLUES ACROSS: (1) 111, (100) 11, (110) 1001, (111) 101, (1000) 10, (1001) 1101, (1011) 11001, (1101) 10, (1110) 100.

CLUES DOWN: (1) 111011, (10) 100, (11) 10110, (101) 1101, (1000) 1010, (1010) 101, (1100) 10.

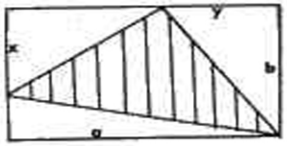
ALGEBRA CORNER

3. The addition signs in the second expression should be multiplication signs. B.A.

is a member of two great families of curves : the caustic curves (formed by reflection) and the epicycloids (formed by circles rolling on circles).

Jean's son, Daniel, developed the theory of Wave Motion from the vibration of stretched strings. This is applicable to many problems from musical instruments to electron microscopes and the design of television aerials. On his work on Fluid Dynamics is based the design of the modern jet engine and indeed the aircraft itself owes much to Bernoulli's Theorem. We shall meet Daniel Bernoulli again in our next Time Chart. R.M.S.

ALGEBRA CORNER No. 2



1. What profit % is made by buying articles at x for y pence and selling at y for x pence ?
2. What is the area of the shaded triangle ? The answer contains 3 terms.

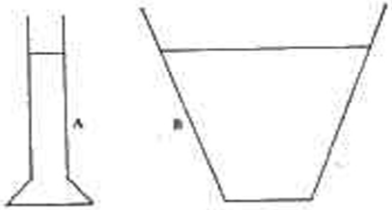
y^2-4ax	$a+bx$	$a-b$
h^2-ab	$ax^2+2hxy+by^2$	$\frac{c}{a\ b}$
g^2-bc	$a+b$	$ax+hy+g$

$\equiv$

4	17	1
29	637	1
1	9	80

3. The diagrams show an algebraic cross diagram and its solution. Find the values of a, b, c, x, y, g and h.
4. In a race of x yards, A beats B by b yards and C by c yards. If B and C only ran the race, by how many yards would B beat C ? J.G.

A WEIGHTY PROBLEM



The diagram shews two vessels filled to the same depth with water. The area of the base of vessel A is the same as that of vessel B and vessel B holds 100 times as much water as vessel A. Which base has the greater load on it ? R.H.C.

MATHEMATICAL JUGGLER

A man carrying 3 bowling balls came to a bridge that would only carry his weight and one ball at a time, so he decided to juggle the balls as he crossed so that 2 are always in the air. Smart ?

THE AVERAGE FAMILY

The average family is composed of 1 underpaid male, 1 overworked female, and 2·2 underfed children. *(From Moroney : Facts from Figures)*

FROM A MATCHBOX

The Right Angle for tackling problems is the TRY ANGLE.

26193 36416 92936 97783 37178 40550 29073 62691

cycle once in every revolution of the rotor.

To ensure smooth operation of the rotor it has on one side a circular recess with internal teeth which engage with a stationary gear of two-thirds the diameter, see Fig. 4. On the other side the 'centre' of the rotor, which moves twice round the circumference of a small circle in every revolution of the rotor, is connected by a crank to the output shaft.

C.V.G.

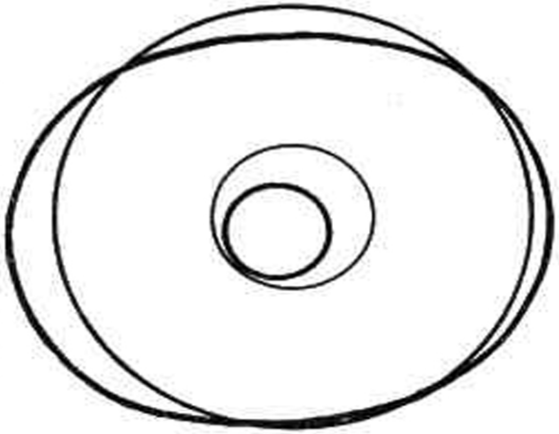


Figure 4

" I make it 22 revs per minute "

FUN WITH NUMBERS No. 7

(Suggested by Miss Margaret M. Gow, Sedgley Park College, Prestwich, Manchester).

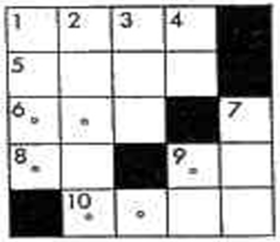
$4^2=16$ and $17^2-15^2=(2\times4)^2$

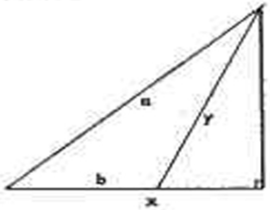
$5^2=25$ and $26^2-24^2=(2\times5)^2$

Generalise these statements and verify that the generalised result is true.

SENIOR CROSS DIAGRAM No. 2

HINT for arrangement ; the small circles represent squares.





CLUES ACROSS :

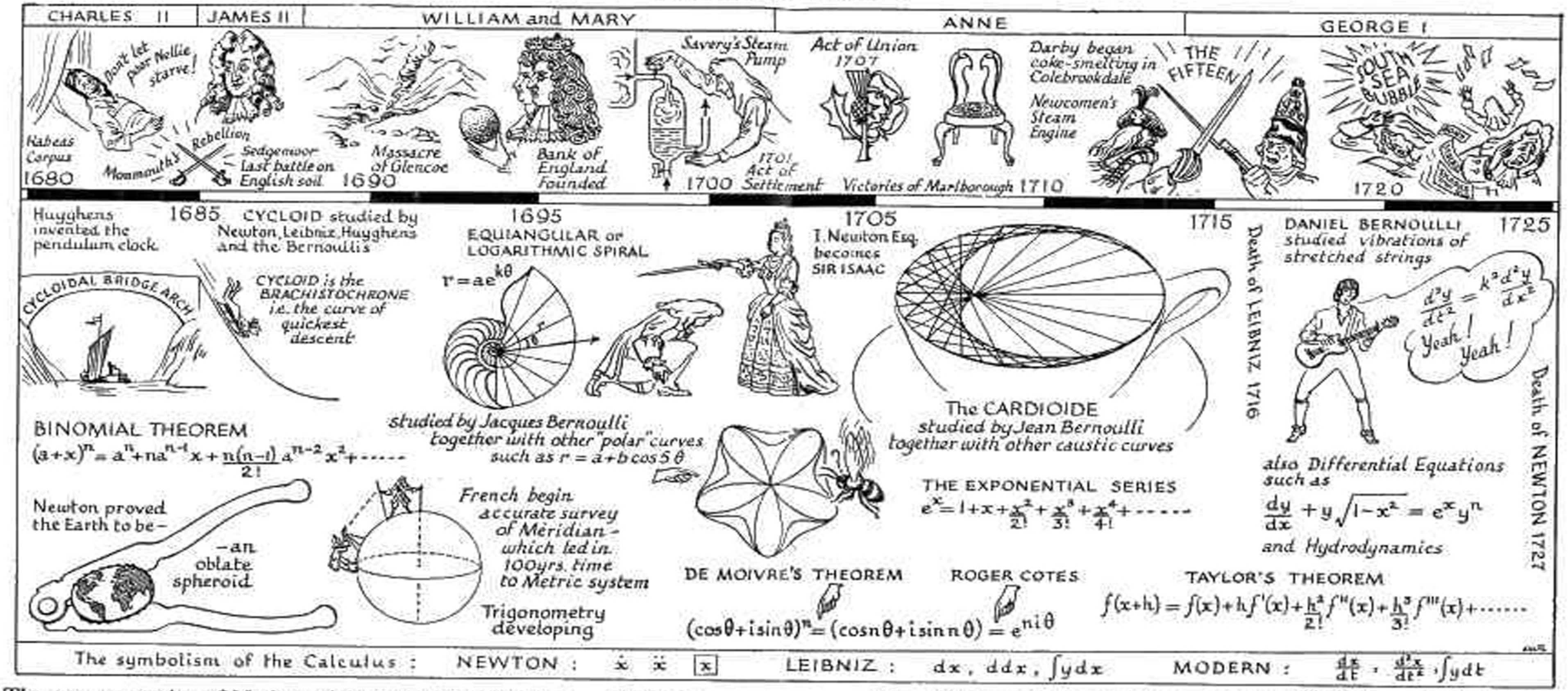
1. Divided by ax^2+bx+c, the quotient is x and the remainder is d.
5. $(a+b+c)\ (a^2+b^2+c^2-bc-ca-ab)$.
6. Value of y^2 when $\frac{x}{a}=c$ in figure.
8. B^2-AC when $B=a+b$; $A=a+2b+c$ and $C=a$.
9. $(f+1)^2-(1-2f)$.
10. $(x-g)^2+(y-f)^2-(g^2+f^2)$.

CLUES DOWN :

1. Numerator of $\frac{a}{a^2+b^2}+\frac{1}{x^3+a^2}$ written as a single fraction.
2. Value of $K+J$ when $\frac{K}{x^2+b(b+1)}=b$ and $\frac{x^2-J}{a}=c$.
3. $c\ (c^2+x)-2bx$.
4. Square root of $d^2-6\ abcd+9a^2b^2c^2$.
7. Area of a border f ft. wide around a rectangle x ft. long and y ft. wide.
9. Added to $x^2+2fx+x^2+g^2$ gives the sum of two squares. J.G.

77684 06484 74521 50457 98113 97853 08940 90850

TIME CHART No. 11



The great expansion of Mathematical Knowledge continues. Up to the time of Newton it might have been possible for one man to master all the then known Mathematics but from then on it was an impossibility. Men had to select but many were proficient in several fields. A genius has the ability to seize upon the essential idea in a situation and to express it simply. Following up the consequences of such an idea may provide work for other men for a couple of centuries or so. Newton was one such genius and Leibniz was another.

Now that the Calculus was allied to Algebra, mathematics had a wonderful new tool. It was used to investigate functions of all sorts : the Binomial and Exponential functions in particular. The Binomial function finds uses in approximations and in probability. It may be regarded as a generalisation of Pascal's triangle. (Use it to work out $(1+1)^5$ and compare with Pascal's triangle). The exponential function is sometimes called the law of Organic Growth ; it has the property that it is its own rate of change and e^1 or e itself is a Universal constant like π, appearing in many strange situations. Both the Binomial and Exponential Functions are special cases of Taylor's Theorem. This is as fundamental to the advanced mathematician as the twice times table is to someone playing double or quits.

Many of the developments of this period were due to the work of many

78445 28929 10005 61622 76585 54834 00215 70757

men. For example, the curve known as the CYCLOID was studied by Newton, Liebniz, Huyghens, the Bernoullis and others. This is the curve that would be traced out by a spot of paint on the outside of a cycle tyre as the cycle follows a straight path. Cycloidal arches are sometimes used for bridges giving great strength. Inverted, the cycloid is the curve down which a body will slide under gravity in the least time and in fact the time taken for a body to slide from any point to the lowest point of the inverted arch is always the same.

The Bernoullis appear again and again in the mathematical history of the next 100 years. In 3 generations this family produced eight mathematicians of whom Jacques I, Jean I and Daniel are outstanding. Jacques I worked in many fields : the calculus of variations, probability, the study of Polar Curves, *i.e.*, curves whose equations are best expressed in terms of r, the distance from a given point and θ, the angle turned through from a given direction. Such a curve is the equiangular spiral $r = ae^{k\theta}$, a curve which is beautifully illustrated by the Pearly Nautilus shell and is also found in the arrangements of the florets in the head of the daisy, aster or sunflower.

His brother Jean I wrote on the theory of tides, the mathematical theory of ship sails, the principle of Virtual Work in Mechanics and also studied the CARDIOIDE. You have probably noticed this curve traced out by the reflection of the sun's rays on to the surface of a cup of tea. The cardioide

77098 96714 96234 74501 43415 10963 72473 24005

SHOW ME THE WAY TO GO HOME
(or That Eternal Triangle)

Al Gebra lives in one of those American towns built up of rectangular blocks of buildings. His home is at *A* (the S.W. corner) and he works in an office at *B* (the N.E. corner). He drives each morning from *A* to *B* and being of an enquiring sort of mind it occurred to Al to wonder in how many different ways he could do it. Needless to say he was always late and always in a hurry so that he couldn't afford to waste time and all his moves must be Northwards or Eastwards.

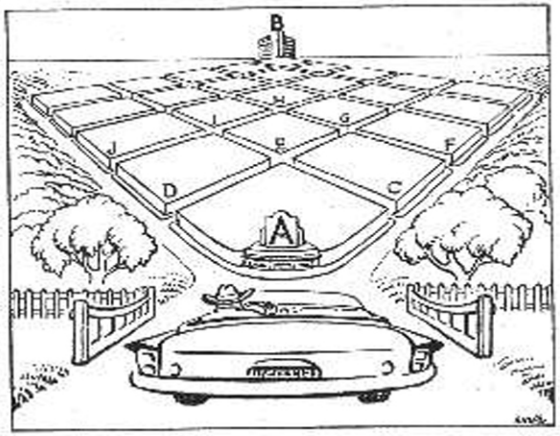


Fig. 1

Starting from *A* he has only one way of reaching *C* and only one way of reaching *D*, thus giving him 2 ways of reaching *E* (via *C* or via *D*). Let's mark on our map the number of ways he has of reaching each road junction as in figure 2.

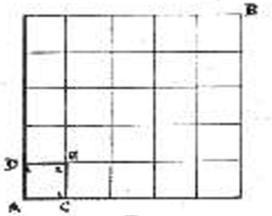


Fig II

From *D* there is one route to *J*. Thus Al has 3 ways of reaching *I*—2 via *E* and 1 via *J*. Similarly there are 3 ways of reaching *G*. (Fig. 3).

Now, how many ways are there of reaching *H*? You're absolutely right !! 3 via *I* and 3 via *G* makes 6 in all. Can you continue this for each intersection until Al reaches *B*. How many ways in all? You'll find the answer at the foot of page 338, but don't turn over until you've had a go.

Some of you may see a shorter way of arriving at this answer but that is not our immediate purpose. Let's have another look at those numbers we've written at the intersections:

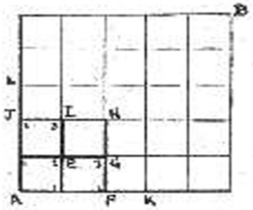


Fig III

After 1 move (*i.e.*, *C* or *D*) 1 1

After 2 moves (*F*, *E* or *J*) 1 2 1

After 3 moves (*K*, *G*, *I*, *L*9) 1 3 3 1

You will remember that we get each number by adding together the route totals at the two previous points from which it can be reached. We should carry on in the same way and get the next line

1 3 3 1

1 4 6 4 1

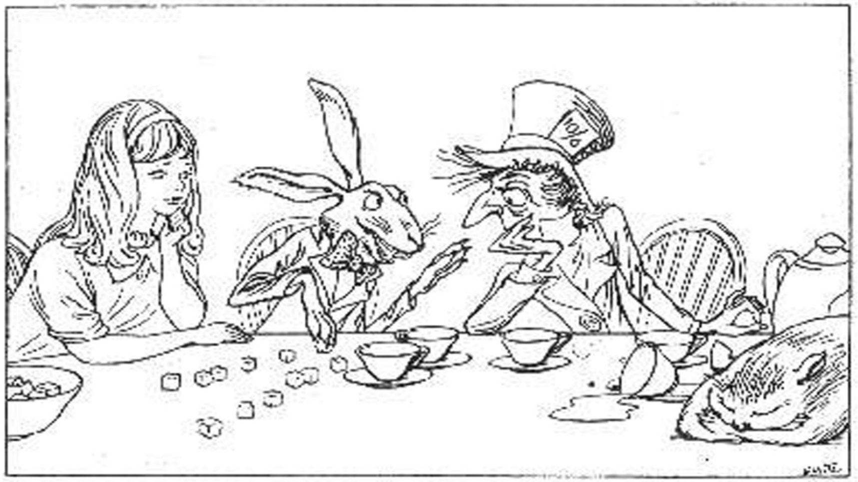
and so on until we have solved Al's problem. (Have you done it yet?)

Continued on previous page

MATHEMATICAL PIE

No. 43 — *Editorial Address*: 100, Burman Rd., Shirley, Solihull, Warwicks, England — OCTOBER, 1964

THE MAD HATTER'S TEA PARTY
(Revised Minutes)



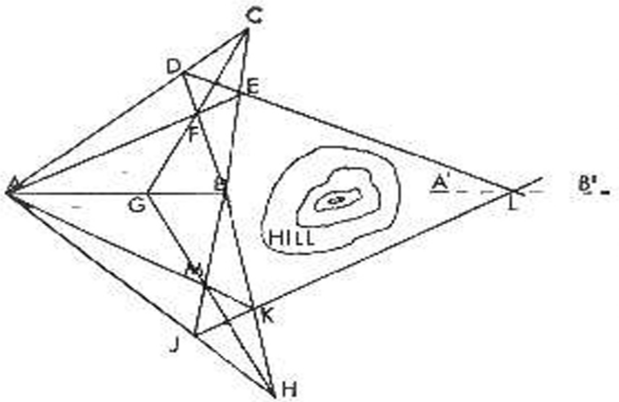
"What did they draw?" said Alice, quite forgetting her promise. "Treacle," said the Dormouse, without considering at all this time. "I want a clean cup," interrupted the Hatter, "let's all move one place on." Before he could move on, he in turn was interrupted by the March Hare, who demanded not one but *three* clean cups. While Alice collected the cups, the March Hare took twelve lumps of sugar from the basin and laid them on the table. "Since you couldn't answer the riddle about the raven and the writing desk," he told her severely, "perhaps you can tell me how to divide these twelve lumps among the three cups so that there is an odd number in each cup." "Yes," said the Dormouse, "that's just how Elsie, Lacie and Tillie like their tea," and added, "it's much easier with lumps of treacle!"

Alice thought hard and finally showed the tea-party how she would solve the problem. The March Hare looked at what Alice had done and remarked "I suppose it *does* work that way, but I *do* like to dip the Hatter's watch and this makes it harder." He then showed how *he* would arrange the sugar lumps in the cups.

Can you think how Alice and the March Hare solved the riddle? Remember that Alice was quite a clever girl and the March Hare was quite mad! (*See page 338 for their solutions*).

The figures at the foot of each page of MATHEMATICAL PIE are the decimal equivalent

OBSTRUCTION



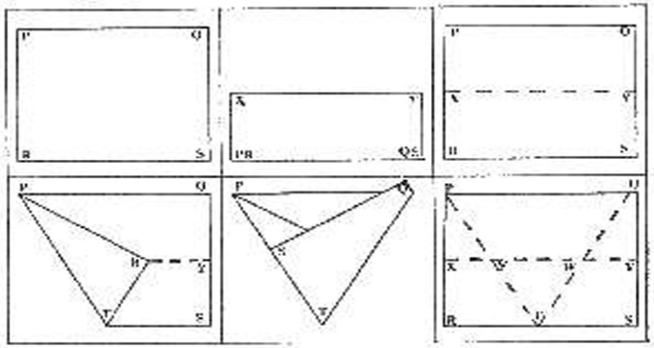
Take a point C sufficiently far from AB so that a line DE produced does not intersect the hill. Join AC and BC. Draw the line DE, joining one point D on AC and a point E on BC. Join AE and BD; let them intersect at F. Produce CF to cut AB at G. Join G to any point H which is sufficiently far from AB on the opposite side of AB to C. Join AH and BH. Let M be any point on GH. Produce AM to K on BH and BM to J on AH. Produce JK to meet DE produced at L. L is then one point on $A'B'$. Repeat the process with new points C', etc., to find a second point on $A'B'$.

J.F.H.

PAPER FOLDING No. 2
An Equilateral Triangle

Take a sheet of paper $PQSR$, about nine inches by eight inches. Fold it in half, along XY, parallel to the longer side. Open out again. Keeping P stationary, fold when R lies on XY along PT. Fold along TR, TS will lie on PT.

If the folds PT and the other from T meet XY in V and W, then TVW is an equilateral triangle.



B.A.

A DOG'S LIFE

Assuming that the following statements are true :—

(1) All dogs bark
(2) Some dogs are terriers
(3) All bulldogs bite
(4) Some terriers bite
(5) All terriers are dogs

Which of the following statements are true and which are false?

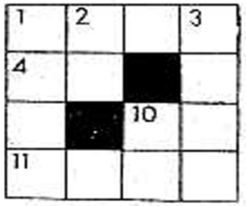
(A) All terriers bark
(B) Some terriers do not bite
(C) Some dogs are not terriers
(D) All barking dogs are bulldogs
(E) All biting bulldogs bark
(F) All barking terriers bite
(G) Dogs that do not bite are bulldogs
(H) All barking bulldogs bite.

R.H.C.

of π. The figures began in Number Eighteen, but those on pages 320 onwards are

QUICKIE CROSS FIGURE

All the clues and answers are in the scale of five.



Across
1. $\sqrt{21{,}313{,}304}$
4. 12×3
10. $3 - 2$
11. 123×12

Down
1. 234×13
2. $12 + 4$
3. 4^4
10. 2×4

B.A.

SOLUTIONS TO PROBLEMS IN ISSUE No. 42

FUN WITH NUMBERS No. 7

$(n^2+1)^2 - (n^2-1)^2 = 2n^2+2n^2 = 4n^2 = (2n)^2$.

SENIOR CROSS DIAGRAM No. 2

Clue 9 across should have read $\left(f + \frac{x}{2}\right)^2 - \frac{x}{2}\left(\frac{x}{2} - 2f\right)$.

Clues Across: (1) ax^2, bx^y, cx, d; (5) a^3, b^3, c^3, $3abc$; (6) a^2, b^2, $2bx$; (8) b^2, ac; (9) f^2, $2fx$; (10) x^2, y^2, $2gx$, $2fy$.

Clues Down: (1) ax^2, a^2, a^2, b^2; (2) bx^y, b^x, $b^2!ac$, x^2; (3) cx, c^x, $2bx$; (4) d, $3abc$; (7) $4f^2$, $2fx$, $2fy$; (9) f^2, $2gx$.

A WEIGHTY PROBLEM

Each base carries the same load.

MATHEMATICAL JUGGLER

The force on his hand to stop the falling balls would be greater than the mass of one ball, so he would not cross the bridge safely.

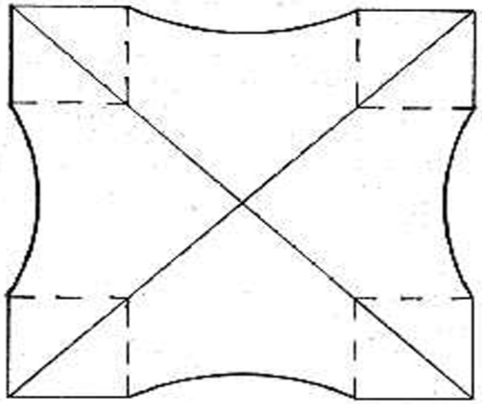
JUNIOR CROSS DIAGRAM No. 2

In the diagram the lengths should be $(x+a)$, $(x-a)$.

Clues Across: (1) $6a$, $6b$, $6d$; (4) y^2, a^2, $2ab$, b^2; (5) $4ax$, $4ay$, $4bx$, $4by$; (6) a^2, b^2, $2abc$.

Clues Down: (1) $6a$, a^2, $4ay$, b^2; (2) $6b$, $2ab$, $4bx$, $2abc$; (3) $6d$, b^2, $4by$; (4) y^2, $4ax$, a^2.

POINTS OF VIEW



The section shows two intersecting equal pipes.

ALGEBRA CORNER No. 1
From Issue No. 41

2. $F = \frac{a(bX-c)}{a-b} + b$

Where X is an arbitrary expression.

3. $\frac{100(a+b+c)}{100a+10b+c}$;

$\frac{0.a \times 0.b \times 0.c}{0.abc} = \frac{abc}{100a+10b+c}$

ALGEBRA CORNER No. 2

1. $\frac{(x^2-y^2)}{y^2}\,100$
2. $\frac{1}{2}(ab - yb + xy)$.
3. $a=5$, $b=4$, $c=20$, $x=3$, $y=8$, $z=9$, $h=7$.
4. B would beat C by $\left(\frac{c-b}{x-b}\right)x$ yd.

DROP A BRICK

The level of the water will rise.

B.A.

SHOW ME THE WAY TO GO HOME—from page 340

If we suppose that the city goes on and on and we can continue our table as far as we like we get a triangle of numbers which has some very striking properties. It was known to Omar Khayyam about 1100 AD and was published in China about 1300 AD but it is usually known as Pascal's Triangle from the French mathematician, Blaise Pascal, who investigated many of its properties around AD 1650. Next time we'll have a look at some of its properties, but perhaps you'd like to spot a few for yourself in the meantime.

R.M.S.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$

but considering a roof section as a warped plane Fig. 4(a) with origin at the centre of that plane, the equation simplifies to $z=kxy$ where k is a measure of the amount of warp.

Vertical sections of a Hyperbolic Paraboloid roof parallel to the high diagonal are parabolas concave upwards (Fig. 4b) and parallel to the low diagonal sections are parabolas concave downwards. Thus a supported roof is in compression one way and in tension the other. Thus strong edge beams are necessary to absorb the shear forces in the structure. These beams can be seen in the photograph Fig. 5 of a most attractive dual purpose bandstand and restaurant now erected in a park at Keynsham, Nr. Bristol. This roof is of timber and felt construction and mounted on two large concrete buttresses at the low corners. These support the vertical load and restrain the outward thrust. Such buttresses avoid an unsightly tie bar between the low corners.

Examples of this type of roof can be found in many places: Fig. 6 shows a plywood playground shelter in Aberdeenshire comprising four such roofs linked together. The new Redcliffe Methodist Church in Bristol has a roof made up of two large concrete sections prefabricated as described above and placed in position by a large crane.

The Spanish architect, Felix Candela, has done much from the aesthetic point of view to foster the application of exciting forms in concrete and his Chapel of San Vicente at Coyoacan (Fig. 7) incorporates hyperbolic paraboloid roof structures.

In conclusion, we might quote Eric Ambrose writing in the *Ideal Home* magazine February 1962: " I welcome the use of the hyperbolic roof for its monumental qualities. This roof makes the discussion of flat roof versus sloping complete nonsense."

We leave the reader to decide whether Ambrose was influenced most by the practical advantages, by the relative simplicity of the mathematics involved or by the sheer beauty of form present in the hyperbolic paraboloid.

K.A.

The Sugar Solution

Alice put five lumps in one cup and four in another cup. She placed the third cup so that it rested inside the second one and put the remaining three lumps into it. " There now," she said, " this cup has five lumps and this one has three; the cup below it has seven lumps in it all told." " How do you make seven?" asked the Hatter. " Four and three," answered Alice, " but I could make it from two and five or six and one."

" In *my* shop," said the Hatter, " four and three is not the same as two and five; you get a *much* better hat for four and three!"

The March Hare then showed them his solution. He emptied out the lumps and put the cups in a row. He put one lump in the first cup; one lump in the second cup and *ten* lumps in the third cup. " That won't do!" exclaimed Alice, indignantly: " I know one is an odd number, but what about ten?" " I should have thought," replied the March Hare, " that even the young lady would know that ten is an *exceedingly* odd number of lumps to put in a cup of tea!" J.F.H.

Show Me The Way To Go Home

Al has 252 ways of travelling to his office. R.M.S.

in the Apostle's name, should we bother with all the decimals?

THOSE CAR NUMBERS—The Final Word

In issues No. 36 and 39, we suggested counting car numbers to prevent duelling at your Editorial Board meeting. The response was extremely good and we thank all our readers who sent in cards. The analysis shows :—

Score	0	1	2	3	4 or more
Frequency per 100	35	37	21	6	2

This uggests that the distribution is binomial. B.A.

"I think we've found the spot where they divided up the loot!"

DIVIDE AND RULE

What are the necessary conditions that an isosceles triangle may be divided into two isosceles triangles? J.F.H.

FOOTBALL RESULTS

At the end of the season, the top club in a league of 18 teams had obtained 48 points (2 for a win, 1 for a draw), and had drawn two matches more than they had lost. How many matches did the club win during the season. J.G.

NUMBER THEORY

If $n = 2$, then $n+2 = 4$, and $2n - 1 = 3$. Hence $n(n-2)(2n-1) = 2\times4\times3 = 24$. Now take n equal to another *even* number and again find the value of $n(n-2)(2n-1)$. Is this number also divisible by 24?

Can you prove that this will always be true?

Now take n to be an odd number. Is $(n-1)(n+1)(2n+3)$ divisible by 24?

Explain your answer. R.H.C.

FOR BIRD WATCHERS

How is it possible for four sparrows to arrange themselves so that each one is the same distance from the other three? J.G.

SENIOR CROSS FIGURE No. 41

Submitted by T. J. McKee, Alleyn's School, Dulwich.

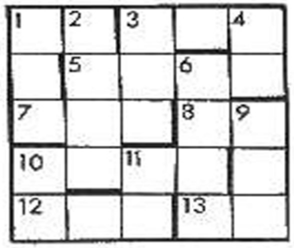
When the answer has 'N' digits, it is to the base 'N', (*e.g.*, 1 Across is in the binary scale).

CLUES ACROSS

1. 2×8 Across—10 Down.
3. Square root of 10 Across.
5. 6×9 Down $-$ 7.
7. Prime.
8. $\frac{1}{2}$(1 Across + 10 Down).
10. 3 Across squared.
12. 6/11 Down.
13. 1 Down $-$ 3 Down $-$ 1.

CLUES DOWN

1. 3 Down $-$ 13 Across $+$ 1.
2. 6 Down $-$ 4 Down $+$ 4.
3. 1 Down $-$ 13 Across $-$ 1.
4. 6 Down $-$ 2 Down $-$ 4.
6. 4 Down $+$ 2 Down $-$ 4.
9. $\frac{1}{6}$ (5 Across $-$ 7).
10. 2×8 Across $-$ 1 Across.
11. $\frac{1}{6}$ (12 Across).



incorrect; the sequence will be continued in the next issue. P. J. South, of Hounslow,

A FACET OF THE NEW LOOK IN ARCHITECTURE

Suggested by Maureen Darbey and Valerie Jones, Brislington School, Bristol.

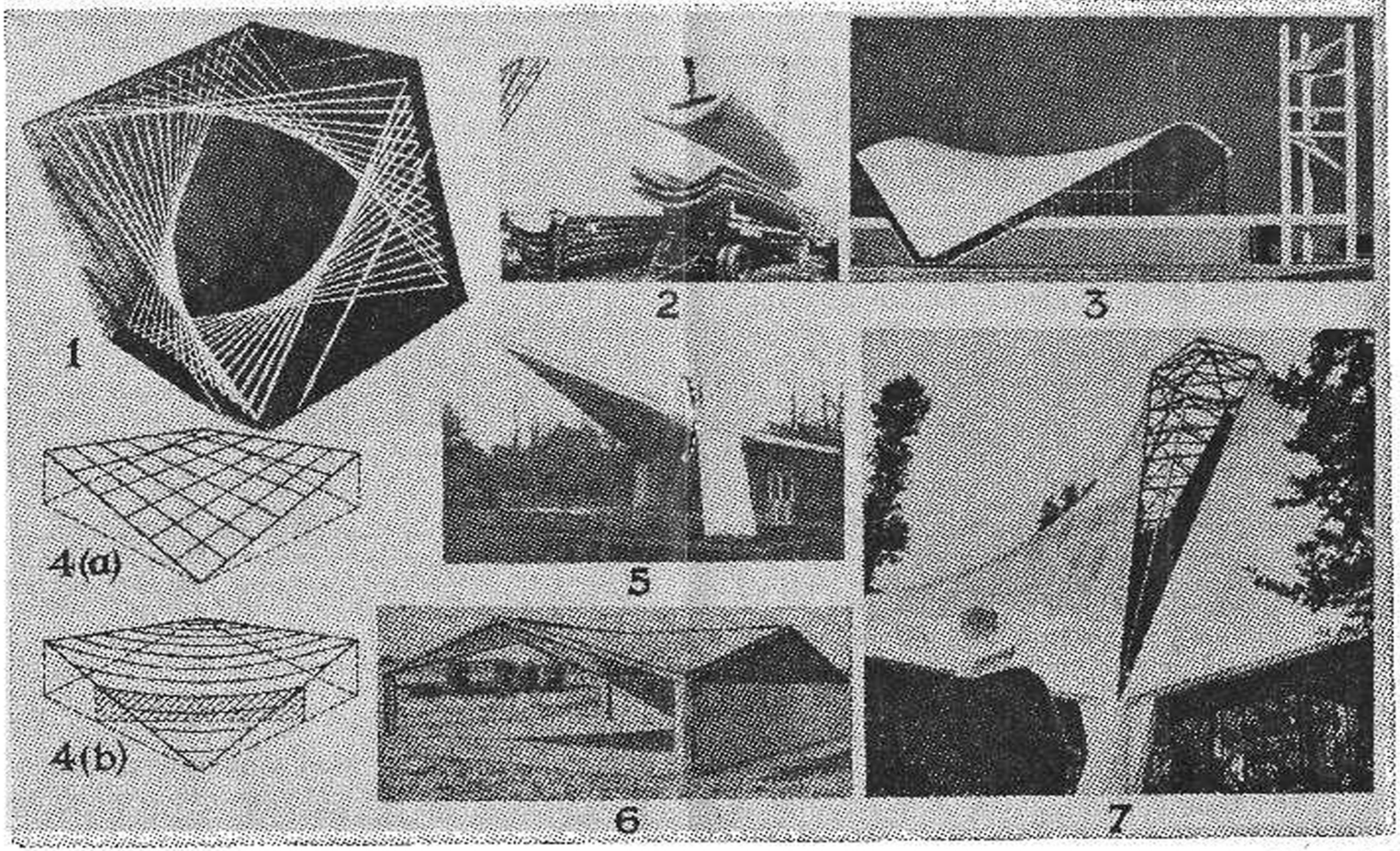


Fig. 1. A ruled surface.
Fig. 2. Prefabricated sections of a hyperbolic paraboloid.
Fig. 3. St. John's Church, Lincoln.
Fig. 4a and 4b Properties of a hyperbolic paraboloid.
Fig. 5. Bandstand and restaurant, Keynsham, Bristol.
Fig. 6. Playground shelter, Aberdeenshire.
Fig. 7. Chapel of San Vicente, Coyoacan

All over the world, buildings of exciting new shape and form are appearing and architects are helped in their new approaches by the use of the Hyperbolic Paraboloid roof. This ruled surface is shown simply in Fig. 1 where a Mathematical Pie curve stitching set has been used.

Roofs in general are conventionally either flat, pitched or spherically domed. The flat roof being two dimensionsal can only support stresses in its own plane and cracks or buckles under forces perpendicular to that plane. A pitched roof, *e.g.*, two sloping planes and a ridge is rather stronger, each plane helping to oppose movement of the other by means of internal trusses. Domed roofs answer most of the architect's problems, being able to withstand considerable load when of quite light construction. However, the difficulties of forming such a doubly curving surface which contains no straight lines are considerable. The hyperbolic paraboloid roof contains all the advantages of a doubly curved surface but is much simpler to construct

as straight 'beams' can be incorporated. A Bristol firm, for example, prefabricate enormous prestressed concrete sections of this type and transport them to the building sites. They are made by stretching steel wires (the elastics of Fig. 1) and pouring a thin layer of concrete around them. Fig. 2 shows one of these sections on its transporter. The fact that the shape of the roof is formed by straight lines helps the manufacturers to produce them easily with consequent low costs. The nature of the roof excludes the need for many internal supports and so large areas can be covered without internal obstruction. Maintenance costs are low as no steelwork need be exposed. There is opportunity here for extensive and attractive roof lighting. (Fig. 3).

Apart from the aesthetic value of this shape, the hyperbolic paraboloid is the only compound surface which can be analysed by simple statistics to evaluate stress and strain. The equation of this central quadric surface is frequently encountered in text books in the form

submitted the mnemonic for the first twenty-seven figures. For a time I stood

wondering to myself about the inane accuracy involving decimal fractions. Why,

ROCKET PROPULSION

Submitted by D. Hurden, Esq., Bristol Siddeley Engines Ltd.

A rocket engine propels an aircraft or missile by converting the chemical energy stored in a fuel into kinetic energy. It does this by burning the fuel at a moderately high pressure in a combustion chamber (which is essentially a cylinder with a hole in one end) and allowing the hot gases so formed to escape as a high-speed jet. According to Newton's Third Law of Motion, the force that accelerates the gases backwards is accompanied by a reaction that accelerates the rocket forward.

This description of the principle on which a rocket engine operates could apply equally well to an aircraft turbo-jet engine, which also converts fuel into a propulsive jet of hot gas. Air is swallowed by a compressor which raises its pressure and feeds it into a combustion chamber in which the oxygen it contains is used to burn a liquid fuel. The hot gases so formed flow through a turbine that drives the compressor and then escape from the jet pipe. The essential difference between such an engine and a rocket is that the turbo-jet relies on oxygen from the surrounding air to burn its fuel whereas the rocket carries its own supply of oxygen with it. A rocket used to propel an aircraft or missile or spacecraft will use either liquefied oxygen or an oxygen-rich liquid like nitric acid to burn liquid fuels such as alcohol or kerosine. The firework rockets launched in great numbers on November 5th burn a solid fuel with oxygen from potassium nitrate.

Fig. 3

Fig. 2

The fundamental difference between these two kinds of jet propulsion engines is emphasized by the two sketches. In Figure 1 the " engine " is seen to be picking up its " oxygen " from its surroundings before accelerating it aft, while in Figure 2 the " oxygen " is being carried in the vehicle. These pictures also underline two other important facts ; first, any jet engine works on the principle set out in Newton's Third Law and not by pushing against the atmosphere. Secondly, although a turbo-jet will not work in space because there is no oxygen for it to swallow, a rocket will work just as well out of the atmosphere as in it, so some kind of rocket must be used to propel spacecraft.

LETTERS TO THE EDITOR

From MATHEMATICAL PIE No. 43, page 339, Solutions to " Mathematical Juggler." The *force* on his hand to stop the falling balls would be *greater than the Mass* of one ball so he could not cross the bridge safely. Since when has a force been greater than a mass ? ? ? ?

P. R. Huish, Shirley, Croydon, Surrey.

Is my face red ?

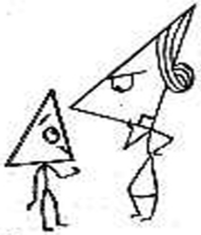
98116 14101 00299 60783 86909 29160 30288 40026

MATHEMATICAL PIE

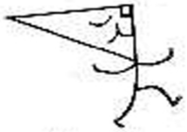
No. 44 *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England **FEBRUARY, 1965**

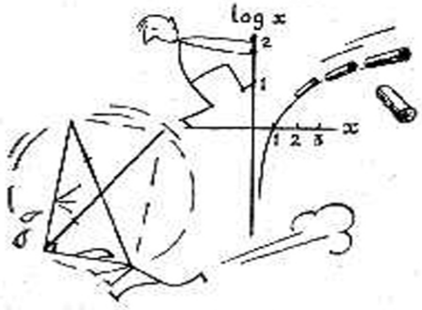
A STORY COMPLETELY UNCONCERNED WITH THE SINE FORMULA

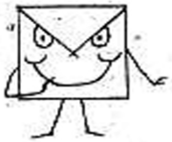
Submitted by Elizabeth M. Rann, Herts. and Essex High School.



There was once an acute angled triangle who wanted to become a square. Worried by his problem he consulted his aunt, Right Angled Pythag., who suggested a topological transformation but, unfortunately, she only had the power to change him until he was similar to herself. In this changed form he set off for a walk round the perimeter of the wood near his aunt's house to think, and suddenly he came across a man cutting logs with axes. This man accidentally cut his hypotenuse and he discovered he was a cyclic quadrilateral and an isosceles triangle. But still he was not a square. He caught an aeroplane and flew via the land of the Triangle of Velocity to a point on the longitudinal circle 45° W. There he found a wise old owl who directed him round the earth on a compass bearing of West for 3,960 miles. Sadly, because he was not on the equator this did not get him far ; only a few thousand yards. By now the cyclic quad., because it had had no pies, had the triangle inscribed in it because it had eaten it owing to extreme hunger. Moving millimetre by millimetre the cyclic found that his other corners were right-angled and, because of an unknown theorem, he was a square ! It was tragic that on his way home to his aunt, Right Angled Pythag., he fell down a Three Dimensional Well to the land of Forgotten Facts and dwelt there, forgotten, evermore.







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WHO IS FROM WHERE?

Three men, let us call them A, B, and C, went out to dinner in Paris. One was English, one Scottish, and one Welsh. The waiter was asked if he could guess their respective nationalities. He said that A was English, B was not English, and C was not Welsh.

Only one of his answers was correct. What were the nationalities of A, B, and C? J.G.

JIG-SAW PUZZLE

Submitted by Canon D. B. Eperson, Christ Church College, Canterbury.

Any trapezium, when cut into two pieces along the line joining the mid-points of its two non-parallel sides, can be made into a parallelogram in two different ways. Can you find them? Answers on page 346.

Any trapezium can also be cut into two pieces that can be made into a triangle. How should the cut be made?

In how many ways can this be done, each providing a triangle of a different shape?

PYTHAGOREAN TRIADS

It is a common experience when using the theorem of Pythagoras to find that when the hypotenuse is calculated, its length is found to be irrational, it is the square root of a number which is not a perfect square. To find the sides of certain right angled triangles if all the sides are to be rational, the following rule is worthy of note:—

(1) Write down any fraction, using any two numbers whatever, *e.g.*, $\frac{9}{17}$. Invert it and double; $\frac{17}{9} \times \frac{2}{1} = \frac{34}{9}$

(2) Add 2 to each: $\frac{9}{17} + 2 = \frac{43}{17}$. $\frac{34}{9} + 2 = \frac{52}{9}$

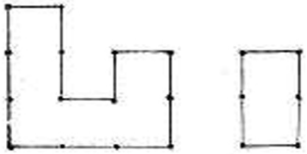
(3) Multiply each by the L.C.M. of the numerator and denominator of the original fraction.

$$\frac{43}{17} \times 17 \times 9 = 387; \quad \frac{52}{9} \times 9 \times 17 = 884.$$

The resulting numbers (387 and 884) when used as the shorter sides of the right angled triangle will always yield a hypotenuse which is rational. The sum of the squares of 387 and 884 is 931225. The square root is exactly 965.

The numbers were purposely made large, try the method with smaller numbers and prove that the result is always true. J.G.

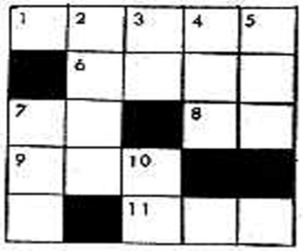
CAN YOU MATCH THIS?



Here you see twenty matches arranged in two groups so that the larger encloses three times the area of the smaller. Can you now transfer *one* match from the larger group to the smaller and rearrange them so that the thirteen matches again enclose an area three times as large as that enclosed by the seven matches. If it is any help to you, twelve of the matches are not touched at all and, of course, none is bent or broken. R.M.S.

JUNIOR CROSS FIGURE No. 37

Submitted by Peter Packard, Felixstowe Grammar School.



CLUES ACROSS

1. Product of the first three perfect numbers.
6. sine 83° 30′.
7. $3x + 2y = 22$
   $x - y = -1$. Find xy.
8. Muliply the square of the second prime number by the third prime number.
9. Product of two consecutive prime numbers.
11. Square of 0.9.

CLUES DOWN

2. cosine 67° 1′.
3. $x(y + 1) + 3y$. See 7 across.
4. The first three terms of an A.P. of which the first three terms add up to 9.
5. If 540 half-pennies are put end to end, how far short are they of half a mile (in yards).
7. Exact square root of 47089.
10. Two sides of a right-angled triangle are 40 and 50 units. Find the number of units in the third side.

DICTIONARY MATHS.

(*genuine extracts*)

GEOMETER 1. A student of geometry. 2. A kind of moth, the caterpillars of which appear to measure out the ground as they move by drawing up and extending the body in loops.

BUN Small round sweet spongy cake with convex top and too few currants.

SPIRAL 1. Forming a curve that winds continually about a centre from which it constantly recedes. 2. Winding constantly about a centre while undergoing continual change of plane.

(Mathematicians may not agree with some of these !). J.F.H.

DOUBLE OR WIN

Three men play a game with a rule that the loser is to double the money of the other two. After three games each has lost one game and each ends with 24/-. How much had each at the start of the game? R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 43

A DOG'S LIFE

The statements A, C, E, H are true.

DIVIDE AND RULE

The triangle must be one with angles 36°, 72°, 72° or 45°, 45°, 90°.

FOOTBALL RESULTS

The club won 20 games, drew 8 games and lost 6 games.

FOR BIRD WATCHERS

The four birds must be at the vertices of a regular tetrahedron.

SENIOR CROSS FIGURE No. 41

CLUES ACROSS: (1) 11, (3) 101, (5) 1111, (7) 201, (8) 11, (10) 1210, (12) 110, (13) 11.
CLUES DOWN: (1) 122, (2) 1102, (3) 111, (4) 11, (6) 1101, (9) 111, (10) 11, (11) 10.

QUICKIE CROSS FIGURE

10 Across should read 3 × 2. 1 Down should read 134 × 13.

CLUES ACROSS: (1) 3202, (4) 41, (10) 11, (11) 2031.
CLUES DOWN: (1) 3402, (2) 21, (3) 2011, (10) 13.

B.A.

40430 27253 86076 48236 34143 34623 51897 57664 11872 67675 62204 15420 51618 41634 84756 51699

Bugia in N. Africa and it was through this connection with commerce that Leonardo became acquainted with the various arithmetical systems in use around the Mediterranean. Convinced of the superiority of the decimal system which we still use today he wrote the *Liber Abaci* to make it more widely known. His other works include algebra, practical surveying, trigonometry and a rather remarkable series: 0, 1, 1, 2, 3, 5, 8, 13, 21, ...in which each term is the sum of the previous two. This series has a habit of cropping up in all sorts of strange places.

Towards the end of our period we have *Oresme* (c.1360) who used a form of coordinate geometry, *Peurbach* (1423–1461) and his pupil *Regiomontanus* (1436–1476) who published a table of sines and the latter a Trigonometry far in advance of anything done before. Indeed the end of the 15th century was remarkable for the number of text-books published on various branches of Mathematics.

R.M.S.

CHARLIE COOK INVERTS

Solve $\dfrac{1}{x-9} + \dfrac{3}{x-10} = \dfrac{6}{x-8} + \dfrac{2}{x-6}$

Invert $\dfrac{x-9}{1} + \dfrac{x-10}{3} = \dfrac{x-8}{6} + \dfrac{x-6}{2}$

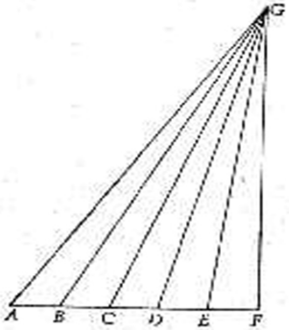
Multiply by 6, $6(x-9)+2(x-10)=x-8+3(x-6)$

$$x = 12.$$

Answer checked by substitution.

J.G.

DOWN MEXICO WAY 1968



Six runners, each capable of running 10 yards per second, are arranged along a straight line AF at equal intervals of 30 yards. Their goal is a point G situated 200 yards from F, and $\angle AFG$ is a right angle. The runners do not start simultaneously, but at time intervals of 1 second; B starts one second after A; C one second after B, and so on. Which of the runners will arrive first at G and in what order will they finish? Where are the runners after 5 secs?

J.G.

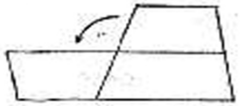
CORNERING

(1) A square of 10-inch side is made into a regular octagon by cutting off four corners. Where should the cuts be made?

(2) The same square is made into an equilateral triangle by cutting off three corners. Where should the cuts be made?

What is the length of each side of the triangle?

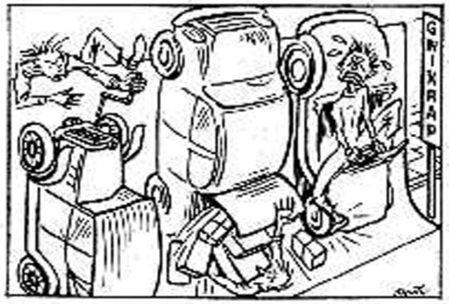
J.G.



JIG-SAW PUZZLE ANSWER

55582 15725 03107 12570 12668 30240 29295 25220

THE GRAVITY OF THE PARKING PROBLEM



In a letter to the Editor of a small newspaper, a citizen suggested that the parking instruction which said 'that cars should be parked at right-angles to the curb' should read 'perpendicular to the curb!'

A PROGRESSIVE DATE

Submitted by Mr. B. K. Booty, Wotton-under-Edge.

On the first day of August, 1964, the day, month, and year (1.8.64) were in Geometric Progression. (G.P., that is, the ratios 1 : 8 and 8 : 64 are equal). Counting the year '01 as 1, etc., how many such dates occur in a century?

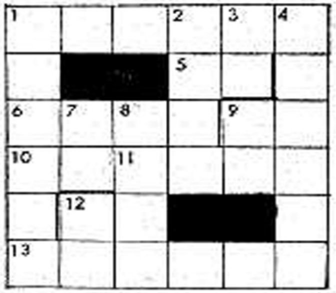
A SUPERIOR ADDRESS

In the number 35,678,243,100, how much greater is the value of the 3 on the left in comparison to the 3 on the right?

R.H.C.

SENIOR CROSS FIGURE No. 42

Submitted by Kenneth Turner, Hutcheson's Grammar School, Glasgow.



Ignore decimal points in the answers.

CLUES ACROSS

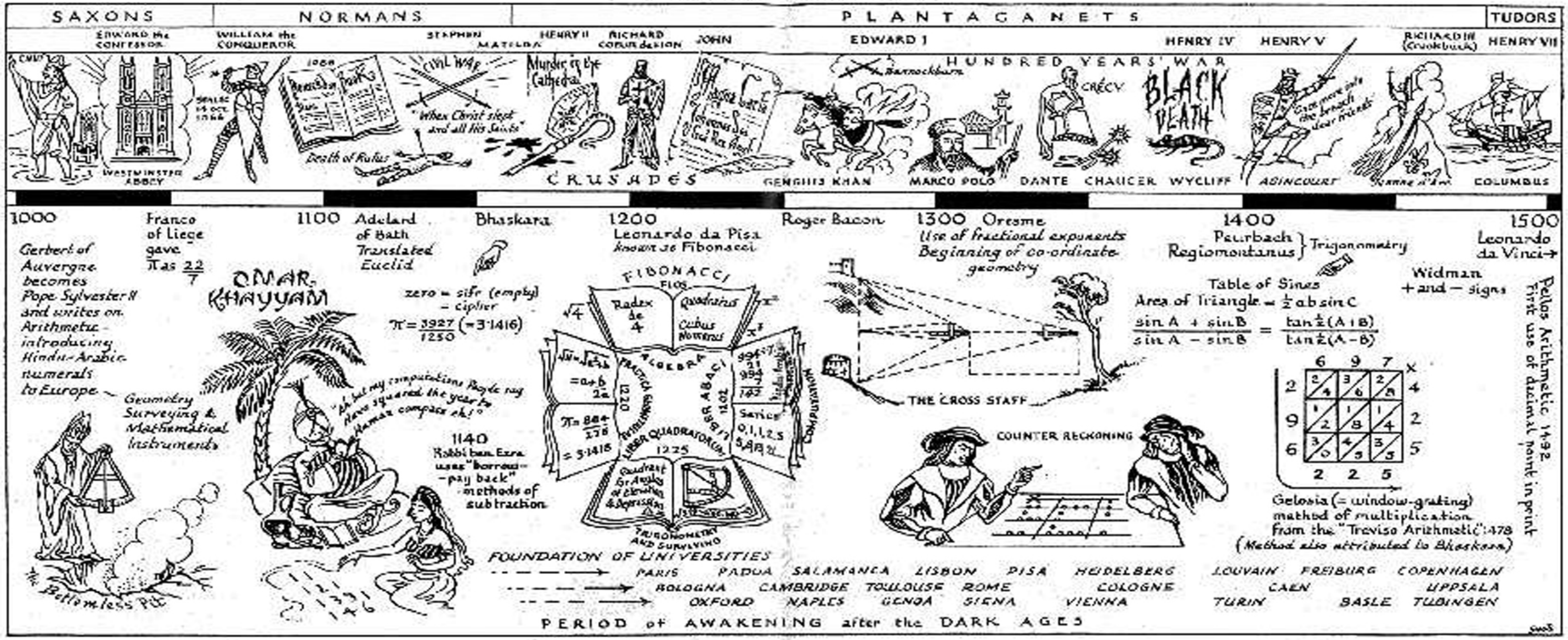
1. Product of the squares of the roots of $x^2 - 12x - 693 = 0$.
5. Area of a right angled triangle with hypotenuse 6 units and an angle whose tangent is 2, over 3.
6. The sine of the angle whose cosine is .5402.
9. The smallest number that is three times the sum of its digits.
10. The smallest number which is the sum of its digits plus the number formed by its digits transposed.
11. The sum of the series $1500 - 750 + 375 - \ldots$
12. The area between the graphs $y = x^2 - 2x + 6$, $y = 2x - 3x^2 - 2$, $x = 0$, $x = 3$.
13. The smallest number ending in 4, which is multiplied by 4 when this 4 is transposed to the first place.

CLUES DOWN

1. The smallest number ending in 1 which is reduced to $\frac{1}{3}$ of its value when this 1 is transposed to the first place.
2. The sum of the infinite series $2025 - 202\frac{1}{2} + 20\frac{1}{4} + \ldots$
3. The circumradius of the triangle with $a = 77.2992$, $\angle B = 28° 12'$, $\angle C = 90° 21'$.
4. The product of the squares of the roots of $x^2 - 77x - 968 = 0$.
7. The side a of a triangle in which $\angle A = 61° 39'$, $\angle B = 30°$, $b = 25$ units.
8. The smallest number which gives remainders of 4, 1 and 2 when divided by 13, 19, and 7 respectively.
9. The value of x to give $x(x^2 - 28x - 80)$, its minimum value.
12. $\dfrac{3x(x^2-2x+3)}{5x-9} + \dfrac{(x^3+3)(x^2+4)}{x+2}$ when $x = 2$.

52164 13767 96903 14950 19108 57598 44239 19862

TIME CHART No 5



The 3rd century A.D. saw the end of the age of Greek Mathematics and from then until the middle of the 11th century there was little mathematical activity in Europe. We now come to the period of awakening after this Dark Age.

There were three factors which accelerated the spread of knowledge in Western Europe at this time, namely, the great expansion of trade between wealthy merchants in many countries, the founding of the great Universities beginning with Paris, Bologna and Oxford and the invention of printing with movable type.

The spread of commerce brought with it a need for greater facility in Arithmetic and we see the development of methods of multiplication and division. Until then all calculation had been done on the counting frame or sand-table and the results recorded in the awkward Roman system of numerals. Now, however, the spread of the Hindu-Arabic system with its place notation and the use of the cypher or zero for the empty column enabled calculation to be done on paper and the counting frame to be discarded like the chrysalis case of a butterfly after it has served its purpose. This was the era of the publication of many books on the Art of Reckoning such as the *Treviso* Arithmetic (1478) and about this time we find the symbols + and — first appearing in print, though they had long been in use to indicate over or underweight bales of merchandise.

Another consequence of this spread of trade was the need for accurate measurement of land. This led to the science of surveying and the use of instruments such as the quadrant, the cross-staff and the astrolabe and to the development of trigonometry.

Omar Khayyam (c.1044–1124) is usually remembered for his poetry, notably the *Rubaiyat* which is well known in its English translation. In addition he was no mean mathematician, writing on Euclid, astronomy and calendar reform as well as a noteworthy book on algebra which contains the triangle of numbers usually attributed to Pascal four centuries later.

Another of the famous names of the era is *Bhaskara* who lived in India in the mid-twelfth century. His most important work is the *Lilavati* (named after his daughter) which deals with the common arithmetical operations, mensuration, commercial rules (*e.g.*, for Interest), proportion, some algebra and a statement of the rules for operating with zero. In other books he deals with negative numbers, surds, simple quadratic equations and Pythagorean numbers (a, b, c such that $a^2 + b^2 = c^2$).

Probably the most famous mathematician of this time is Leonardo da Pisa, often known as *Fibonacci*. His father was the Customs officer for

EASY TO TWELFTH MAN

If you have been given a function of x such as $x^3 - 5x^2+11x - 2$ and asked to obtain its value when different numbers are substituted for x, I expect you will do it as follows $x=2$

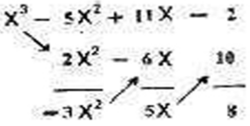
$$\begin{aligned}&(2)^3-5(2)^2+11(2)-2\\ &=8-5(4)+22-2\\ &=8-20+22-2\\ &=8\end{aligned}$$

Have you tried it this way $x^3-5x^2+11x-2$, $x=2$, the first term $x^3=xx^2=2x^2$

the second term becomes $2x^2-5x^2=-3x^2=-3xx=-6x$
the third term becomes $-6x+11x=5x=10$
the constant term becomes $10-2=8$, which is the answer.

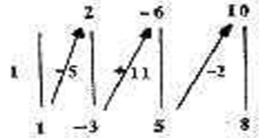
It would be unfair to say that this is a longer method than the first one because it has been set out carefully so that you can follow all the steps. As soon as you have grasped the whole process this method of substitution can be abbreviated something like this

when $x=2$



or better still we can dispense with the x labels

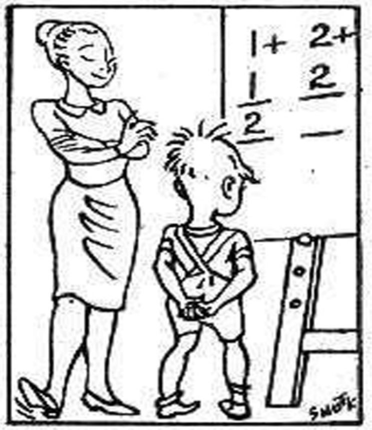
when $x=2$



This dispensing with the x labels is no worse than what we do with our ordinary numerals where 549 really means $5(10)^2+4(10)+9(1)$.

Why not try this method of substitution next time you have to obtain various values of a function before you go on to drawing a graph. R.H.C.

" Yoo-Hoo , Ivy ! "



"Er, now let me make a rough guess."

37182 94127 03389 31378 51786 09040 70866 71149

MATHEMATICAL PIE

No. 45 — Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England — MAY, 1965

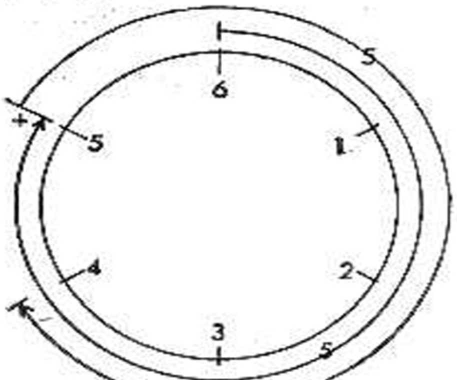
CLOCK ARITHMETIC No. 4

Squares and Square Roots

In issue No. 37, you were shown how to make out a multiplication table for a circular face marked out with the scale of 6. This revealed that in modulo 6

$2\times5=5+5$ $=4$ $\qquad 3\times4=4+4+4$ $=2+4$ $=6$ $\qquad 4\times5=(5+5)+(5+5)$ $=4+4$ $=2$

and from these and similar results, the full multiplication table was constructed.



x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	2	4	6
3	3	6	3	6	3	6
4	4	2	6	b 4	2	6
5	5	4	3	2	a 1	6
6	6	6	6	6	6	6

Now examine the table and pick out those numbers which are the product of equal numbers, *e.g.*, in square (a) we learn that $5\times5=1$, or using symbols $5^2=1$. Similarly in square (b) we discover that $4^2=4$. Listing all the results from an examination of the diagonal of the table produces $1^2=1$, $2^2=4$, $3^2=3$, $4^2=4$, $5^2=1$, $6^2=6$. In normal arithmetic, the fact that $8^2=64$ leads us to invent new symbols and words for an inverse process namely that $\sqrt{64}=8$. Similarly $\sqrt{81}=9$. Using this same idea of an inverse root process for clock arithmetic in modulo 6 gives

$$\sqrt{1}=1 \text{ or } 5,\ \sqrt{3}=3,\ \sqrt{4}=2 \text{ or } 4, \text{ and } \sqrt{6}=6.$$

These latest results raise some interesting issues. Why should $\sqrt{1}$ and $\sqrt{4}$ have two answers whilst $\sqrt{3}$ and $\sqrt{6}$ have only one? What has happened to $\sqrt{2}$ and $\sqrt{5}$ which are so far missing? When you have done your homework properly, you will be led to the conclusion that 1 has *six* square roots, all of them different ! ! ! R.H.C.

91041 40792 88621 50784 24516 70908 70006 99282

TWO FOR A PENNY

1. The scale of an ordnance survey map is 1 mile to 1 inch. A halfpenny is placed on the map, how many acres of ground would be represented by the area covered by the coin?
2. Three halfpennies are placed flat on a table so that each touches the other two. What is the area of the space left between them?
3. Six halfpennies are arranged with their centres on a circle so that each touches the two adjacent ones. What is the radius of the circle which will just enclose the six coins?

J.G.

ALGEBRA CORNER No. 3

1. Square $x + y + z$.
2. If the length of a diagonal of a rectangular solid is 6 inches, and the sum of the areas of all the faces is 64 square inches, what is the sum of all the edges of the solid?
3. If $a^2 = bc$ and $b^2 = ca$, does $c^2 = ab$?
4. The number of inches in a yards b feet is equal to one sixth of the number of pence in £a . . bs. If a and b are whole numbers, what are their values?
5. The parallel sides of an isosceles trapezium are 2 inches and $2n^2$ inches. The slant sides are $(n^2 + 1)$ inches. What is the area?
6. What is the angle in degrees between the hands of a watch when the time shown is x minutes past y?

J.G.

AT THE CROSS WAYS

Two lines cross at an angle α. A point V is fixed in position with respect to the lines. Find a construction for drawing another line through V and cutting the first two lines at P and Q in such a way that PQ is a given length l.

J.F.H.

A PARADOX

" He put 2 and 2 together and made 5," is a phrase that we often hear. Mr. A. R. Pargeter, Blundell's School, suggests $\sqrt{(.2)^{-2}} = 5$. Thus two 2's can make 5.

Can you find other examples of this kind?

THE ROARING FORTIES?

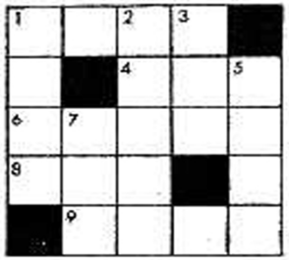
FORTY	Each letter represents
TEN	a different figure in
TEN	the addition sum.
SIXTY	

J.F.H.

12066 04183 71806 53556 72525 32567 53286 12916

JUNIOR CROSS FIGURE No. 38

Ignore decimal points and work to the appropriate number of significant figures.



CLUES ACROSS

1. Area of a square of side 47.2 units.
4. Third side of a triangle, with hypotenuse 10 units and one side 3.4 units, squared.
6. One tenth of 9!
8. Number of litres in one pint.
9. Diagonal of a square of side 1 unit.

CLUES DOWN

1. Number of grams in one ounce.
2. The length of a diagonal of a square of side 2 units.
3. The length of a rectangle whose diagonals are 10 units and breadth 4.6 units.
5. $1^6 + 2^6 + 3^6 + 4^6 + 4$.
7. The breadth of a rectangle whose diagonals are 1 unit and length is $\frac{3}{4}$ unit.

B.A.

SOLUTIONS TO PROBLEMS IN ISSUE No. 44

WHO IS FROM WHERE?

A is Welsh, B is English, C is Scottish.

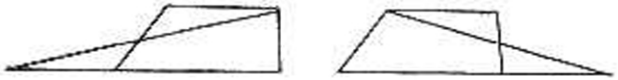
A PROGRESSIVE DATE

There are 51 dates per century which are in geometric progression.

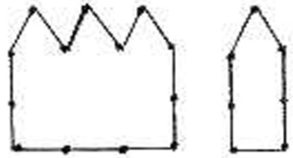
A SUPERIOR ADDRESS

The first 3 is 29,999,997,000 more than the second 3. The first 3 is 10,000,000 times as big as the second 3.

JIG-SAW PUZZLE.



CAN YOU MATCH THIS?



SENIOR CROSS FIGURE No. 42

CLUES ACROSS: (1) 480249; (5) 24; (6) 8415; (9) 27; (10) 54; (11) 1000; (12) 42; (13) 102564.
CLUES DOWN: (1) 428571; (2) 2250; (3) 44; (4) 937024; (7) 44; (8) 1122; (9) 20; (11) 122; (12) 40.

DOWN MEXICO WAY, 1968

The order of finishing is D, C, E, B and A and F tie last.
After 5 seconds, A, B, C, D, and E have travelled 50, 40, 30, 20 10 yards and F is about to start.

CORNERING

(1) The cuts should be made $5(2 - \sqrt{2})$ inches from the vertices.
(2) The cuts are made from one vertex to the points on the opposite sides $10(\sqrt{3} - 1)$ from the opposite vertex.
The length of the sides of the triangle is $20\sqrt{(2 - \sqrt{3})}$.

JUNIOR CROSS FIGURE No. 37

CLUES ACROSS: (1) 83328; (6) 9936; (7) 20; (8) 45; (9) 143; (11) 081.
CLUES DOWN: (2) 3904; (3) 39; (4) 234; (5) 865; (7) 217; (10) 30.

DOUBLE OR WIN

The first loser had 39/-, the second had 21/-, and the third had 12/-.

B.A.

01729 67026 64076 55904 29090 45681 50652 65305

PROBLEM CORNER

Printer's Error

In setting up $2^5.9^2$ the printer put 2592, which surprisingly enough is the same. Can you find another number of four digits which has this peculiar property?

R.M.S.

Enclosed

What is the greatest area that I can enclose using 400 yards of wire netting?

What is the greatest rectangular area that can be enclosed by the netting?

R.M.S.

It is not cricket

The factorial sign (!) is a very convenient way of writing some products, *e.g.*, 5! means $1 \times 2 \times 3 \times 4 \times 5$ and 10! means $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$ which equals 3,628,800. You are surprised how large it is? Hence the (!) sign.

A newspaper headline once said, "Bradman out for 12!".

I doubt if the famous Australian cricketer made 579,001,600 runs in the whole of his career, let alone in one innings!

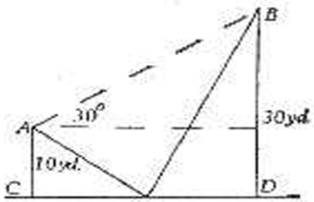
What is the largest number you can make using three matches? (Hint: use one of them upside down for the factorial sign (!)). It is well over Bradman's "score."

R.M.S.

Handicapped

At our School Sports, the Junior girls had to run from a point A, touch a wall CD and then run on to the point B. One bright lass scored an unfair advantage by working out exactly the quickest route to take from A to B fulfilling the condition. How far did she run?

R.M.S.



FIFTH COLUMN SOLUTION (from page 351)

Try plotting the five points. They lie on a straight line. Can you find the equation of the line?

Factorise the expressions on the left hand side of the equations,

$$(x - y)\ (2x + y - 4) = 0 \text{ and } (2x + y - 4)\ (3x - y - 2) = 0$$

Hence the first equation represents the pair of straight lines

$$x - y = 0 \text{ and } 2x + y - 4 = 0$$

and the second equation represents the pair of straight lines

$$2x + y - 4 = 0 \text{ and } 3x - y - 2 = 0$$

so that any point which lies on $2x + y - 4 = 0$ will satisfy both equations.

R.M.S.

57710 28402 79980 66365 82548 89264 88025 45661

FIFTH COLUMN

(Adapted from Mathematics Student Journal)

You may have met the fact that the graphs of two quadratic equations can intersect in at the most 4 points.

For example, the graphs of the two equations

$$2x^2 - y^2 - xy - 4x + 4y = 0$$
$$\text{and} \quad 6x^2 - y^2 + xy - 16x + 2y + 8 = 0$$

meet in the four points (–1, 6), 1,1), (0, 4), (2, 0); this is a fact that can be verified by substitution.

Your teacher will probably pat you on the back for a good effort, so then floor him with the fact that (3, –2) also fits both equations.

Can you see why? Turn to page 354 for the explanation. R.M.S.

SENIOR CROSS FIGURE No. 43

Ignore decimal points and work to the appropriate number of significant figures.

$x + y = p, \quad y + p = q + 1$

$x + p = q, \quad x + y = q - 2$

1		2	■	3		4
	■	5	6		■	
7	8	■		■	9	
■	10					■
11		■		■	12	13
	■	14		15	■	
16			■	17		

CLUES ACROSS

1. $\dfrac{pq}{x}$
3. $\dfrac{xp + q}{q - y}$
5. $\dfrac{y}{q}$
7. $x(3q + p)$
9. $(q - x)^2$
10. One tenth of 9!
11. $(p + x)(p + q)$
12. $\dfrac{p^3 + p}{x}$
14. $\dfrac{1}{x + p}$
16. $\dfrac{p}{q}$
17. $x^2 y^2 + 2pxy + p^2$

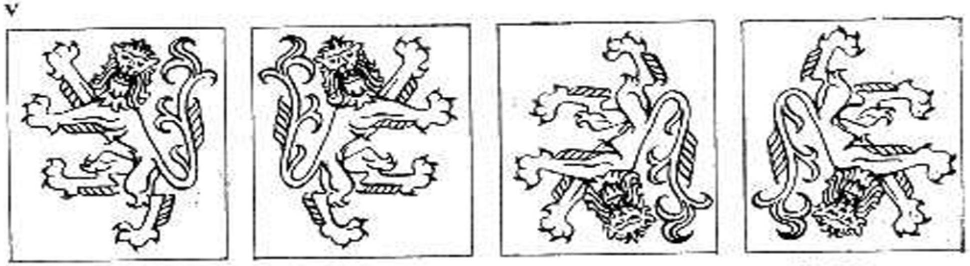
CLUES DOWN

1. pqy
2. xy^3
3. q^2
4. $\dfrac{xy(p + x)}{y + p}$
6. $\sqrt{8}$
8. Consecutive figures.
9. $\dfrac{x}{q}$
11. $\dfrac{xy}{q}$
13. $q^3 + (y + p)^2 + y$
14. xq
15. $p^2 + xy$

B.A.

42487 76182 58297 65157 95984 70356 22262 93486

ABOUT TURNS



Lion V. Lion H. Lion R. Lion

On the little sketch of the heraldic lion (rampant gardant argent) are drawn two axes labelled OH and OV and you must imagine a third axis OR at right angles to the paper. If you will copy the little lion and the axes on tracing paper, you can perform three simple operations. First place your tracing so that it fits exactly on the printed sketch, then turn it through 180° about OV as if you were turning the page of a book. You now have a lion facing to the right. The letter **V** can be used to describe this operation. Heavy type has been used to emphasize that **V** represents an operation, not a number as in ordinary algebra.

If the operation **V** is performed twice the lion is back where he started. It is convenient to introduce another symbol, **I**, (*I* for identical) which means that the picture of the lion is unchanged. Using this symbol we can write **V.V** Lion = **I**. Lion or $\mathbf{V}^2$. Lion = **I**. Lion. A similar statement would be true of a unicorn or of any picture. This can be expressed by writing $\mathbf{V}^2 = \mathbf{I}$, which is a very compact way of saying that the result of turning a picture twice through 180° about the axis OV is to return it to its original position.

If the symbol **H** is used to represent the operation of turning the picture through 180° about OH and **R** to represent turning through 180° about OR, we have $\mathbf{H}^2 = \mathbf{I}$ and $\mathbf{R}^2 = \mathbf{I}$. Here is the first oddity which shows that this algebra is not quite the same as ordinary algebra, $\mathbf{V}^2 = \mathbf{H}^2 = \mathbf{R}^2$ but **V**, **H** and **R** are all different. The same operation performed twice gives **I**. What happens if the operation **V** is performed and then the operation **H**? Experiment shows that the result is the same as performing the operation **R**, that is **H**. (**V**. Lion) = **R**. Lion or **H.V** = **R**.

Just as we can write out a multiplication table for numbers, so we can write out a table for operators. When two operations are performed in succession, the one performed first is printed last.

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Table 1.

	I	V	H	R
I	I	V	H	R
V	V	I		
H	H	R	I	
R	R			I

Table 2.

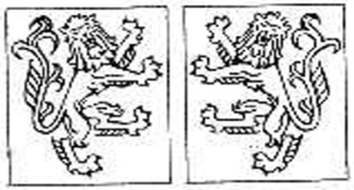
Perform the operations **R.V**, **V.R** and so on and complete the second table.

00341 58722 98053 49896 50226 29174 87882 02734

In *Mathematical Pie* No. 38 and No. 39 all possible types of repeating patterns were listed. Using operational algebra, properties of these patterns can be deduced. To describe all the possible patterns more operators are required but we can make a start with **I**, **V**, **H** and **R**.

Consider pattern A of two rampant lions face to face. It consists of the original Lion and **V**. Lion and can be described by

A = Lion + **V**. Lion = (**I** + **V**). Lion.



Pattern A

If we apply the operation **V** to A, we have **V**.A = **V**. (**I** + **V**). Lion
= ($\mathbf{V.I} + \mathbf{V}^2$). Lion
= (**V** + **I**). Lion
= **I**.A

In words, this means that the pattern A is symmetrical about OV. The four lions of pattern B are made by adding pattern A to its reflection in OH.

Pattern B

B = (**I** + **H**).A
= (**I** + **H**).(**I** + **V**). Lion
= ($\mathbf{I}^2$ + **I.V** + **H.I** + **H.V**) Lion
= (**I** + **V** + **H** + **R**). Lion

This result expresses in symbols that the pattern B consists of four lions in different positions. To test that our operational algebra works **V**.B = **V**(**I** + **V** + **H** + **R**). Lion = (**V.I** + $\mathbf{V}^2$ + **V.H** + **V.R**). Lion = (**V** + **I** + **R** + **H**). Lion = B

Similarly **H**.B = B
Now **R**.B = **H.V**.B (because **R** = **H.V**)
= **H**.B (because **V**.B = B)
= B (because **H**.B = B)

This is a formal proof of the theorem that a pattern with two perpendicular axes of symmetry has also rotational symmetry. Perhaps this is using a sledge hammer to crack a nut but it would be very difficult to prove that there are only seven types of linear pattern and seventeen types of plane pattern without using operational algebra. C.V.G.

SOLUTIONS TO SENIOR CROSS FIGURE No. 43

In the scale of 26, A=1, B=2, C=3 . . . X=24, Y=25, Z=0. The solutions are given in the scale of 26 using these letters.

ACROSS: 1. FS, 3. PI, 5. PM, 7. BZ, 9. Y, 10. BAQR, 11. CF, 12. BM, 14. EM, 16. AAL, 17. DQ.

DOWN: 1. DA, 2. BB, 3. AW, 4. TE, 6. AOAR, 8. IZ, 9. KZ, 11. AFY, 13. Uy, 14. N, 15. AE.

SOLUTIONS TO JUNIOR CROSS FIGURE No. 38

ACROSS: 1. CGQ, 4. AHZ, 6. BAQR, 8. VF, 9. BBJ

DOWN: 1. DHZ, 2. AOAR, 3. ADH, 5. GFF, 7. YU. B.A.

20922 22453 39856 26476 69149 05563 84250 39127

EARLY DIVISION

STAGE 1	STAGE 2	STAGE 3
17978	17978 12 5978 21 3878 6 3818	17978 12 5978 21 3878 6 3818 32 618 56 58 16 42
472	472 472	472 472 472
0	03	038

In the sixteenth century, division was a feat which could only be performed by a skilled mathematician. The method favoured by the Arabs and Persians (800—1400 A.D.) is shown in the example $17978 \div 472$.

472 is larger than 179, so a 0 is put in the 2 column for the answer. 472 is now written in the next three columns; 4 times 3 is 12, 7 times 3 is 21 and 2 times 3 is 6, these are written in the appropriate columns, and subtracted in turn. The 472 is then moved one column to the right and the process is repeated. Hence $17978 \div 472 = 38$ remainder 42. It will be noticed that the part you usually do in your head is all written down. B.A.

CLOCK ARITHMETIC No. 5

Powers and Roots

In the last issue you learned that $\sqrt{1}$ has *six* answers in modulo 6 arithmetic. If we now extend the idea of powers and roots, we begin by listing the results that $1^2=1$, $2^2=4$, $3^2=3$, $4^2=4$, $5^2=1$, $6^2=6$.

Then

$$1^3 = 1^2 \times 1 = 1 \times 1 = 1 \qquad 2^3 = 1^2 \times 2 = 4 \times 2 = 2 \qquad 3^3 = 3^2 \times 3 = 3 \times 3 = 3$$

$$4^3 = 4^2 \times 4 = 4 \times 4 = 4 \qquad 5^3 = 5^2 \times 5 = 1 \times 5 = 5 \qquad 6^3 = 6^2 \times 6 = 6 \times 6 = 6$$

In turn, this reveals that in modulo 6 arithmetic

$\sqrt[3]{1}=1$, $\sqrt[3]{2}=2$, $\sqrt[3]{3}=3$, $\sqrt[3]{4}=4$, $\sqrt[3]{5}=5$, $\sqrt[3]{6}=6$.

Are there any other answers for each of these cube roots?

Now use this method to complete the following table of powers for modulo 6 arithmetic

Number	Square	Cube	Fourth	Fifth	Sixth
1	1	1			
2	4	2			
3	3	3			
4	4	4			
5	1	5			
6	6	6			

What conclusions can you draw about a^{2n+1}? R.H.C.

82446 25759 16333 03910 72253 83748 18214 08835

MATHEMATICAL PIE

No. 46 Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England OCTOBER, 1965

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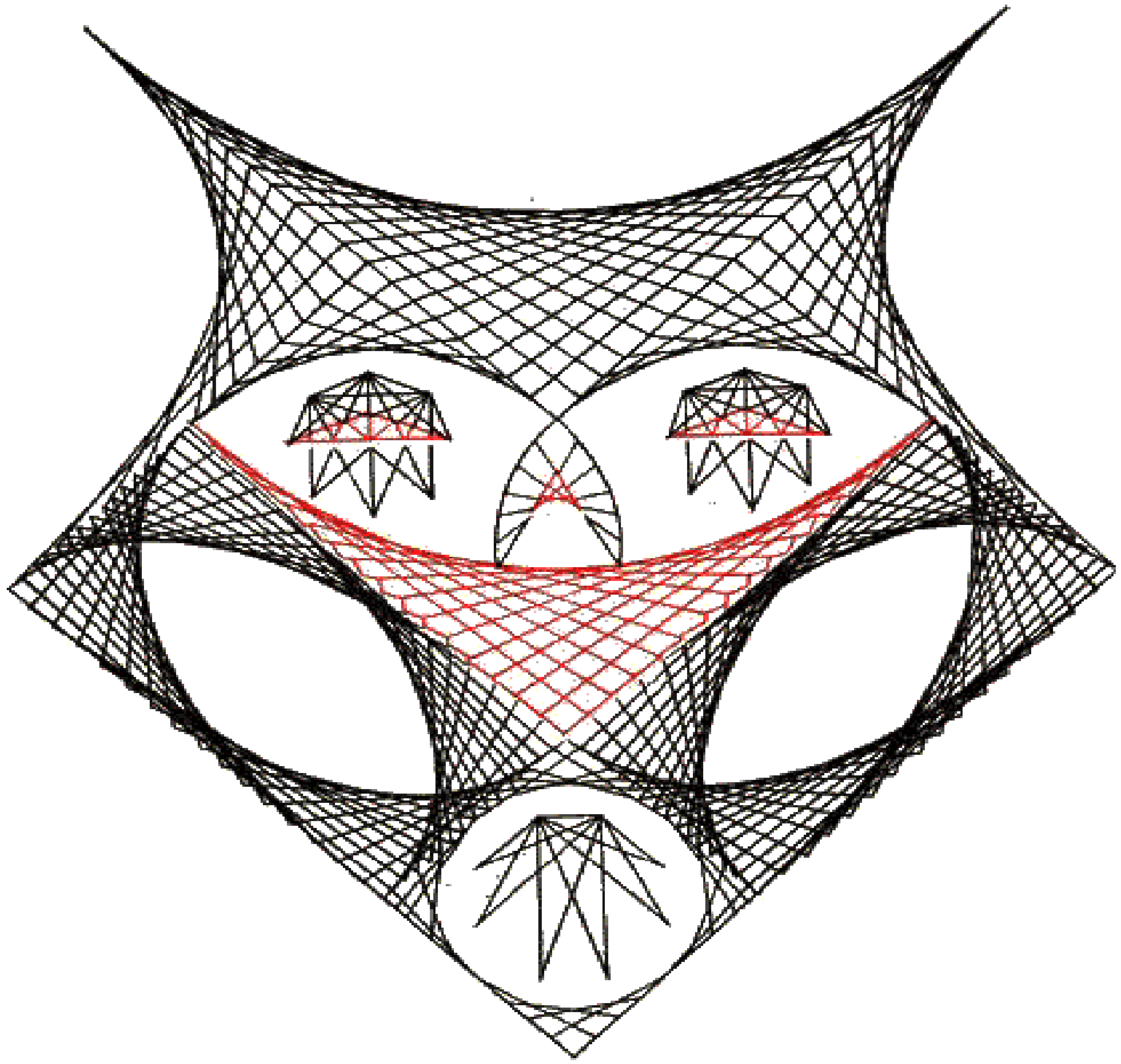
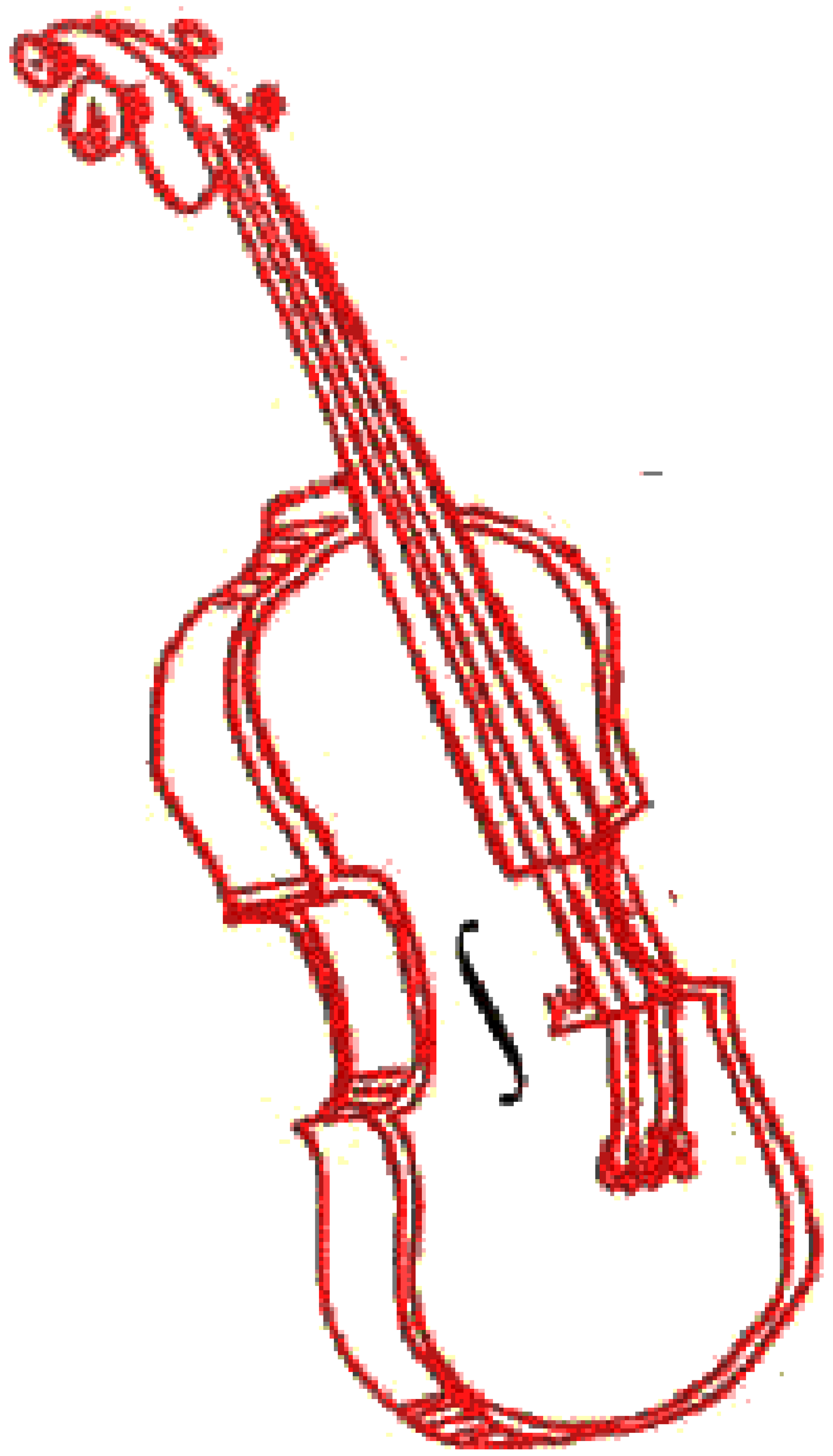


Submitted by J. R. Fox, The City School, Lincoln

65583 43434 76933 85781 71138 64558 73678 12301

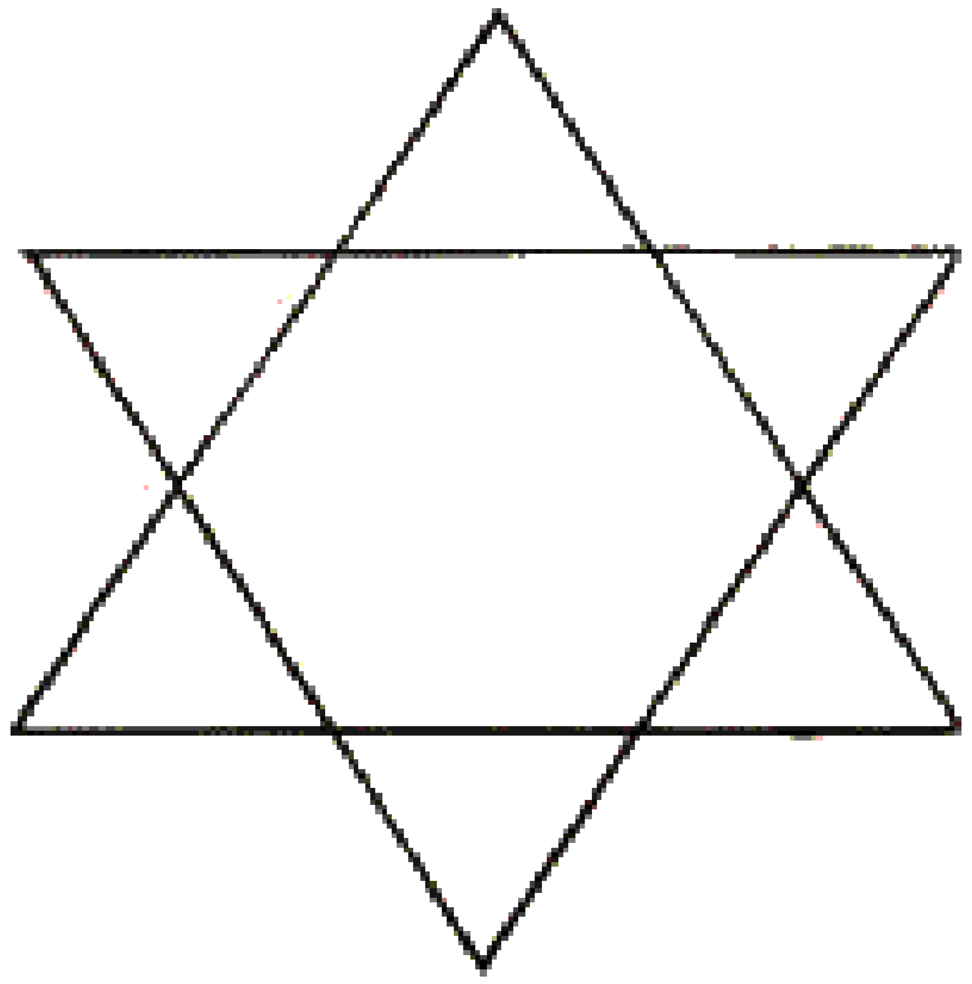
THE ORIGIN OF $\int$ — MATHEMATICAL FIDDLING?

The beginner in algebra can be compared to a traveller making a cross-country walk. Every now and then, the traveller has to exert himself to climb over a stile. The first 'stile' that must be overcome in algebra is that of learning to calculate with letters instead of figures. There is no reason why figures should not be used in some algebra but, if long numbers are involved, a lot of time would be wasted in just writing: the use of symbols, therefore, is a time-saver.



The other symbols such as +, —, ×, and ÷ should already be well known to the beginner because it is customary to teach arithmetic before algebra. The next stile, therefore is probably the use of brackets, to be followed in due course by other (and maybe higher) stiles such as equations, indices, factorisation, and so on. The beginner will not meet any new symbols for the next two or three years until he is one day faced with $\int$.

If the beginner becomes an engineer or a physicist, he will use $\int$ many times until he regards it as an old friend. It is a symbol first used by the celebrated Leipzig mathematician, Gottfried Wilhelm Leibniz (1646–1716) in a manuscript dated 29th October, 1675. The significance of $\int$ is that a large numb^er of very small quantities placed behind the sign are to be added together according to certain rules to provide what is called an *integral*. The integral is not always easy to find and sometimes a little 'inspired guesswork' is helpful. For this reason non-mathematicians may think of the process as 'fiddling.' J.F.H.



STAR DISSECTION

Submitted by Canon D. P. Eperson.

A regular hexagon can be arranged in a square by cutting it into five pieces. A hexagonal star is more complicated but it is surprising that this can be re-arranged to form a square by cutting it into five pieces.

Can you find the cuts that must be made?

45876 87126 60348 91390 95620 09939 36103 10291

JUNIOR CROSSFIGURE No. 39

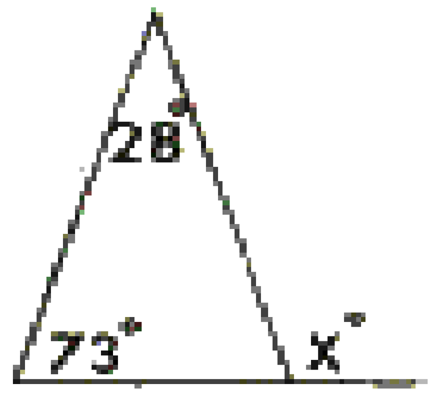
CLUES ACROSS:

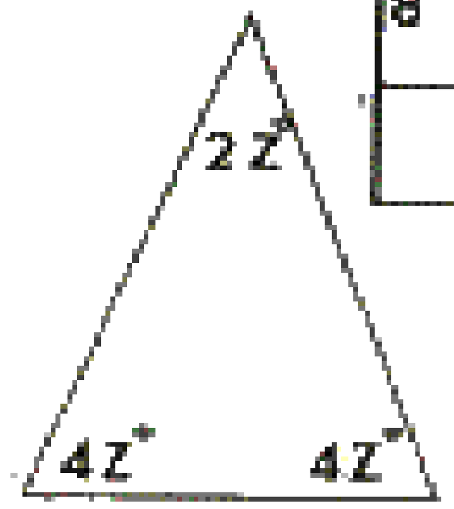
1. A perfect square.
4. $2p+3=61$, find p.
5. Average of $(63-y)$, 20, $(y+1)$.
6. Consecutive digits.
8. A multiple of 41.
10. Find x.

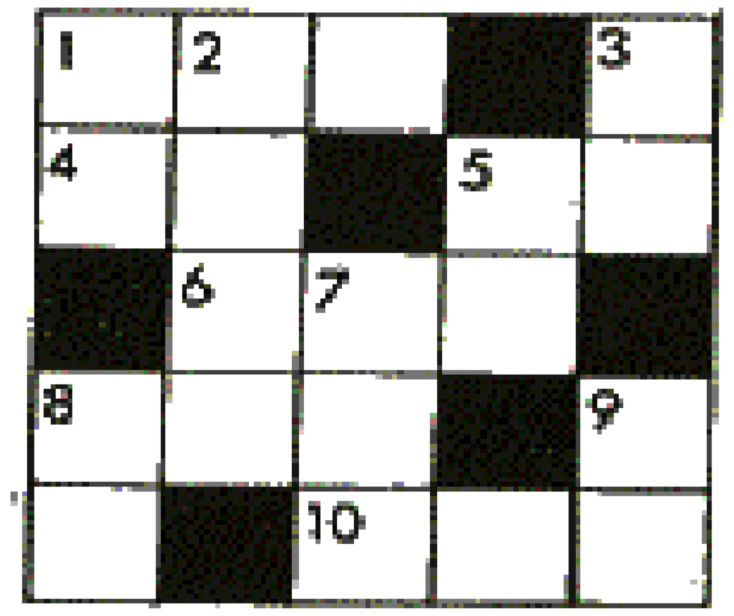
CLUES DOWN:

1. 2^5.
2. $7 \times 103 - 1$.
3. Find z.
5. Evaluate $3t^2+5t+5$ when $t=2$.
7. a^2-b^2 when $a=40$ and $b=27$.
8. Correct 62.4 to two significant figures.
9. $\sqrt{121}$.

B.A.







ENCIRCLING MOVEMENT

Three sides of a cyclic quadrilateral are 3″, 4″, 5″. The angle between the first two sides is 120°. What is the length of the remaining side? J.G.

SOLUTIONS TO ISSUE No. 45

TWO FOR A PENNY

1. 503 acres. 2. $\frac{1}{4}(2\sqrt{3}-\pi)$. 3. $1\frac{1}{2}$ inches.

ALGEBRA CORNER No. 3

1. $x^2+y^2+z^2-2xy+2yz+2zx$. 2. Using the result of part 1, the sum of the edges $=\sqrt{(6^2+64)}=10$ inches. 3. $c^2=ab$ if a and b are not both zero. 4. $a=5$, $b=2$. 5. The area is $2a(a^2+1)$ square inches. 6. The angle between the fingers is $\frac{1}{2}(60y-11x)$ or $360-\frac{1}{2}(60y-11x)$.

AT THE CROSS WAYS

The solution will appear in the next issue.

THE ROARING FORTIES

I = 1, F = 2, S = 3, X = 4, E = 5, Y = 6, R = 7, T = 8, O = 9, N = 0.

SENIOR CROSSFIGURE No. 43

CLUES ACROSS: (1) 175, (3) 425, (5) 429, (7) 52, (9) 25, (10) 36288, (11) 84, (12) 65, (14) 143, (16) 714, (17) 121.

CLUES DOWN: (1) 105, (2) 54, (3) 49, (4) 525, (6) 28284, (8) 234, (9) 286, (11) 857, (13) 571, (14) 14, (15) 31.

The clue to 13 Down should have read $q^3+(yp)^2=y$.

PROBLEM CORNER

Printer's Error.—No other set of four digits with this property is known.

Enclosed. The greatest area is a circle, $40{,}000/\pi$ square yards. The square of area 10,000 square yards encloses the greatest rectangular area.

It is not cricket

12! = 479,001,600 and not 579,001,600 as stated.

HANDICAPPED

The girl touched the wall $5\sqrt{3}$ yards from C. The distance covered was $20\sqrt{7}$ yards. It has been suggested that she should be promoted to the Sixth.

JUNIOR CROSSFIGURE No. 38

CLUES ACROSS: (1) 2228, (4) 884, (6) 36288, (8) 568, (9) 1414.
CLUES DOWN: (1) 2835, (2) 28284, (3) 888, (5) 4884, (7) 661.

B.A.

03375 11173 54719 18530 46449 02636 55128 16228

EXTENDING THE MULTIPLICATION TABLES

Submitted by W. J. Davies, Greenhill Grammar School, Tenby.

Robert Recorde, in his " Grounde of Artes," explained how to multiply by a number between 5 and 10 because he did not advise his readers to learn the tables for multiplying beyond five, in the early stages of Arithmetic.

Fig. 1

To multiply 6 by 7, place the 6 and the 7 at the left hand corners of a cross, as in figure 1, subtract 6 and 7 from 10 and place these results at the right hand corners. Multiply 4 by 3 giving 12. The 2 is put in the units column and the 1 is carried into the tens column. Take 3 from 6 (or take 4 from 7), and place the result 3 in the tens column making 4 with the 1 that was carried. The final answer is 4 tens and 2 units, or 42.

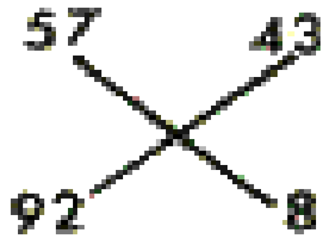
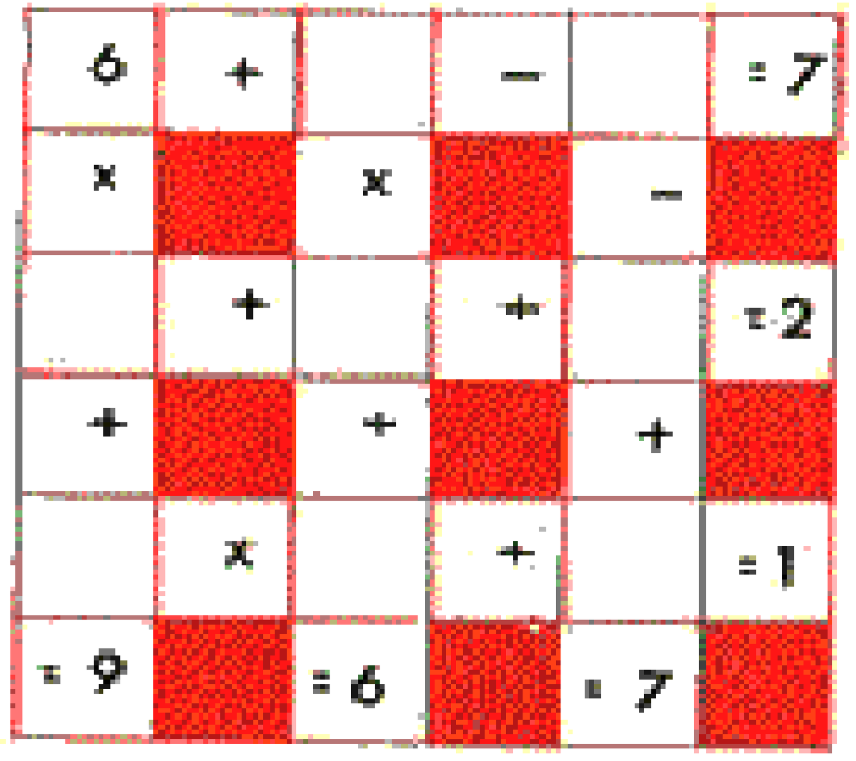


Fig. 2

It is interesting to note that this method may be extended to the multiplication of numbers between 50 and 100 or 500 and 1,000, etc. Thus to multiply 57 by 92, we could proceed as follows : $43 \times 8 = 344$. This time we put down 44 and carry 3 into the hundreds column. $57-8$ (or $92-43$) $=49$, to which the 3 must be added to give 52. The answer is therefore 5244.

This extension of Recorde's method was " discovered " by a Form IIIA at the Greenhill Grammar School, Tenby. Can you prove why it works?



WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. B.A.

A STRIKING PROBLEM

If a clock takes six seconds to strike six, how long does it take to strike (1) eleven, and (2) twelve?

A BREAKFAST TIME PROBLEM

What is the quickest way to toast three slices of bread on both sides, using a double-sided electric toaster?

BULL'S EYE

Ten concentric circles are drawn, so that the areas between two consecutive circles are all equal to the area of the inside circle. The radius of the inner circle is 1 inch, what is the radius of the outer circle? J.G.

UPWARDS EVER UPWARDS

A cube of six inch edge stands on a table. An insect crawls from one of the lower corners to the middle of the top face, but cannot climb at an angle greater than 30° with the horizontal. What is the least length of its journey? J.G.

33467 68514 22342 77379 30375 87034 43661 99106

FOR EXPERIENCED MATHEMATICIANS

It is a great pleasure for us to know that *Mathematical Pie* is read with interest by many mature readers. The following problem is directed mainly towards such readers.

Three circles, radii 3", 4", 5" touch each other. They are enclosed by the external common tangents forming a triangle. It is required to calculate the lengths of the sides of this triangle.

Younger readers will find it interesting to solve the problem by scale drawing. A book token will be given for the best solutions. J.G.

STEPPING IT UP !

Find a value for x that satisfies the equations (a), (b), (c), (d) :—

$$\text{(a) } x^{2^3}=64,\ \text{(b) } 2^{x^3}=64,\ \text{(c) } \left(2^3\right)^x=64,\ \text{(d) } 2^{\left(3^x\right)}=64$$

Now find a value for y that satisfies (e), (f), (g) :—

$$\text{(e) } y^{2^3}=4096,\ \text{(f) } 2^2.2^{2^y}=4096,\ \text{(g) } 2.2^{2^{2^y}}=4096.$$

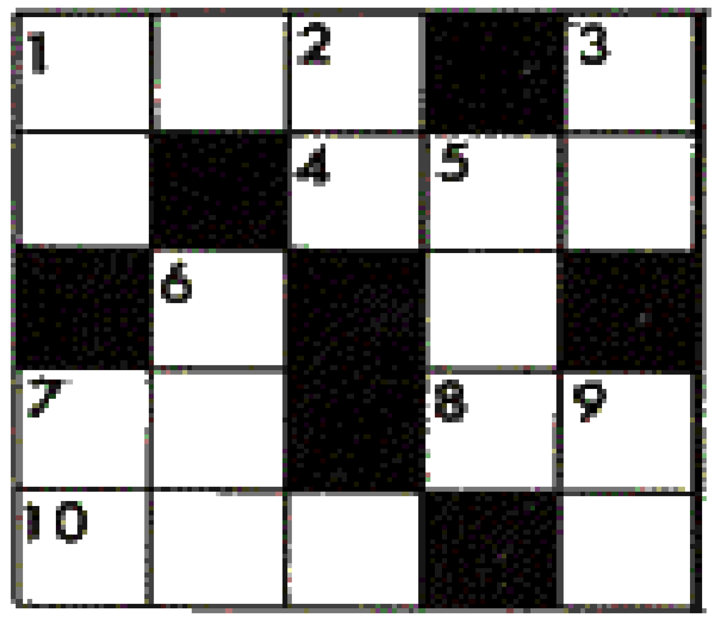
J.F.H.

Hip-hip-hoo-rah. Hip-hip-hoo-ray,
A tin of fruit for tea today !
First the label round you'll see :
In size it's π times h times d.
Next the contents you will try—
Of course they're r^2 h times π.

PASSING BY

Two trains of equal length pass each other on parallel tracks each travelling at 50 miles per hour. An observer on one of the trains finds that it takes the other 4 seconds to pass him. How long are the trains? R.H.C.

SENIOR CROSSFIGURE No. 44



CLUES ACROSS :

1. $\dfrac{3}{.01} + \dfrac{1.1}{.02}$
4. Convert 624.8 ft. per sec. to miles per hour.
7. Area in sq. in. of a trapezium with parallel sides 7" and 4", 6" apart.
8. Find the shortest side of a right-angled triangle, two of whose sides are 35" and 28".
10. Taking $\pi = 3\frac{1}{7}$, find the volume of a cylinder of radius $3\frac{1}{2}$" and height 6".

CLUES DOWN :

1. If $a=2$ and $b=-3$, evaluate a^3-b^3.
2. Find x when $\dfrac{x+2}{7} - \dfrac{x-6}{8} = 2$.
3. The simple interest on £360 for 4 years at $2\frac{1}{2}$ per cent. per annum.
5. Find the width, in yards, of a rectangular field of 22 acres which is $\frac{1}{4}$ mile long.
6. $7x$ where $3x+4y=-11$ and $5x+6y=-7$.
7. The smallest angle of a triangle whose angles are $8x°$, $17x°$ and $20x°$.
9. The larger root of the equation $2x^2-21x-36=0$. B.A.

62615 28813 84379 09904 23074 73363 94804 57593

MATHEMATICAL EMBROIDERY

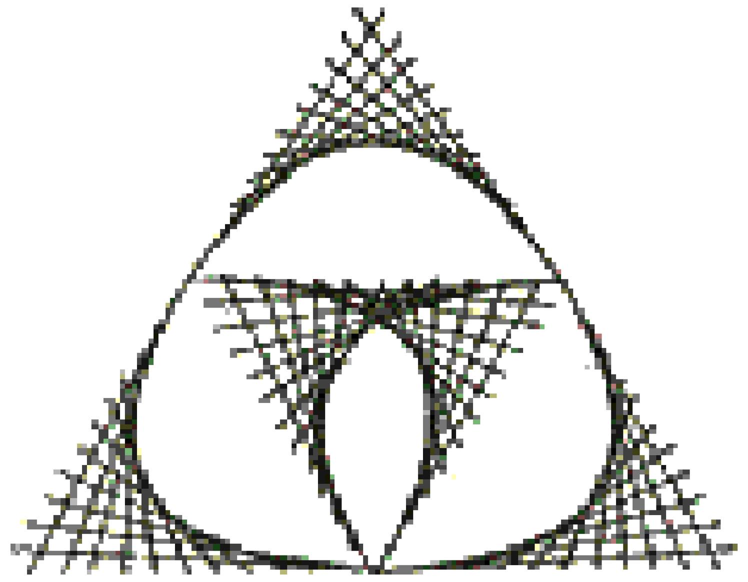


Fig. 1

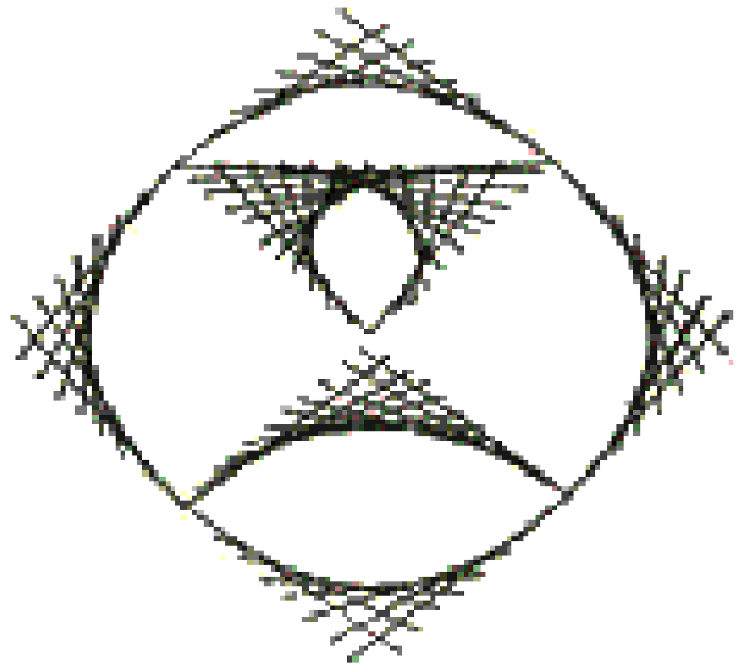


Fig. 3

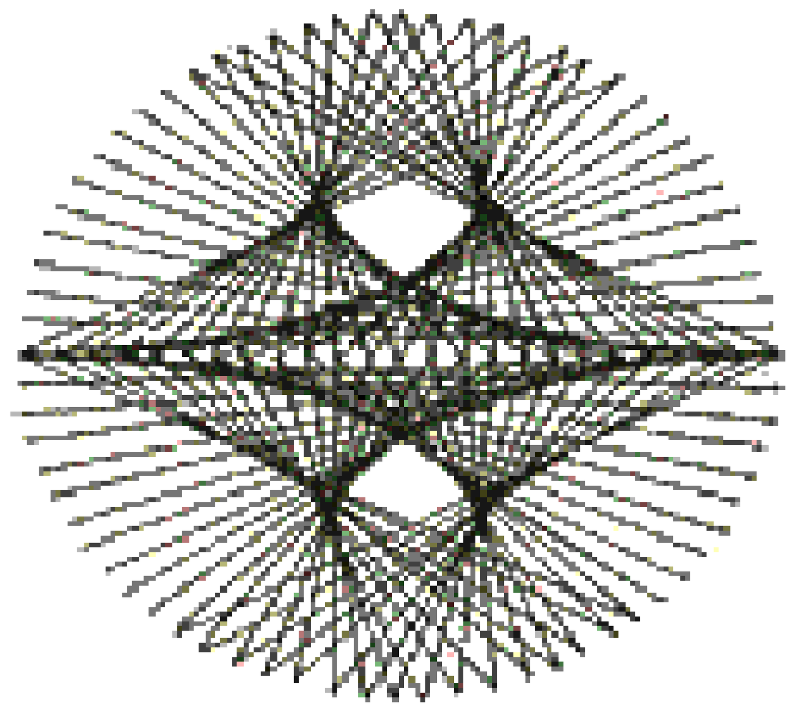


Fig. 5

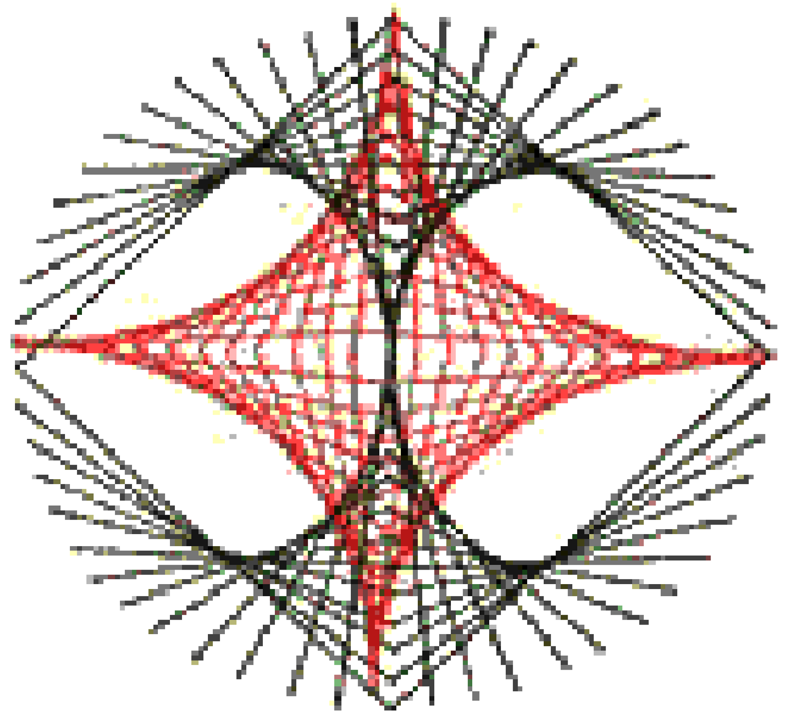


Fig. 7

The centre pages of issue No. 25 were devoted to Mathematical Embroidery and such interest was aroused that the issue was quickly sold out. So many requests have been made for copies that the basic ideas are repeated with new illustrations. We are indebted to Mrs. B. J. Atkins and last year's IIB form of the Newland High School, Hull, for the basis of the examples.

Many interesting patterns may be produced by stitching to order on card or more substantial material. Holes are made, usually along a straight line or a circle, to a rule; they may be equally spaced but this is not essential. The holes are then joined to a pre-determined pattern with thread. The final design is usually attractive but can often be improved by using blending or contrasting colours of thread for different parts.

Figures 1, 3, 7, 8 and 9 are all based on parabolas which are the easiest curves to produce. Two lines are drawn and the same number of equally spaced holes are made along each. The first hole of one line is sewn to the last hole of the second line, the second hole of the first line is sewn to the next to the last hole of the second line, and so on until each hole on one line is sewn to one hole of the second line. The " envelope " of the " stitches " is a parabola, (a curve like $y = x^2$), as sixth formers may be able to prove mathematically.

Figures 4, 5 and 6 consist of holes on circles sewn in various ways which are obvious. Figure 2 consists of three " curves of pursuit." These are the paths, (*i.e.*, loci) followed by three dogs each starting from a vertex of a triangle and running directly towards the next dog taken in order around the triangle.

Chainstitch may be used to pick out lines and various types of " filling in " stitch may be used to produce areas, which can be worked into patterns. The standard text on the subject is one published in 1906 by Miss E. L. Somervell, *A Rhythmic Approach to Mathematics*, but a simplified explanation can be found in " *Curve Stitching* " by C. Birtwistle of the Association of Teachers of Mathematics. B.A.

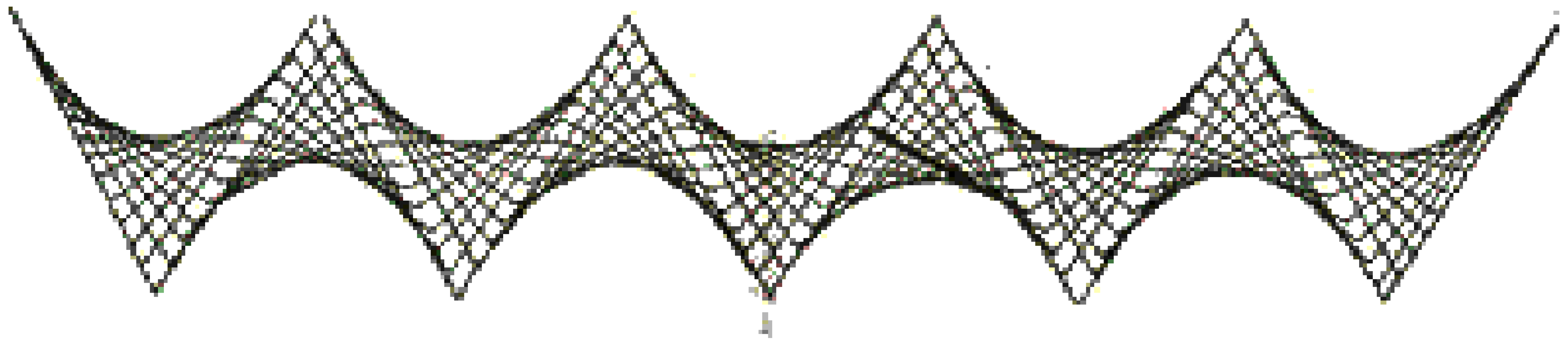


Fig. 8

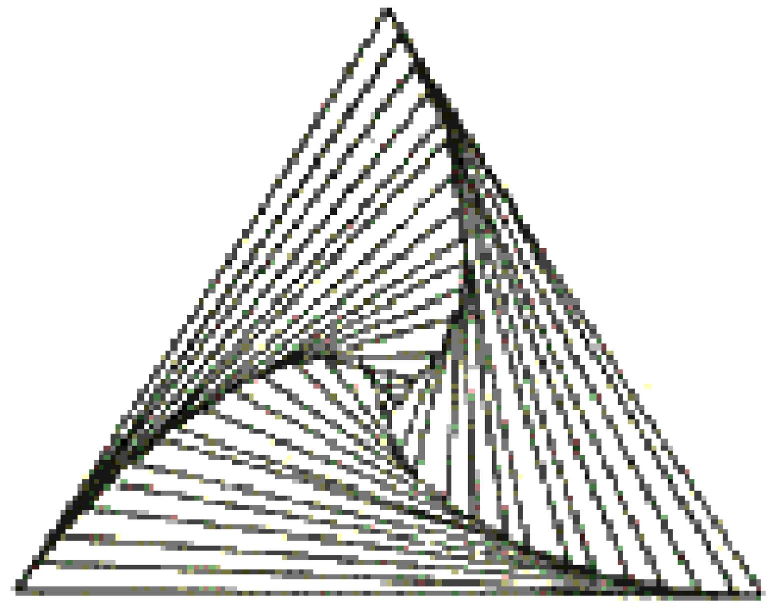


Fig. 2

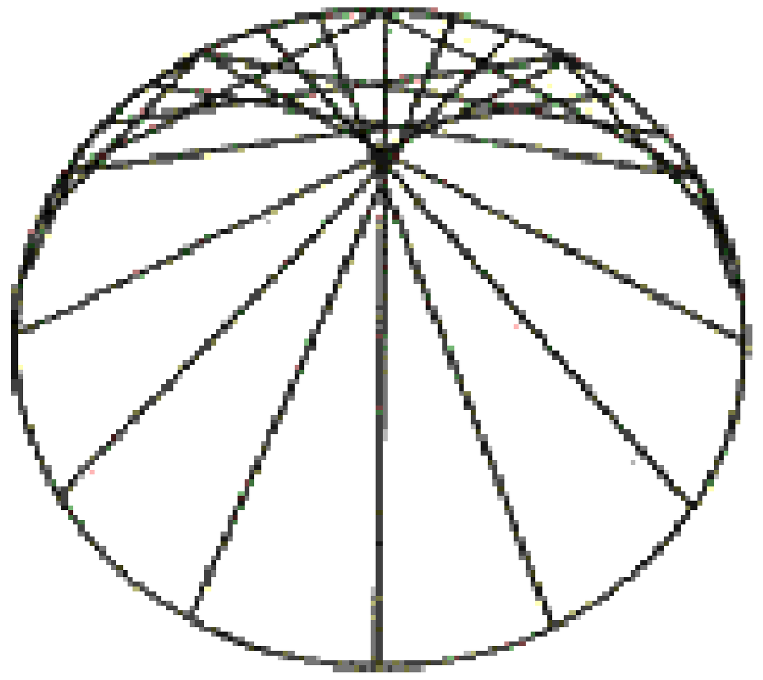


Fig. 4

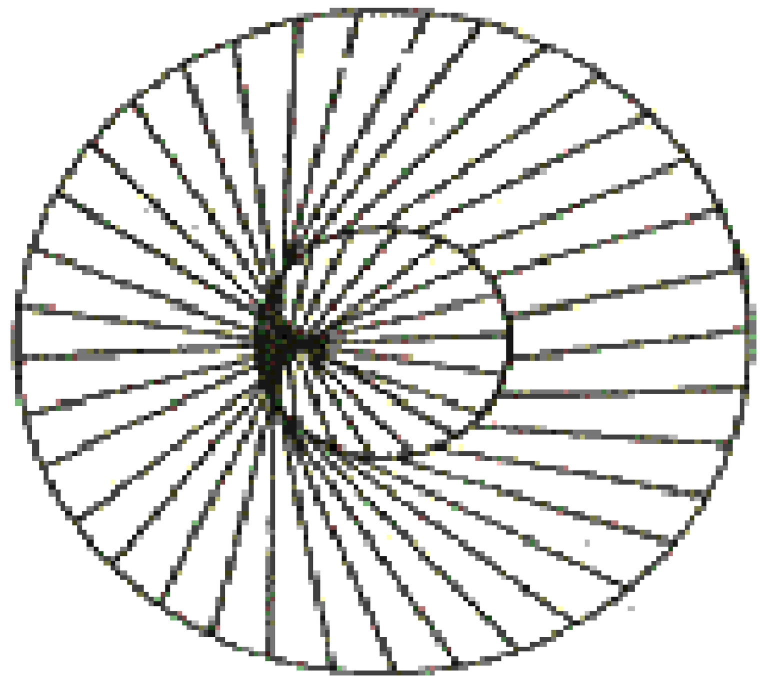


Fig. 6

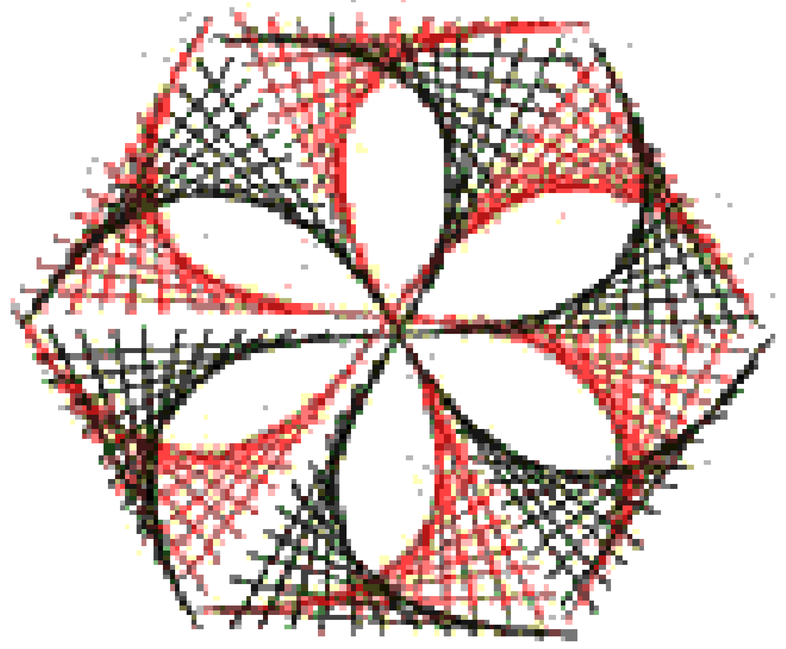


Fig. 9

14932 40529 76347 57481 19356 70911 01377 51721 00803 15590 24853 69066 92037 67192 20532 29094

QUADRATIC EQUATION SOLVER: A 'Slide Bottle' which revolutionises the solving of quadratic equations:

Materials required: a plastic cylindrical bottle (several washing-up liquid brands are most suitable); a sheet of transparent plastic or thick cellophane paper; two sheets of graph paper.

Construction:

1. Cut a sheet of graph paper to completely cover the curved surface of the bottle.

2. Draw on the horizontal and vertical axes for x and y, marking in values. A satisfactory range can be obtained by letting $-8 \leq x \leq +8$ and $-16 \leq y \leq +16$. (The range of x will limit the value of 'x' which can be found in $ax^2 + bx + c = 0$, while the range of y will limit the value of 'c').

Fig. 1

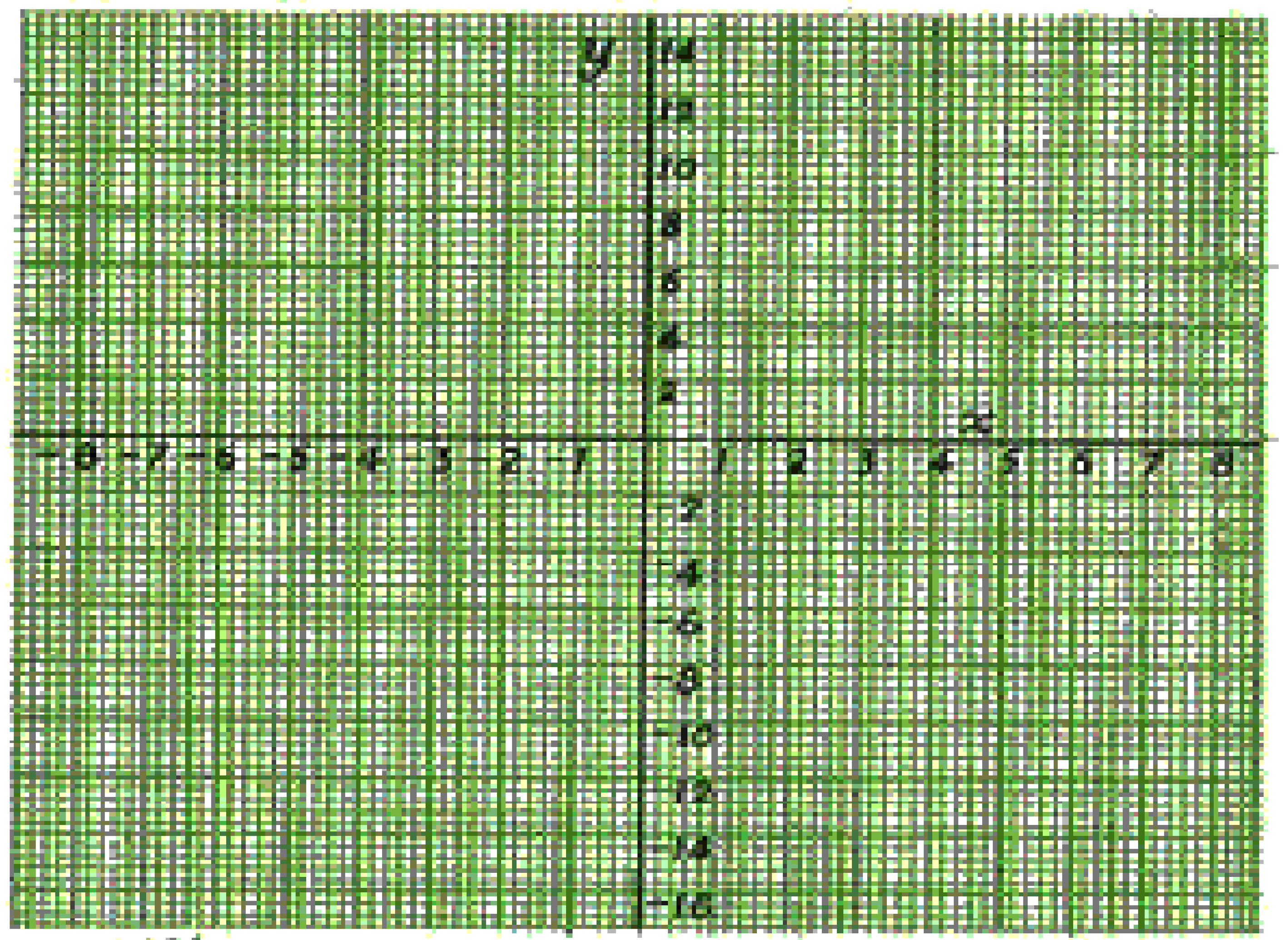


Fig. 3

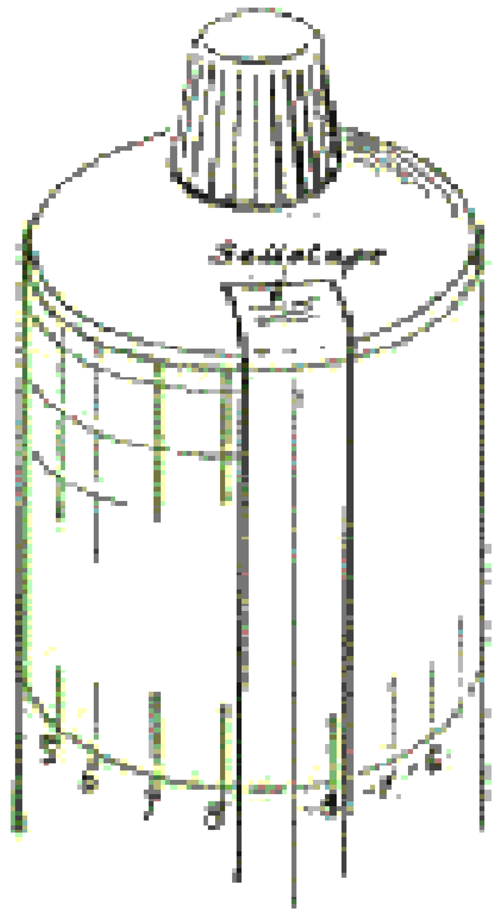


Fig. 2

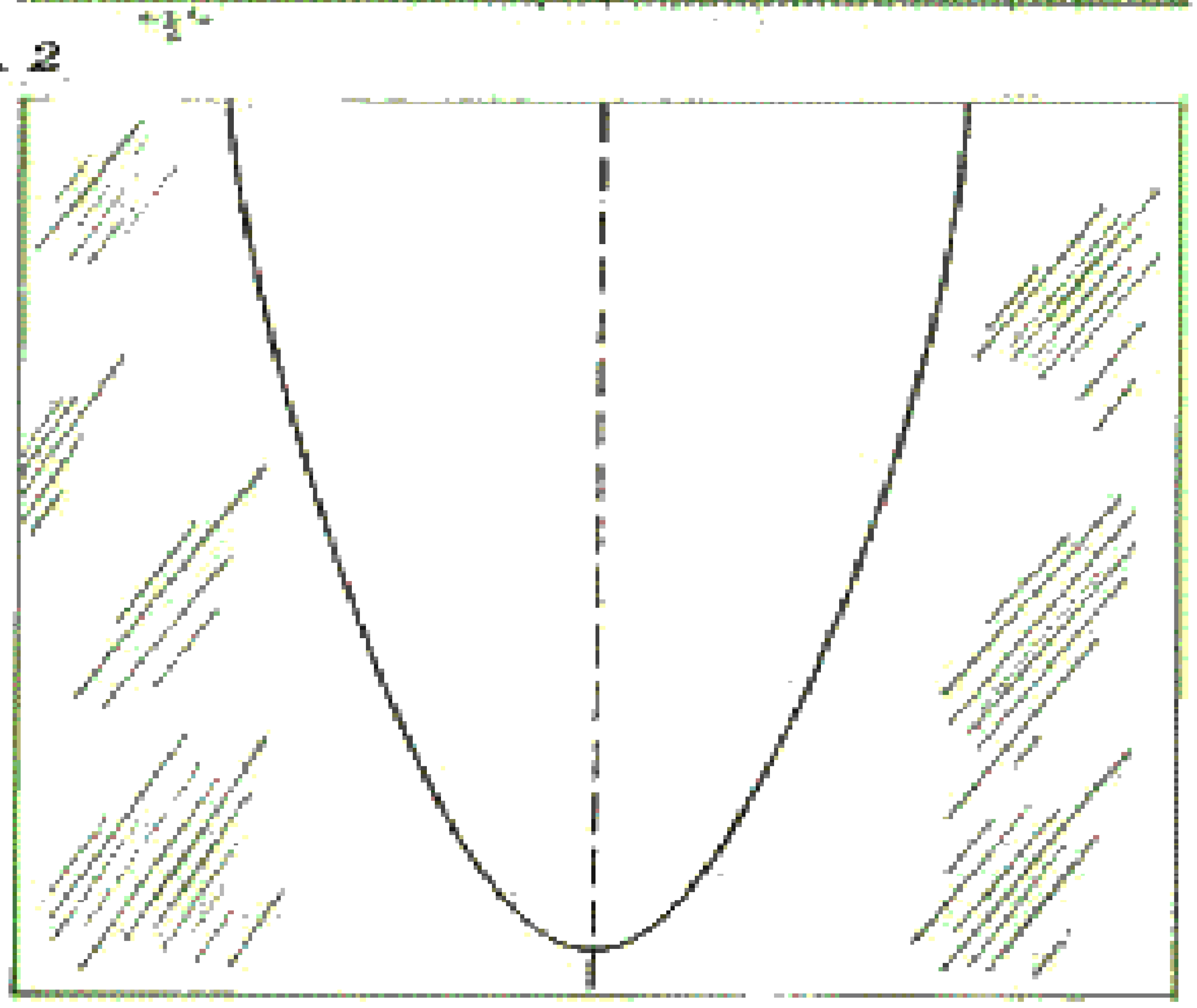
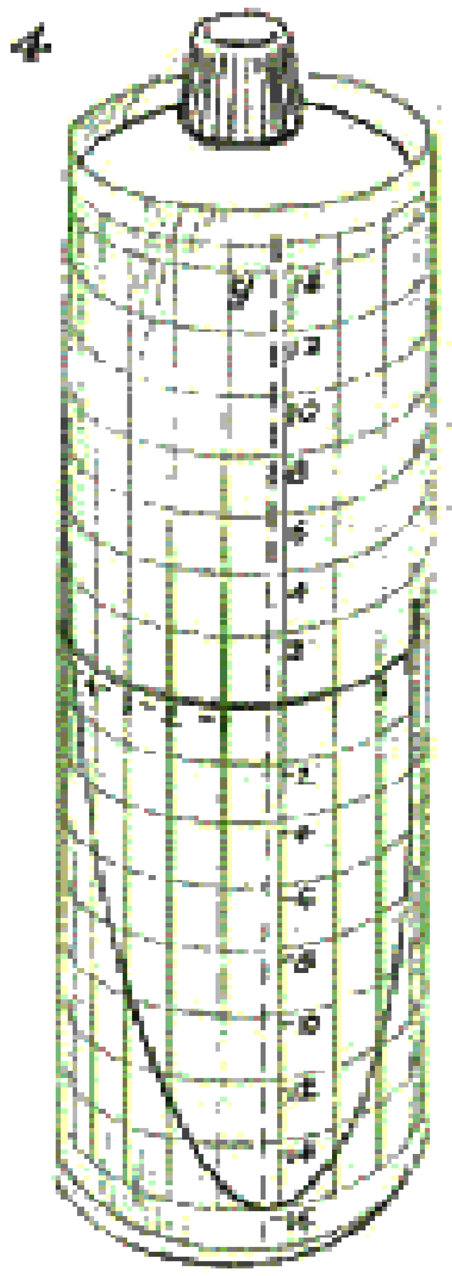


Fig. 4



3. Wrap the graph paper around the bottle securing with adhesive tape.

4. On the second sheet of graph paper mark in identical axes and, very carefully, draw the graph $y = x^2$.

continued on page 366

mathematical pie

No. 47 — *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England — FEBRUARY, 1966

PASCAL'S TRIANGLE

The diagram shows a Chinese version of the triangle of Pascal, which was known to the Chinese in 1303 and rediscovered by Pascal some 300 years later.

Each line in the diagram corresponds to the coefficients of a binomial expansion, $(1+x)^n$, in ascending powers of x, the values of n varying from 0 to 8.

Use the diagram to write out the Chinese equivalent of the Arabic symbols for 1, 2, 3, 4, . . .

B.A.

(2,25) is a pair of numbers such that the difference between the product and the sum of the pair is 23, *i.e.*, $2.25 - (2+25) = 23$. Find three more pairs of numbers which have the same property. (Hint—the numbers you use are less than 14).

Now find a pair of numbers such that their product-sum difference is 215.

R.H.C.

3	−		×	4	=
+		+		×	
	+		÷		= 3
÷		÷		÷	
2	−		+	6	=
=		= 1		= 2	

WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. B.A.

If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the approximate weight of water falling on an acre of land for a rainfall of 1 inch?

What is the weight of water falling on a city of area 50,000 acres?

J.F.H.

A snail crawls up a post which is 20 ft. high. During the day it climbs 3 feet, but slips back 1 foot during the night. How many days will it take to reach the top? S.T.P.

EQUALLY DISPERSED

My "Gardeners' Handbook" says that onions should be planted 1 foot apart. How many could I plant inside and on the perimeter of a circle 6 feet in diameter if each is planted exactly 1 foot from its immediate neighbours? J.G.

A CUTTING PROBLEM

A wooden hemisphere is reduced to the largest possible cube by 5 plane cuts. If the hemisphere has a radius of 12 inches, what is the volume of the cube? J.G.

continued from page 372

5. Place the transparent sheet over this curve and trace the impression of the curve and the vertical axis of symmetry in Indian ink.

6. When dry, wrap this sheet around the bottle superimposing the axis of symmetry exactly over the y axis on the graph paper, with the turning-point as a minimum. Secure with adhesive tape to form a sliding sleeve which can be moved horizontally and vertically.

Use: to solve $x^2 - x - 6 = 0$.

1. Find minus half the coefficient of 'x', *i.e.*, $-\frac{1}{2}(-1) = +\frac{1}{2}$, and move the sleeve so that the axis of symmetry is in this position, *i.e.*, $x = \frac{1}{2}$.

2. Taking care to move the sleeve only vertically, arrange it so that the curve intersects the 'y' axis at the value of 'c,' *i.e.*, -6.

3. The roots of the equation can be found where the curve intersects the 'x' axis, *i.e.*, $x = -2$ and $x = +3$.

For equations of the general type $ax^2 + bx + c = 0$, first divide through by 'a' and then continue as in 1 — 3 above. D.I.B.

JUNIOR CROSS FIGURE No. 40

Submitted by Terence Morgan, Filton High School, near Bristol.

1	2	3		4
	5			
6				
7			8	
9				

Ignore decimal points and work to the appropriate number of significant figures.

CLUES ACROSS:

1. Volume of a cylinder, radius 7", height $2\frac{1}{4}$", take $\pi = \frac{22}{7}$.
5. Sine of 22°36'.
6. Find the principal, in £, which yields £15 at 5% in 2 years.
7. The radius of a circle of area 24·62 sq. in., log π = ·4971.
8. Area, in sq. units, of a triangle with $a = 7$, $b = 20$, $\angle C = 30$.
9. Length of the diagonal of a square with sides 9".

CLUES DOWN:

2. Square of 8·578.
3. Number of gallons in 60 bushels.
4. $n = 13$, $x = 3$, and $y = 4$. Evaluate $\dfrac{nx + ny}{x + 4}$.
6. The square of 11.
8. The number of yards in $1\frac{1}{2}$ chains.

SOLUTIONS TO PROBLEMS IN ISSUE No. 46

FOR EXPERIENCED MATHEMATICIANS

Owing to the large number of solutions, the answer will be given in issue No. 48.

STEPPING IT UP!

(*a*) $x = 2$, (*b*) $x = 2$, (*c*) $x = 2$, (*d*) $x = \log 6/\log 3$, (*e*) $y = 4$, (*f*) $y = \log 10/\log 2$, (*g*) $y = \log 11/\log 4$.

PASSING BY

Each train is $\frac{1}{11}$ mile, or a little over 162 yards.

SENIOR CROSS FIGURE No. 44

CLUES ACROSS: (1) 355, (4) 426, (7) 33, (8) 21, (10) 231.
CLUES DOWN: (1) 35, (2) 54, (3) 36, (5) 242, (6) 133, (7) 32, (9) 12.

WITHOUT A WORD

$6 + 4 - 3 = 7$, $1 + 3 + 2 = 2$, $3 \times 2 \div 6 = 1$.

A STRIKING PROBLEM

Each pause takes six-fifths of a second; eleven strokes takes ten pauses, hence twelve seconds. Twelve takes thirteen and one-fifth seconds.

A BREAKFAST-TIME PROBLEM

Toast side 1 of slices 1 and 2. Turn slice 1 and replace slice 2 by slice 3 side 1. Turn slice 3 and replace slice 1 by side 2 of slice 2.

BULL'S EYE

The radius of the outer circle is $\sqrt{10}$ inches.

UPWARDS EVER UPWARDS

The least journey is twice the height of the cube to the edge of the top plus half to the centre.

JUNIOR CROSS FIGURE No. 39

CLUES ACROSS: (1) 361, (4) 29, (5) 28, (6) 987, (8) 697, (10) 101.
CLUES DOWN: (1) 32, (2) 6999, (3) 18, (5) 27, (7) 871, (8) 62, (9) 1.

ENCIRCLING MOVEMENT

The fourth side is $\frac{1}{2}(5 + \sqrt{73})$ or 6.77 inches.

CLOCK ARITHMETIC No. 5

$a^{2n+1} = a$ B.A.

intersect at V. X is distant d_1 from V as before. Using X as pole, draw a series of rays intersecting AA^1 and BB^1. Choosing one of these lines, say AA^1, mark off distances $=d_2$ along the rays on both sides of the line giving a series of points Y_1 Y_2 - - - and Z_1 Z_2 - - -. By joining the two sets of points as shown, two parts of a curve (called a *conchoid*) are drawn, one on either side of AA^1. If the curve cuts BB^1 at P_2 and Q_1, the rays XP_2Q_2 and XP_1Q_1 provide solutions of the problem. Note that a second conchoid can be drawn by marking off d_2 from BB^1. This conchoid, shown in broken lines, provides the same solutions as the first.

For this method of solution, which can be worked by anyone capable of using a ruler and dividers, nothing has to be known about higher mathematics. It is a useful exercise to construct a set of conchoids by keeping XR (Fig. 3) constant and taking different value for RS $(=d_2)$: when RS is made greater than XR, the part of the curve on the same side of AA^1 as X assumes an interesting form.

Which method of solution appeals to you as (*a*) more elegant, (*b*) more practical, (*c*) having taught you most? J.F.H.

The operation * is defined by

$$a*b = \frac{a+b}{1+ab}.$$

Show that the operation is associative and evaluate

(i) $1*2*3*\ldots\ldots\ldots*n$,

(ii) $\frac{1}{2}*\frac{1}{4}*\frac{1}{6}*\ldots\ldots\ldots*\frac{1}{28}$. C.V.G.

AL'S ANALYSIS

Do you remember our American friend Al Gebra, from issue No. 43, and his problem about the number of ways of going to work? Do you also remember how Pascal's triangle gave him the solution?

Al has a son, Little Alph, and last night Al found him struggling with his Mathematics Assignment (homework to you). Alph was groaning over the fact that old Chalky had given him $(a+b)^7$ to work out and it was going to take him hours, and he'd fixed to go to a movie with his friend Trigger......Al had a look at what his son had written and it struck him as vaguely familiar. Here it is:

$$a+b$$
$$(a+b)^2 \qquad a^2+2ab+b^2$$
$$(a+b)^3 \qquad a^3+3a^2b+3ab^2+b^3$$

Then the penny dropped and seizing a pencil Al wrote it out again without the *a*'s and *b*'s.

1 1

1 2 1

1 3 3 1

and he was able to add the next line with very little work:

1 4 6 4 1

It was his old friend Pascal's Triangle again. Do you remember how to go on from line to line? If you do you will be able to write down the expression for $(a+b)^7$ as easily as Al did. Alph and Trigger said it was a good movie. Al's efforts will help you with the problem on the front page. R.M.S.

58382 92041 97484 42360 80071 93045 76189 32349

ODDS ON

Everything was going with a swing at the Prefects' Valentine Dance. In the Paul Jones, David was pleased to find that his first partner was Jacqueline. He could not believe his luck when they were partners again for the second time and when the circles stopped for the third time, he was staggered to find himself face to face with Jacky once more.

"It's a chance in a million," he said to her (amongst other things).

"Don't be silly, Dave," said she, "it's only about 1 in 6,000 and in any case I'm the lucky one because there are three more girls here than boys."

The rest of the conversation was rather private so we'll skip it. Actually, Jacky was wrong because she had not noticed that Rob and Lindsey were sitting out in the corner behind the piano and this reduced the odds to 4912 to 1 against. How many prefects of each sex were at the party? R.M.S.

Pythagoras and his Squares!

Archimedes and the Twist!

If a bag contained twelve balls, four red, four white and four blue, what is the smallest number you would have to withdraw to ensure that you had

(i) Two balls of the same colour

(ii) One ball of each colour

(iii) Three red balls? S.T.P.

SENIOR CROSS FIGURE No. 45

Submitted by Paul J. Castle, King Edward's School, Birmingham.

CLUES ACROSS:

1. $2y^2+y$.
3. a.
5. $(b+2)c$ (palindromic).
6. $xz+\frac{1}{2}y$.
8. $d(x-y)^2$.
10. $\left(\frac{y^2}{2}\right)^2$
11. $2x+z+2(d-x)^2$.
12. $2y^2+z+2$.

CLUES DOWN:

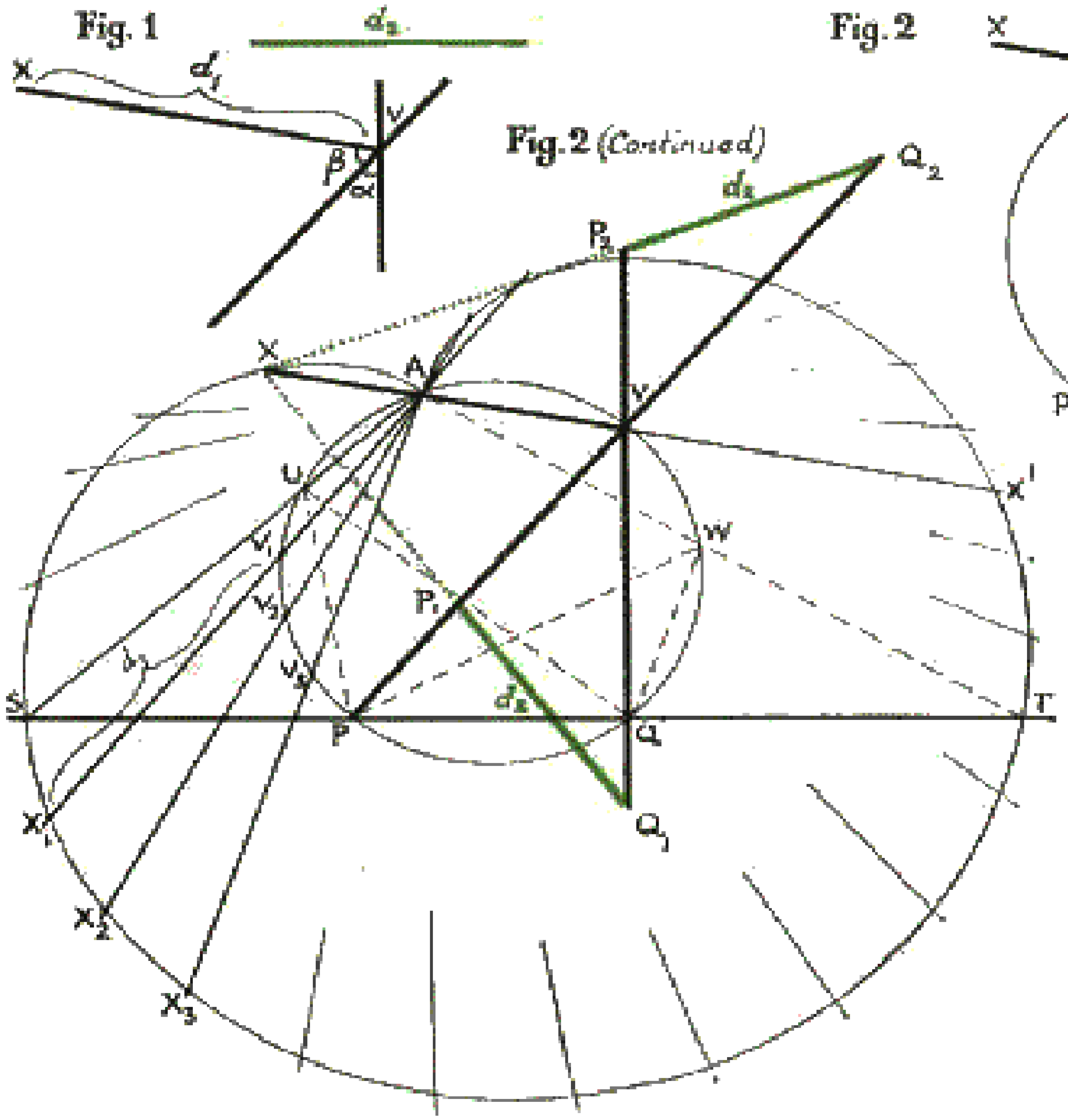
1. $y+3z$.
2. bz.
3. cx.
4. $3a$.
7. fx.
8. $(z-1)(x-2)$.
9. y^3.
10. $z-2y$.

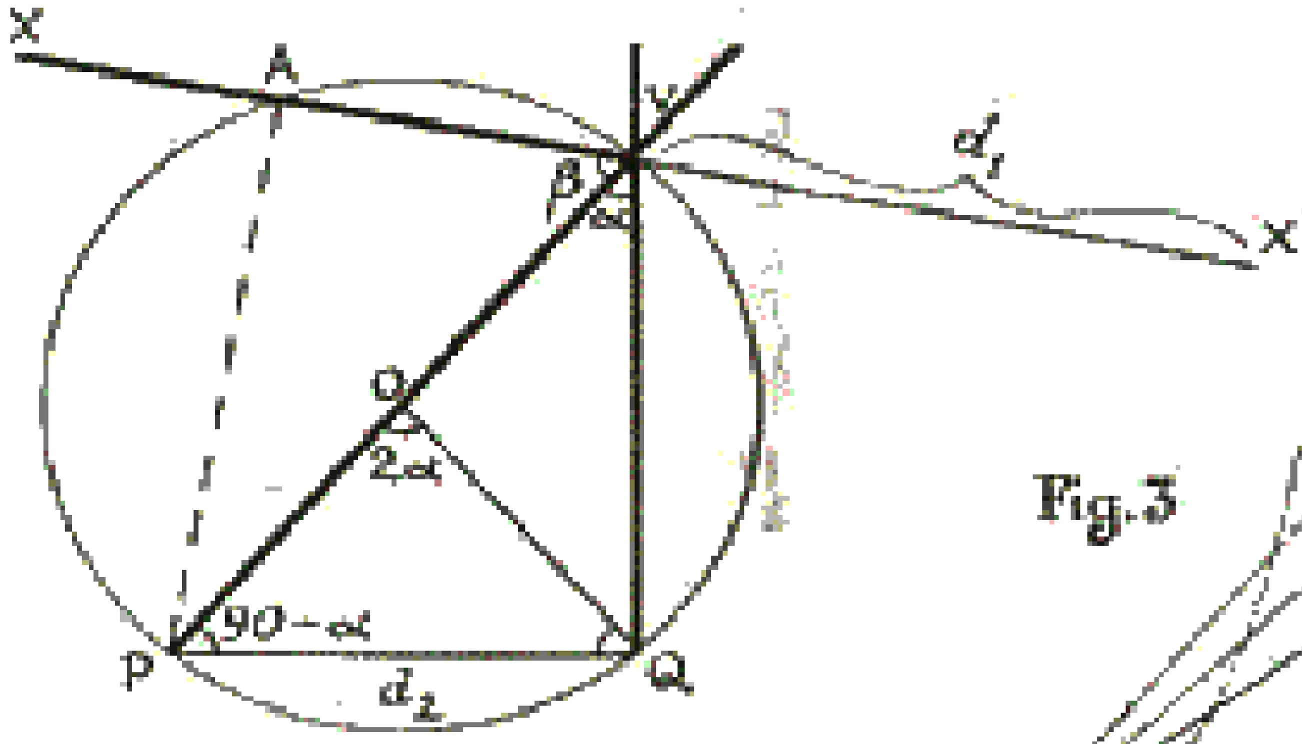
1		2	■	3	4
	■	5			
6	7		■		■
■		■	8		9
10				■	
11		■	12		

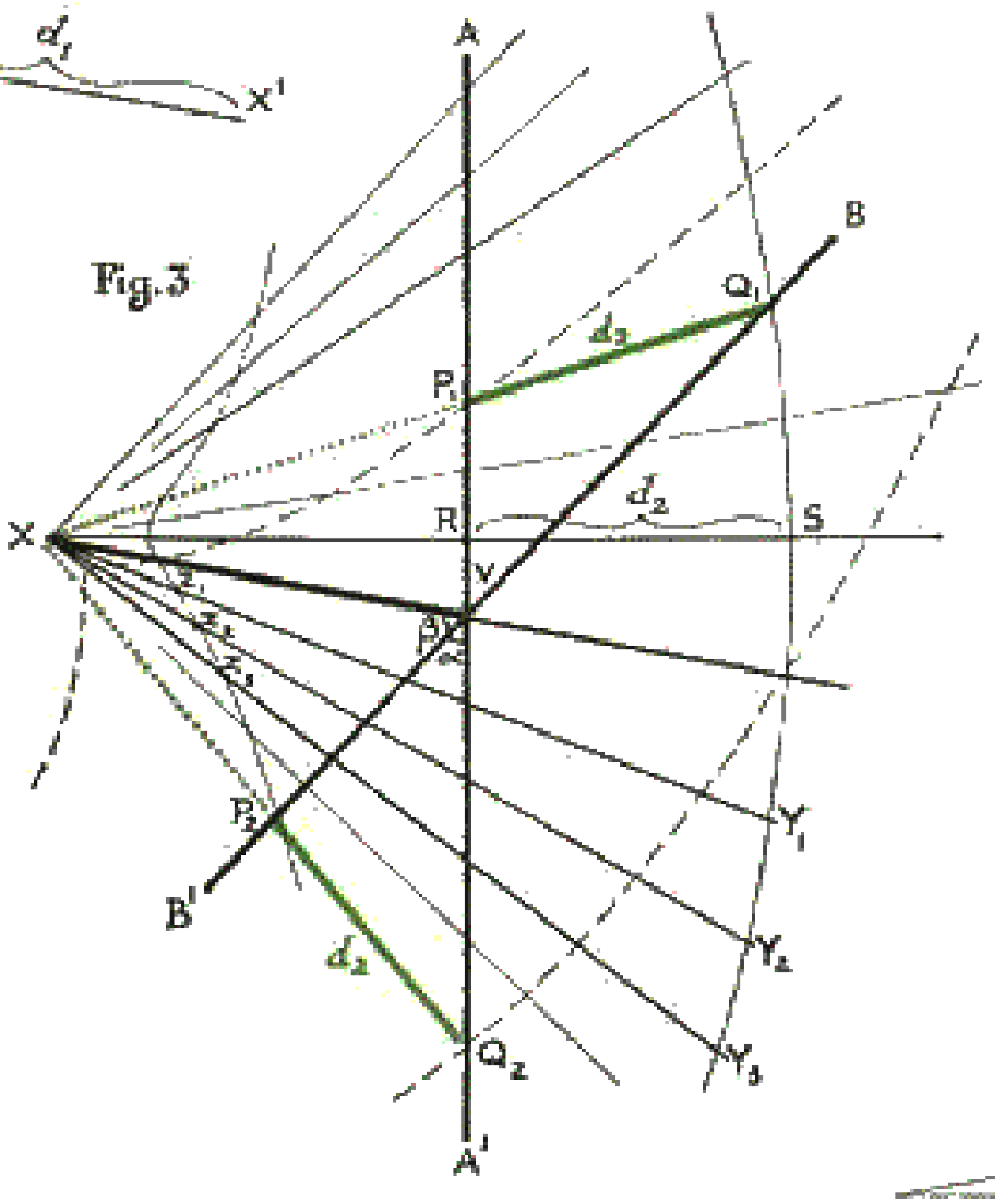
Only whole, positive numbers are involved in the working and the solutions. No guess work is necessary. The values of a, b, c, d, e, f, x, y, and z can be derived from the clues.

38188 39994 46974 86762 65516 58276 58483 58845

Mathematical Elegance







One of the interesting features of mathematics is the fact that, if a problem *is* soluble, there are usually several ways of arriving at a solution — some of these ways may be more direct than others. As you become more proficient in your mathematics, you will learn to appreciate " elegance " in working for a solution.

An elegant solution may be described as one that is reached in the fewest possible steps — each step being clear and logical.

When you are learning mathematics, however, the solving of problems serves a useful purpose in gaining familiarity with a useful theorem or method; a longer way round, therefore, may sometimes provide a better exercise.

As an example, take the problem illustrated in Fig. 1. Two straight lines intersect at V making an angle α. A point X is distant d_1 from V and XV makes an angle β with one of the lines. A line is to be drawn through X, intersecting the two lines in such a way that they cut off a length equal to d_2.

Method 1.—making use of circular segments and subtended angles. Draw a baseline and cut off a distance PQ equal to d_2 (Fig. 2). At P and Q draw lines making an angle $(90^0-\alpha)$ with PQ so that they meet at a point O: then angle $POQ=2\alpha$. With centre O and radius OP $(=OQ)$ describe a circle passing through P and Q. PQ subtends an angle equal to α at any part of the segment on the same side of PQ as O.

Take any point V at random on this segment and join VP, VQ. Through V draw a line making an angle β with VP and mark X and X^1 on it—distant d_1 from V, one on either side. If VX cuts the circle at A, then AP will subtend an angle β in the segment AVP. If the point V is moved round the circumference so that XVX^1 always passes through A, the points X and X^1 trace a curve or locus; this is the heart-shaped curve $AXSTX^1$. An easy way is to mark a series of points V_1 V_2 V_3 - - - on the circle and to draw lines through A and these points. With a pair of dividers mark off the distance d_1 on each to give points such as X_1 X_2 X_3 - - -. Join the points.

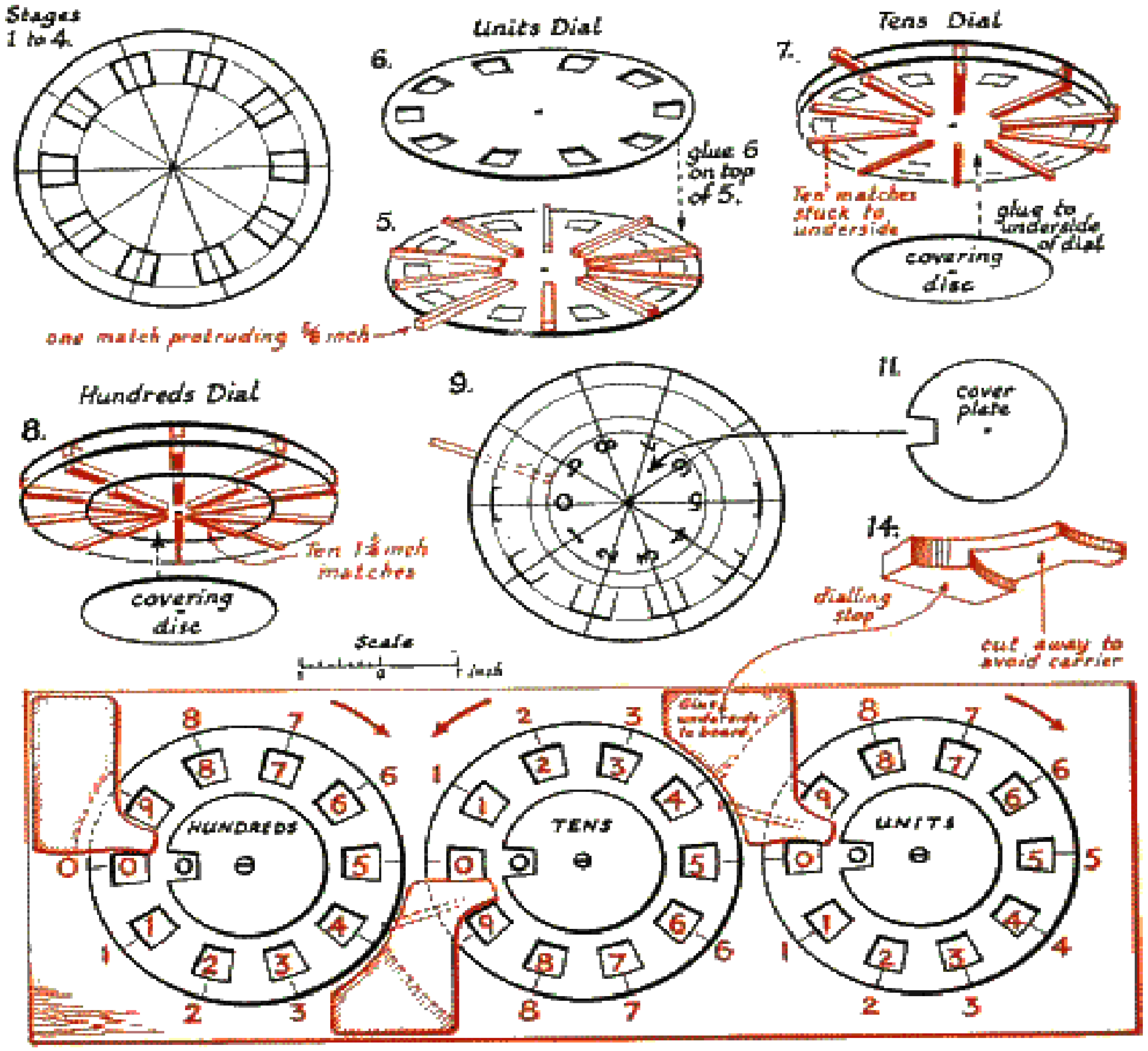
The curve $AXSTX^1$ is called a *limaçon*; the point A is called the *pole*. The curve cuts PQ produced at S and T. Since US is equal to d_1 and the angle PUQ is equal to α, the configuration $SPQU$ is one solution of the problem; similarly, $TWPQ$ is another. Mark off $VP_1=UP$ and $VQ_1=UQ$ also $VP_2=WQ$ and $VQ_2=WP$. The lines XP_1Q_1 and XP_2Q_2 provide the two solutions.

Method 2.—much shorter; more elegant? (Fig. 3). Let AA^1 and BB^1

Continued on next page

31427 75687 90029 09517 02835 29716 34456 21296

40435 23117 60066 51012 41200 65975 58512 76178



A DIAL-ADDER

Material : Fairly stiff card, 50 used matches, three ¾ inch screws, wooden backing-board about 14 inches by 5 inches, balsa wood, glue.

Construction :

1. Cut five 4 inch diameter discs from the card.
2. Use a protractor to divide each disc into ten equal sectors.
3. Draw two circles of radius 1¾ and 1¼ inches radii on each disc, concentric with the disc.
4. With the radii of the sectors as guides, cut equally spaced dialling slots between the two circles.
5. With matches stuck radially between the slots, stick together two discs. Position one of the matches to protrude ⅜ inch beyond the edge of the disc to act as a carrier. This is the units dial.
6. Cut three discs of diameter 2¼ inches.
7. Repeat instruction 5. On the underside of the double disc, stick ten matches radially, all just reaching the circumference. Partly cover these by gluing on a 2¼ inch disc. This is the tens dial.

Continued on page 378

63288 22505 45255 64056 44824 65151 87547 11962

mathematical pie

No. 48 *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England **MAY, 1966**

GEORGE BOOLE, 1815—1864

Nowadays students' grants ensure that no bright boy or girl is denied a university education. A hundred and fifty years ago, John Boole, a Lincoln cobbler, could provide no more than a primary education for his son, but the end of school did not mean the end of learning for George Boole. Mr. Brooke, a bookseller, taught him Latin and from borrowed books he taught himself Greek, French and German. At 16 he was able to obtain a post as an assistant in a small private school at Doncaster. At 20 he established his own school in his native Lincoln.

He became interested in Mathematics and, without help, he worked through all the books he could obtain. When he was 24 his first paper was published in the Cambridge Mathematical Journal. Before his death at the age of 49, he had published over 50 important papers and several books. He developed original ideas in the treatment of differential equations and finite differences, but his greatest contribution was his work on formal logic.

As lawyers know to their cost, it is difficult to make complex statements in words which can be understood, and understood in one way only. Boole conceived the idea of expressing arguments in an algebra which uses letters to stand for statements and symbols to stand for words such as "and," "or," "not." His work attracted such attention that this man whose schooling had finished at 12 was made, at 33, the first Professor of Mathematics of the new University of Cork.

24901 14195 21238 28153 09114 07907 38602 52522

The rules of Boole's algebra of logic differ from the rules of ordinary algebra. If a, b, and c stand for numbers, "$+$" means "plus" and "$\times$" means "multiplied by," we know that

$a \times (b + c) = (a \times b) + (a + c)$ is true and

$a + (b \times c) = (a + b) \times (a + c)$ is false,

but if a, b, and c represent statements, "$+$" means "or" and "$\times$" means "and," both statements are true.

In 1938, C. E. Shannon, a student of M.I.T. saw that Boolean Algebra could be applied to problems in electrical circuits. The Bell Telephone Company developed the algebra as a means of analysing faults in telephone circuits. Now, Boolean algebra is in everyday use in the design of automatic control systems and of computers. Other algebras derived from it have led to important discoveries in nuclear physics. C.V.G.

The Mathematical Pie pamphlet 0 and 1 by T. J. Fletcher includes a section on the application of Boolean Algebra to electrical circuits.—Ed.

(*The drawing of G. Boole is reproduced by courtesy of the Rev. R. H. P. Boole, one of his descendants.*)

8	÷		+		= 9
+		×		+	
	−		+		= 6
÷		−		÷	
	+		÷		= 3
= 2		= 2		= 4	

WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. Ignore the rules that you have been given in ordinary working, BODMAS. Can you find values that obey BODMAS? B.A.

DARK GLASS

You have some pieces of dark glass. Each piece reduces the light passing through it by one half. By how much is transmitted light *reduced* after passing through (*a*) 2 pieces, (*b*) 5 pieces, and (*c*) 10 pieces? J.F.H.

CUTIE

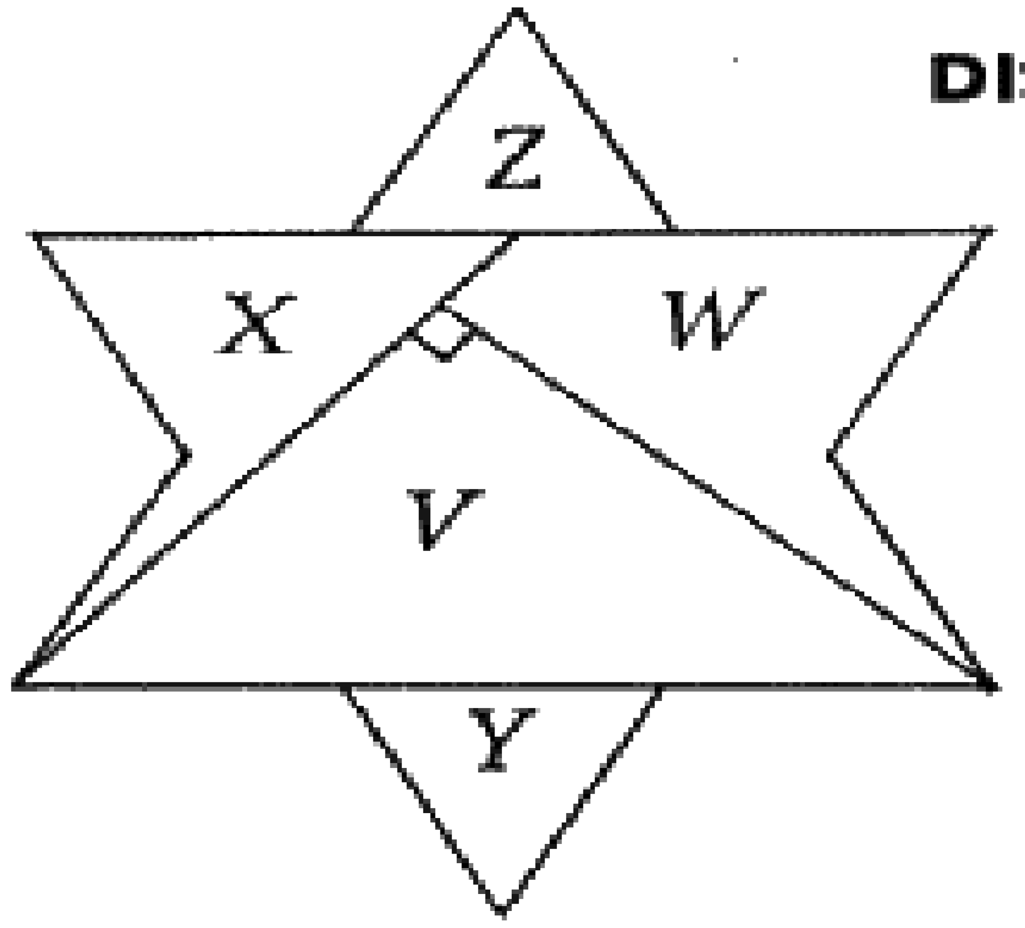
Cutie decides to start saving for her Summer holiday by putting something into her piggy bank every day. On the first day she put in a penny but she realised that a penny a day would not amount to much so the next day she put in tuppence and on the next day she put in fourpence. If she kept this up, doubling the amount every day, for a month, how much would she save? C.V.G.

FOR EXPERIENCED MATHEMATICIANS

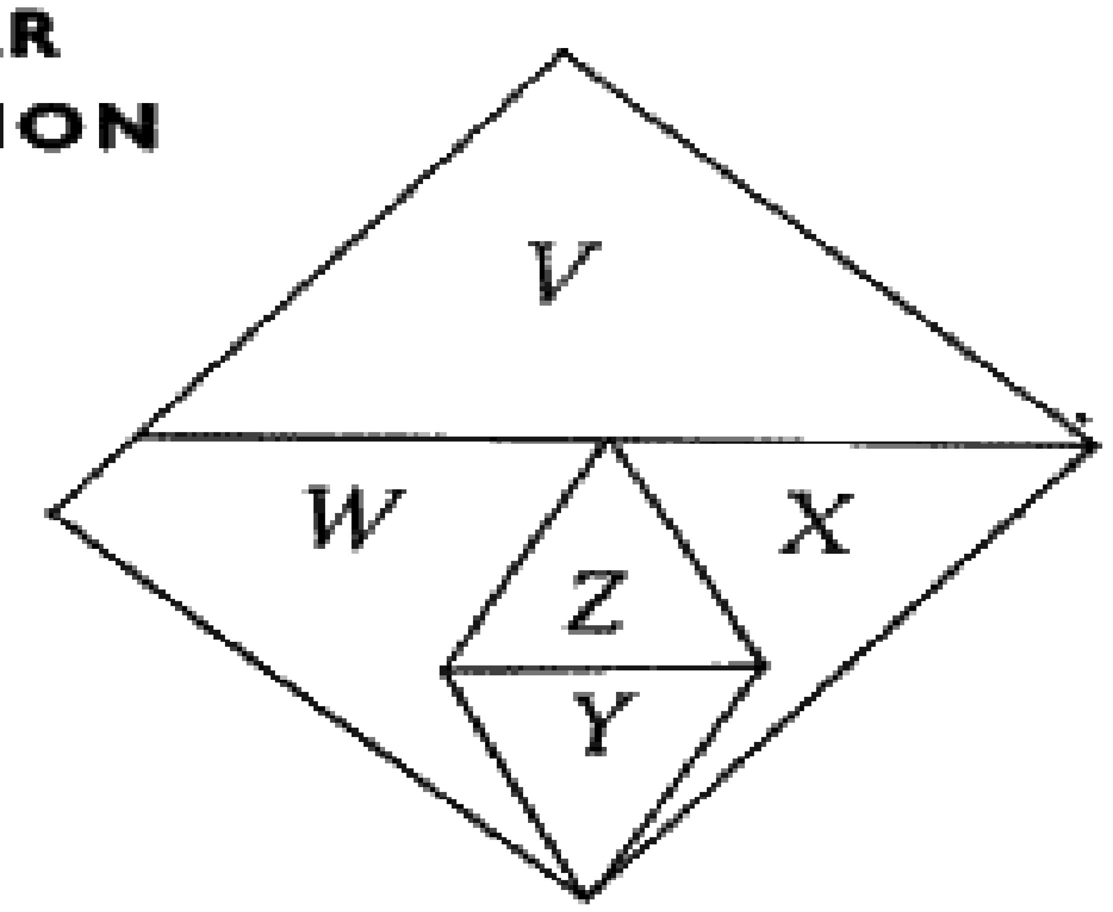
Great interest was shown in the problem of finding the lengths of the sides of the triangle circumscribing three circles, which was given in Issue No. 46. It appears that our "experienced mathematicians" have an age range of eleven to over seventy. The standard of the entries was very high, giving both calculated and drawn solutions. Excellent opportunities existed for slips in the calculation and a number of the attempts took advantage of them. The lengths of the three sides were 17.05, 31.02, and 36.55 inches correct to two places of decimals.

Book tokens have been sent to :—

C. M. Booth, Pinner ; Doreen Broadberry, Shirley ; Mrs. V. W. Carter, Bury St. Edmunds ; Alison Cook, Weston-super-Mare ; A. Cree, New Ollerton ; James T. Donan, Youghal ; G. F. Green, Lancing ; Alan Hills, Faversham ; Vanessa Hamilton, East Kilbride ; Mr. N. S. Jones, Walsall ; Gavin Kelley and William Taylor, Billericay ; Miss R. Olsberg, Sunderland ; Donald M. Salisbury, Baldock ; Mr. C. Sanders, Guildford ; A. Sims, Newport, Mon.



A STAR DISSECTION



SOLUTIONS TO PROBLEMS IN ISSUE No. 47

PAIRS PROBLEM The first part is satisfied by 7, 5; 4, 9; 3, 13.
Let the numbers be x and y, then $xy - (x+y) = 215$.

i.e., $x = \frac{215+y}{y-1} = 1 + \frac{216}{y-1}$. The factors of 216 are $2^3.3^3$.

Hence the possible pairs are 109,3 ; 73,4 ; 55,5 ; 37,7 ; 28,9 ; 25,10 ; 19,13.

WITHOUT A WORD
Three sets of solutions are possible.

$$\begin{array}{ccc} 3 - 1 \times 4 = 8 & 3 - 1 \times 4 = 8 & 3 - 2 \times 4 = 4 \\ 5 + 4 \div 3 = 3 & 7 + 2 \div 3 = 3 & 7 + 2 \div 3 = 3 \\ 2 - 5 + 6 = 3 & 2 - 3 + 6 = 5 & 2 - 4 + 6 = 4 \end{array}$$

WEIGHT OF RAIN falling on one acre is 101¼ tons.

THE SNAIL would reach the top in 9¾ days.

THE EQUALLY DISPERSED onions would total 37.

A CUTTING PROBLEM with the cube would leave a volume of $384\sqrt{6}$ cubic inches.

The number of prefects at the ODD'S ON Valentine Dance was 33 ; 18 girls and 15 boys.

Drawing FOUR balls from the bag would ensure that two were the same colour. NINE balls are required to ensure at least one of each colour. ELEVEN balls would have to be removed from the bag to be certain that three were red.

SENIOR CROSS FIGURE No. 45 5 across and 3 down are not unique.
Clues Across : 1. 136 ; 3. 28 ; 5. 4994 ; 6. 420 ; 8. 375 ; 10. 1024 ; 11. 66 ; 12. 162.
Clues Down : 1. 104 ; 2. 640 ; 3. 2977 ; 4. 84 ; 7. 2106 ; 8. 341 ; 9. 512 ; 10. 16.

$1*2*3* \ldots\ldots *n = 1$ and $\frac{1}{2}*\frac{1}{4}*\frac{1}{6}* \ldots\ldots *\frac{1}{2n} = \frac{n}{n+1}$.

JUNIOR CROSS FIGURE No. 40

Clues Across : 1. 374 ; 5, 3843 ; 6. 150 ; 7. 28 ; 8. 35 ; 9. 1273.
Clues Down : 2. 73582 ; 3, 480 ; 4. 13 ; 6. 121 ; 8. 33.

74299 58180 72471 62591 66854 51333 12394 80494 95889 70695 36534 94060 34021 66544 37558 90045

Continued from page 380

8. Omit instruction 5 but repeat instruction 7 on the remaining 4 inch disc. On the under side of the $2\frac{1}{2}$ inch covering disc glue ten equally spaced matches radially, backing these with the remaining $2\frac{1}{2}$ inch disc. This is the hundreds dial.
9. On each dial mark the digits 0 to 9 about 1 inch from the centre on the radii through the dialling slots as follows:

UNITS DIAL Enter O immediately anticlockwise of the carrier, then continue with the remaining digits in an anticlockwise direction.

TENS DIAL Enter O immediately clockwise of the carrier, then continue with the digits 1 to 9 in a clockwise direction.

HUNDREDS DIAL As for the units dial.

10. Draw a central line along the length of the backing-board. On this line mark the centres for the discs $4\frac{1}{2}$ inches apart.
11. Cut three 2 inches diameter discs as cover plates for the faces of the dials. In each cut out a one-tenth sector so that one digit only can be seen at once.
12. Screw each disc assembly to the backing-board and arrange the cover plates so that the digits are viewed to the left. Glued to the screw head, the cover plates should be immovable.
13. On the board, mark the digits adjacent to the dialling slots as shown.
14. Construct dialling stops from balsa wood and fit them so that they do not obstruct the carriers.
15. Mark in arrows to indicate the direction of dialling.

The discs are designed so that the tens carrier does not operate the units dial.

Having completed this model, try making one for another number system or for pounds, shillings and pence. How could the machine be used for subtraction?

D.I.B.

A LADDER

A ladder stands against a vertical wall, and the foot slides away along the ground (which is horizontal), what is the locus of the mid-point? S.T.P.

Under the integers 1, 2, 3, 4, 5, 6, 7 write another arrangement of the same integers. Find the difference between the two numbers in each column. Prove that these absolute differences cannot be all different.

A book token will be awarded to the best solutions.—Ed. R.H.C.

Write down the next term in the sequence 1, 3, 6, 12, 24, 30, 120, 240.

R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 48

It has been decided that the solutions of the problems shall be given in the current issue except for competitions and Cross figures.

WITHOUT A WORD: $8 \div 2 + 5 = 9$, $2 - 3 + 7 = 6$, $5 + 4 \div 3 = 3$.

TWO DARK GLASSES reduces the light to one half squared, or one quarter; five pieces to one thirty-second, and ten to $(\frac{1}{2})^{10}$ of the intensity.

CUTIE had saved $2^{30} - 1$ pence or over four million pounds in a thirty day month.

THE 5 TERM SERIES which adds up to 153 is $1! + 2! + 3! + 4! + 5!$

A PENNY FOR YOUR THOUGHTS shows the values in pence of the currency of the country so the next term will be 1,200

DISC JOCKEY RIDES AGAIN. The needle travels about $1\frac{1}{4}$ inches along an arc of a circle.

A LADDER falls and the path of the mid-point of the ladder is a quadrant of a circle.

THERE'S NO CATCH. The boy scored 2 runs.

B.A.

60357 99646 92565 04936 81836 09003 23809 29345

PRESIDENTIAL PROBLEM

$$\begin{array}{r} \text{LYNDON} \\ \text{B} \\ \hline \text{JOHNSON} \end{array}$$

Each letter represents a different digit. Can you identify this multiplication problem? R.M.S.

THE 5 TERM SERIES

The 5 term series $1^2+2^2+3^2+4^2+5^2$ involves the first 5 natural numbers in order and adds up to 55. Can you devise another series of 5 terms involving the first 5 natural numbers, one per term, which add up to 153? R.M.S.

DISC JOCKEY RIDES AGAIN

The diameter of the latest top 20 record is 7 inches. The outer $\frac{1}{2}$ inch is blank and the unused centre has a diameter of $3\frac{1}{2}$ inches. If there are 91 grooves to the inch of radius, how far does the needle move during the actual playing of the record? R.M.S.

One track of Tom Lehrer's latest L.P. record is called New Math. We invite you to listen to the recording and write, in about 250 words, a similar monologue on some aspect of the new mathematics. A book token will be awarded to the writer of the best attempts which will be printed in a future issue.—Ed.

THERE'S NO CATCH

There was a boy who played cricket,
But he always used to snick it.
The square of his score,
Plus nine times it — not more,
Equalled the length of the wicket.

How many runs did he make?

MODERN CROSS FIGURE

■	1	2	3	■	■
4	■	5		6	7
8	9		■	10	
11		■	12		
13		14		■	
■	■	15			■

Clues Down:

2. 124 + 33 in base five.
3. Both solutions of $x^2 - 11x + 30 = 0$, smaller first.
4. The next two terms in the series 1, 1, 2, 3, 5, 8, –, –.
6. Ten times the *size* of 1 across.
7. 67.61 francs in lire to 3 sig. fig. when £1 = 13.68 f; and £1 = 1740 lire.
9. Twice the second prime after 131.
12. $\pi \int_0^3 (x^2 + 4)\, dx$ to 3 sig. fig. Take $\pi = 3.14$.
13. Number of combinations of 11 things taken from 13.

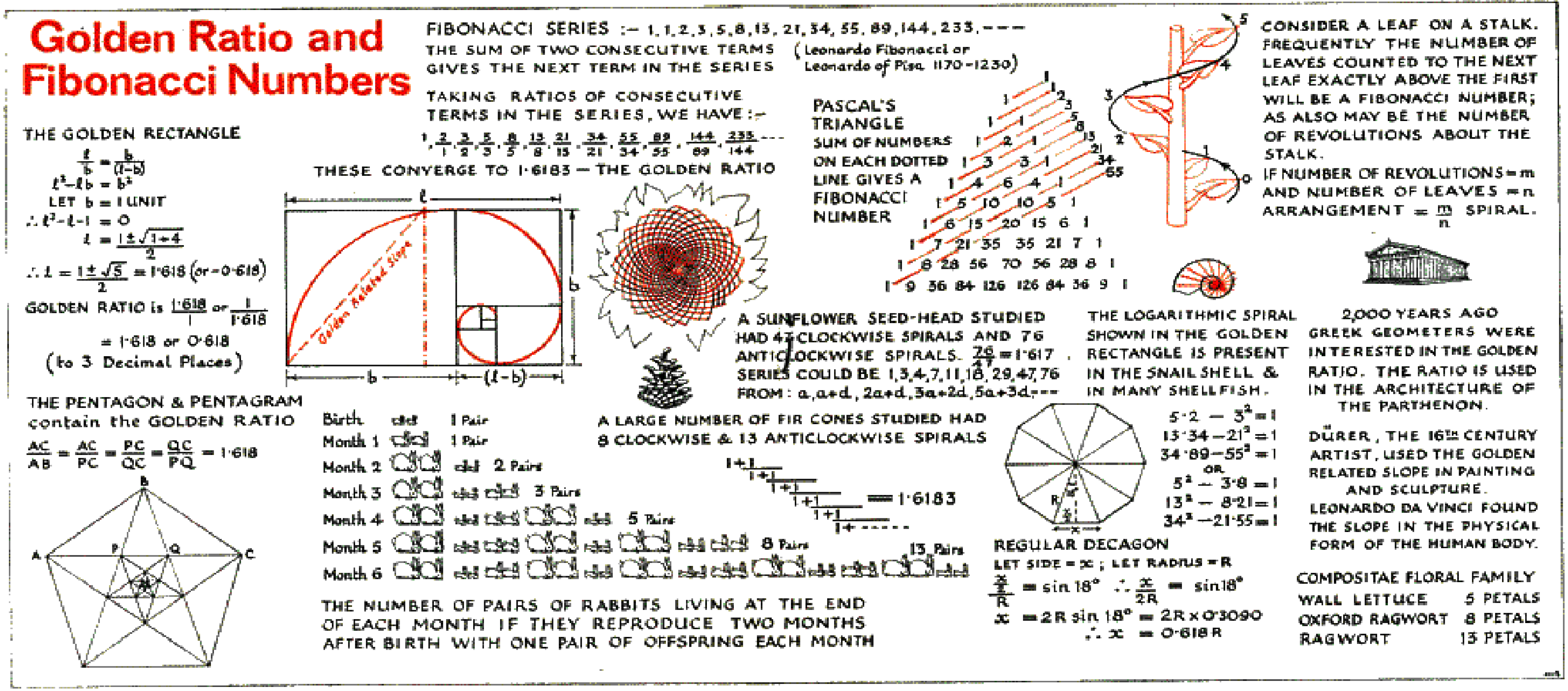
Clues Across:

1. Vectors, $(8, -2) + (4, 7)$.
5. The reciprocal of 0.618.
8. The binary number 101000010 in denary.
10. The number of possibilities with 2 dice.
11. $\{$All perfect cubes$\} \cap \{$Multiples of 3 below 100.$\}$
12. The product of the number of edges and the number of faces of an icosahedron.
13. Tan θ when $\sin \theta = \frac{12}{13}$, as a decimal
15. The speed, in ft. per sec., of a body after 28 seconds falling from rest when $g = 32$ ft. per sec. per sec.

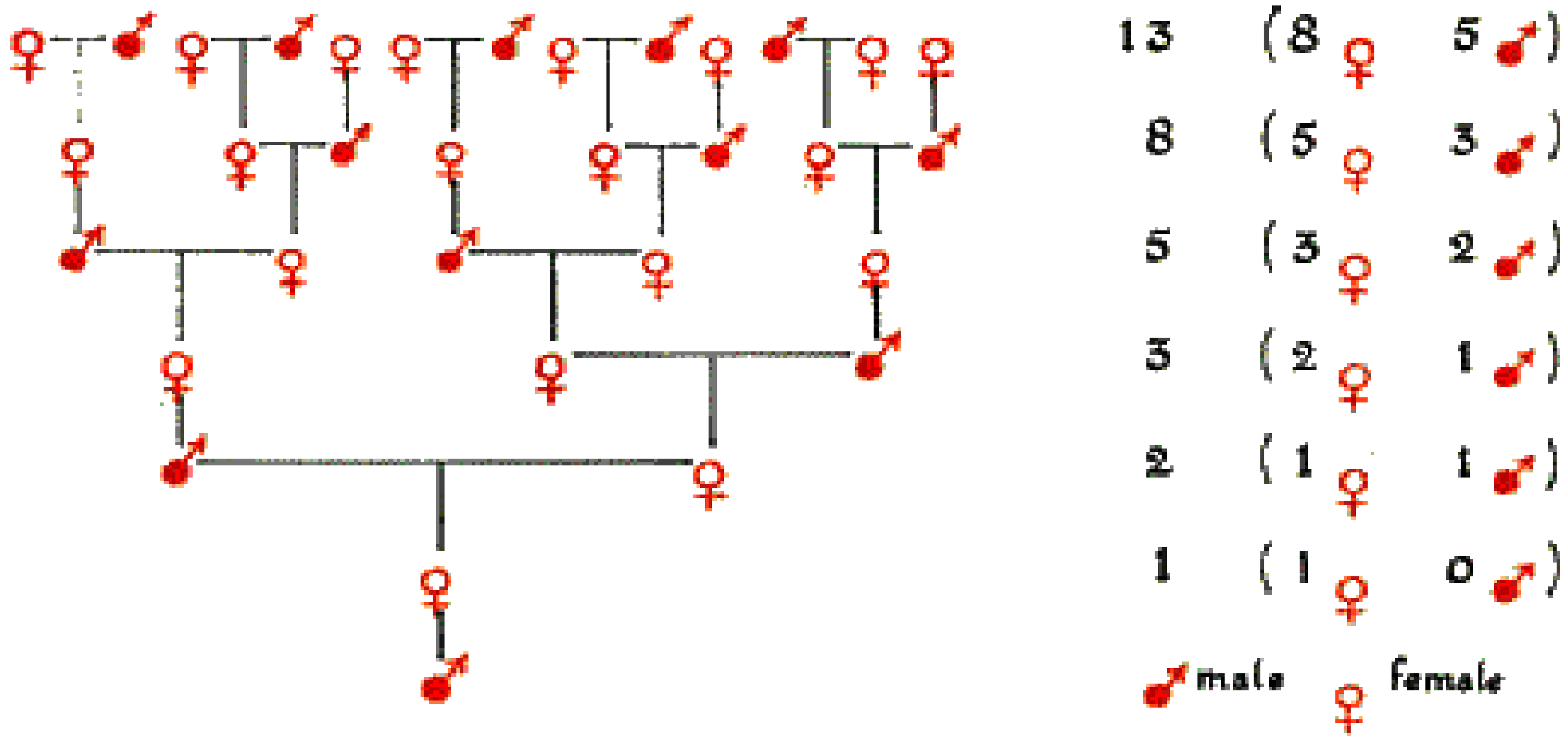
Omit commas and decimal points and work to the appropriate number of figures.

D.I.B.

70791 19153 26734 30282 44186 04142 63639 54800



The Fibonacci sequence of numbers owes its name to the Italian mathematician, Leonardo of Pisa (often abbreviated to Fibonacci). Born in Pisa between 1170 and 1175, he was educated at Bugia and travelled about the Mediterranean, collecting information about mathematics. In 1202 he returned to Pisa and published 'Liber Abaci,' a book which established the introduction of the Arabic notation in Europe and provided a foundation for future developments in algebra and arithmetic. In 1220 he published a book on geometry entitled, 'Practica Geometria.'



04480 02670 49624 82017 92896 47669 75831 83271

A drone bee (male) has a mother but no father, as the queen's unfertilised eggs hatch into drones. The queen's fertilised eggs produce either worker bees or queens. The diagram shows why the number of ancestors of a drone must in any generation be a Fibonacci number.

The sequence is also found in floral families when the number of petals is considered. This has relevance in evolution.

Ranunculaceae family : Buttercup, Larkspurs, Columbines, some Delphiniums, all have 5 petals. Lesser Celandines and other Delphiniums have 8 petals. Globe flower *some Double* Delphiniums 13 petals.

Compositae family :

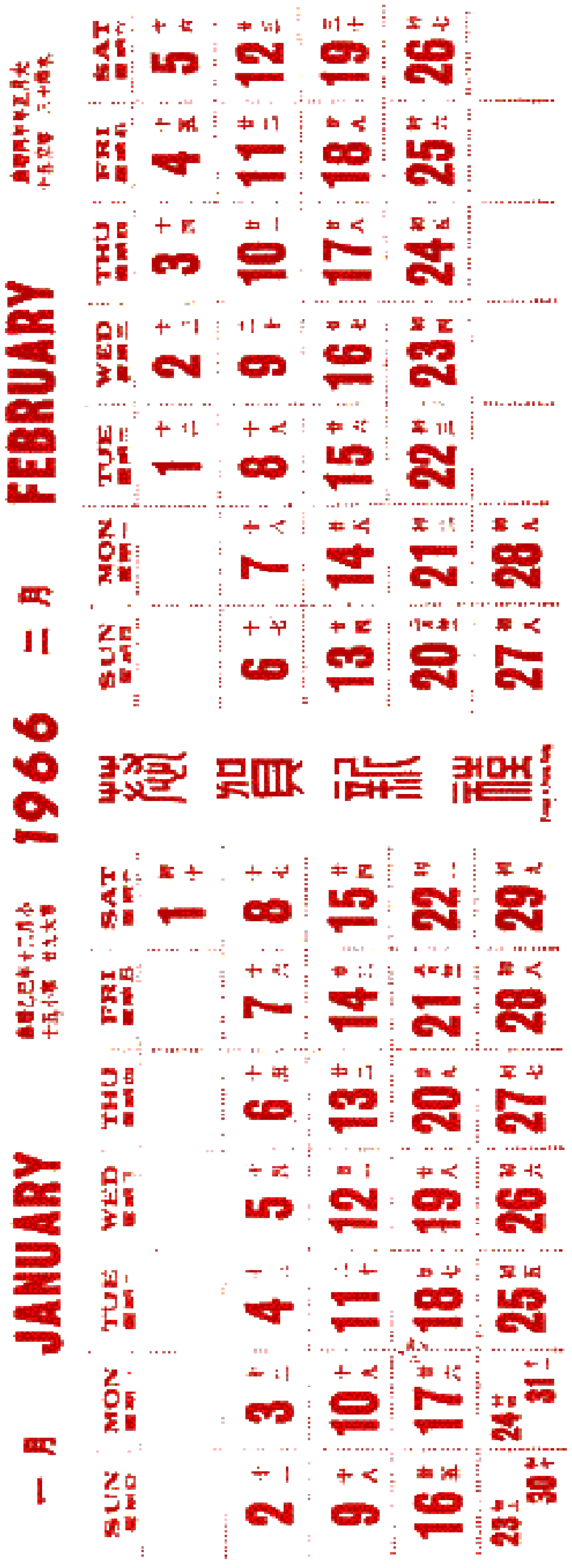
Wall lettuce (rare)	5 petals	*Ragwort* (fairly common)	13 petals
Oxford ragwort	8 petals	*Asters* (fairly common)	21 petals
		Field daisies (most common)	34 petals
		Michaelmas daisies	55 petals or 89 petals

Of course, one cannot expect to find these exact numbers on every example, due to various mutations which might occur, but in general a mean score would definitely tend towards these figures.

References : Fibonacci Numbers by N. N. Vorob'ev, published by Pergammon Press ; On Growth and Form by D'Arcy W. Thompson ; Recurrence Relations by T. J. Fletcher, *Mathematical Pie* ; The Golden Section or Golden Cut by Manning Robertson (Journal of the Royal Institute of British Architects, Vol. 55, No. 1, 1948) ; The Language of Mathematics by Frank Land.

Some indication of the uses of the Golden Section in sculpture, painting, architecture and furniture is given in the book, 'Practical Applications of Dynamic Symmetry' by Jay Hambridge, published by Yale Press, 1932. D.I.B.

31425 17029 69234 88962 76684 40323 26092 75249



CHINESE CALENDAR

The history of the calendar is the story of a struggle to combine lunar months of $29\frac{1}{2}$ days with years of nearly $365\frac{1}{4}$ days. In the West we long ago gave up the attempt to relate the months to the phases of the moon. Our Gregorian Calendar measures the year. The extra days in leap years ensure that the calendar is never more than half a day out.

The best compromise was the calendar formerly used in the Near East. Months of 29 days alternating with months of 30 days took care of the moon, but 12 such months are $9\frac{1}{4}$ days short of a year. To make this up, every third or fourth year had 13 months. There was no regular system of deciding when this extra month should come. The high priest, and later the bishop, in each town would inspect the crops. If he thought the harvest would be late he would order that the year should have an extra month. Perhaps the N.F.U. would like to introduce this system in England.

After the Arab conquests, Mohammed ordered that every year should have twelve months. This means that the Mohammedan "New Year" wanders round the true year and falls in the same season about every forty years.

The Chinese gave up the struggle even earlier. The Chinese "year" consists of twelve lunar months, six of 29 days and six of 30 days. The leaf from a Chinese Calendar shows that China celebrated New Year on January 21st, 1966.

The calendar shows the dates in Chinese numerals. Perhaps you can identify them. The Chinese have numerals from 1 to 9 and symbols for "tens," "hundreds," "thousands." If we wrote 365 as 3h6t5 (just as we might write 3yd.2ft.4in.) we would be using a system like the Chinese. There is just one complication; instead of 2t and 3t there are special combinations for twenty and thirty.

C.V.G.

40041 52970 02878 30766 70944 47456 01345 56417

mathematical pie

No. 49 — *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England — OCTOBER, 1966

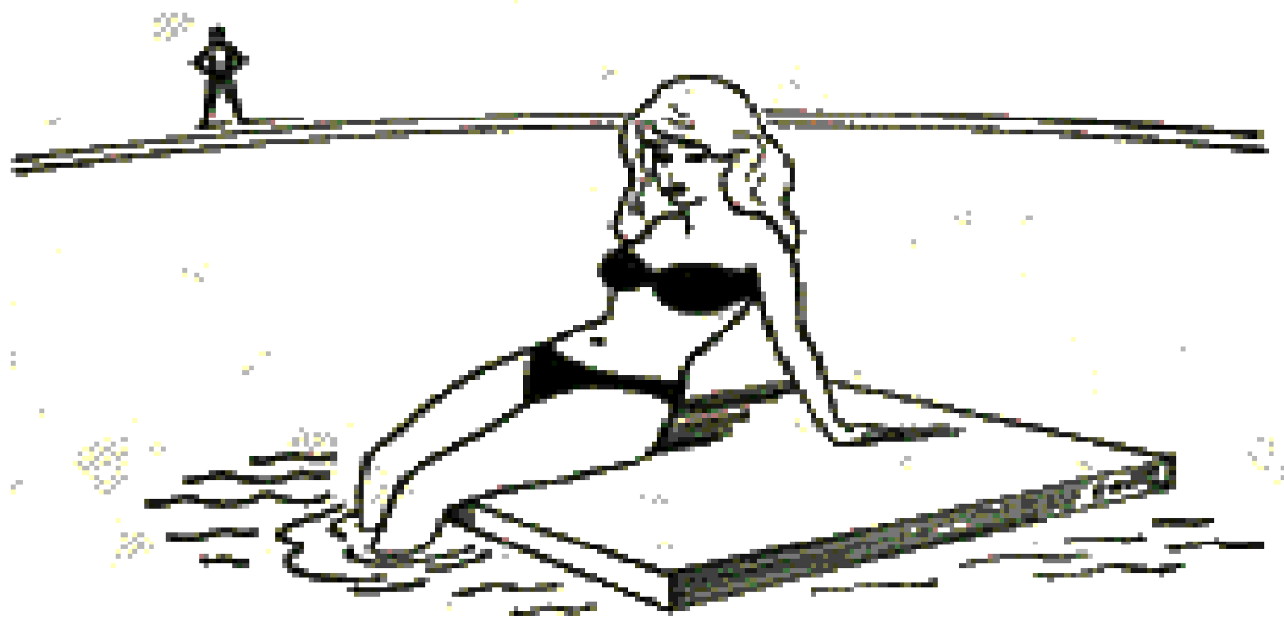
CHASE ME CHARLIE

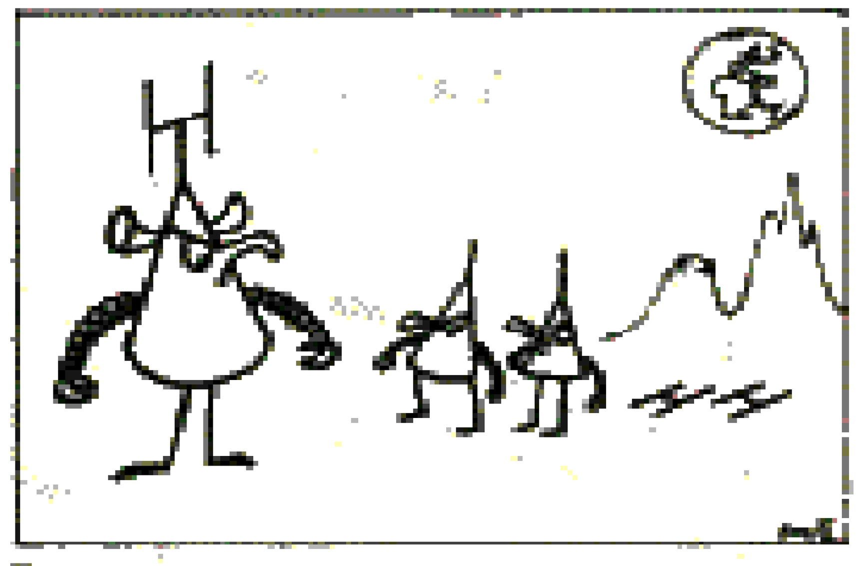
Cutie was sunning herself on a raft at the centre of a large swimming pool when to her annoyance she saw an ex-boy friend waiting at the edge of the pool with the obvious intention of pressing his unwelcome attention on her when she came to the shore. She knew that he could run four times as fast as she could swim but once on land, she could easily outdistance him.

It was beginning to get late and she was hungry as well as annoyed so she devised a plan for outwitting him. What was her best strategy for reaching a point on the land before he could get there?

Assume the pool is circular.

R.M.S.





"Dropping your aitches again, children!"

WHEN WRITING NUMBERS

When writing numbers in the binary system, the various columns correspond to powers base two.

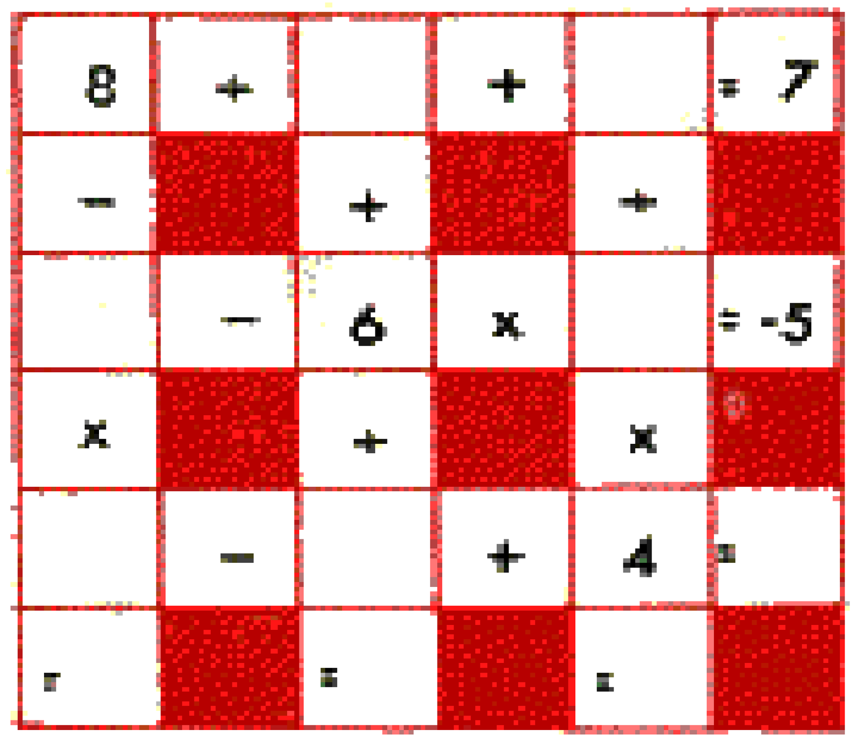
i.e.,	16	8	4	2	1	in the denary notation
					1	means one
				1	0	means two
				1	1	means three, etc.

Hence any number may be expressed in terms of the symbols 0 and 1. Twenty-five is 11001.

Suppose the columns referred to the powers of minus two. Can we express any number by the symbols 0 and 1 when the headings of the columns are 16 −8 4 −2 1?

R.H.C.

18443 96582 51375 43885 69094 11303 15095 26179



WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. BODMAS applies.

D.I.B.

BAND CONSTRUCTION

In my pocket I carry around what used to be a combined ruler and protractor made of white plastic. It is an accurate rectangle 6 in. by 2 in., but the constant rubbing in my pocket has obliterated all the markings from it. Recently, I used it to bisect a straight line 3.7 in. long and also an angle of about 57°. In my usual forgetful way I had left my compasses at home. How did I do the two constructions?

A parallel strip without markings is called a band.

R.M.S.

HOW ODD

Make a table of the values of a^n for $a = 1, 2, 3, \ldots$ working on upwards. For each base inspect the last digit of each power and see what you notice in each case. For example, if we begin with base 4, $4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$, $4^5 = 1024$, etc.

Here you notice that only two digits 4, 6 turn up as the last digit. Which kind of number do you have to have to be sure that 9 digits can turn up at the end. Can you explain why this is so?

R.H.C.

INITIAL ALGEBRA 1

Some shopkeepers use 'private marks' by which the price of an article is stated by letters representing numbers. Usually these letters form a code word if arranged in the order that corresponds to 1, 2, 3, etc. Can you find the code word used in the shop where the following was seen? The trade prices are likely to be between 15 and 25 per cent. of the retail price.

BRUSHES

	B quality		A quality	
	Trade	retail	Trade	retail
1 in.	O/T	3s. 3d.	R/E	3s. 9d.
2 in.	N/A	5s. 6d.	N/T	5s. 10d.
3 in.	A/HH	8s. 9d.	S/L	9s. 6d.
4 in.	HE/R	12s. 6d.	HH/A	14s. 6d.
6 in.	HA/A	20s. 6d.	HT/A	22s. 6d.

Submitted by Mr. G. Edgcombe, Plymstock Comprehensive School.

FRUITY REMARK

What happens when you count apples in two's?

R.H.C.

THE LATEST JAZZ GROUP?

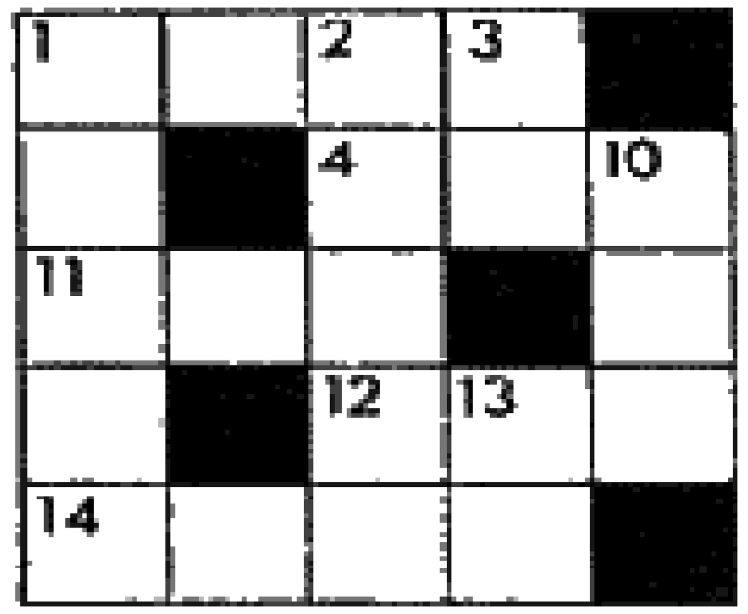
Logarithm. (Say it slowly).

R.H.C.

37800 29741 20766 51479 39425 90298 96959 46995

JUNIOR CROSS FIGURE No. 41

All the clues and answers are in the scale of five.



ACROSS

1. 234×12
4. $212 - 210 \div 24$
11. $4^3 + 3^3 + 2^3 - 1^3$
12. $2.021 - 1.442$
14. $3^{10} - 3^4 - 3^3 - 3^1 - 3^0$

DOWN

1. 243×102
2. 111 reversed + 1
3. $\sqrt{2124}$
10. 4^3
13. $4^2 + 4^1 - 4^0$

B.A.

SOLUTIONS TO PROBLEMS IN ISSUE No. 48

PRESIDENTIAL PROBLEM

LYNDON	570140
B	6
JOHNSON	3420840

MODERN CROSS FIGURE

CLUES ACROSS: (1) 123; (5) 1618; (8) 322; (10) 36; (11) 27; (12) 600; (13) 1875 (15) 896.

CLUES DOWN: (2) 212; (3) 56; (4) 1321; (6) 130; (7) 8600; (9) 278; (12) 659; (14) 78.

SOLUTIONS TO PROBLEMS IN ISSUE No. 49

CHASE ME CHARLIE

She must first swim so that the raft is always between her and the boy, gradually increasing her distance from the raft until she is ¼ radius from the centre. At this distance from the raft he can move round the edge of the pool with the same angular speed that she can swim, so she can gain no more in this way. She must now turn and dash straight for the shore. She has ¾ radius to travel while he has π radii to cover at four times the speed. As ¾ is less than $\frac{\pi}{4}$, she will arrive there first and make her escape.

WHEN WRITING NUMBERS

It is possible to write numbers in this way. For example, three is 111 in the scale of − 2, *i.e.*, 4 − 2 + 1, and seven is 11011, *i.e.*, 16 − 8 − 2 − 1.

BAND CONSTRUCTION

The solutions will be given in the next issue.

INITIAL ALGEBRA I

The code word was HORN?ASTLE, probably HORNCASTLE.

FRUITY REMARK

When apples are counted in two's do they become pears? (pairs!).

ABCD IS A SQUARE

One cut only is necessary, but the folding is left to you.

EIGHT EIGHT'S

$888 + 8(8 + 8) - 8 - 8 = 1,000.$

BACKWARDS

There are six years between 1st January 3 B.C. and 1st January 4 A.D.

SMALLEST NUMBER

The smallest number is 59.

WITHOUT A WORD

$8 + 2 + 3 = 7$, $7 - 6 \times 2 = -5$, $1 - 2 + 4 = 3$.

B.A.

09183 90367 36722 20888 32151 37556 00372 79839

ANY SCALENE TRIANGLE

In any scalene triangle, draw a line perpendicular to the longest side to divide the triangle into two equal areas. Explain your construction.

Book token for best solutions received by 30th November.

NOT A HUNDRED PER CENT FIT

" 25% of patients can be cured completely, and 90% can be sufficiently relieved of their symptoms to make life tolerable."

So said a doctor speaking on television about migraine.

It would be interesting to know what happens to all the others ! S.T.P.

QUOTE

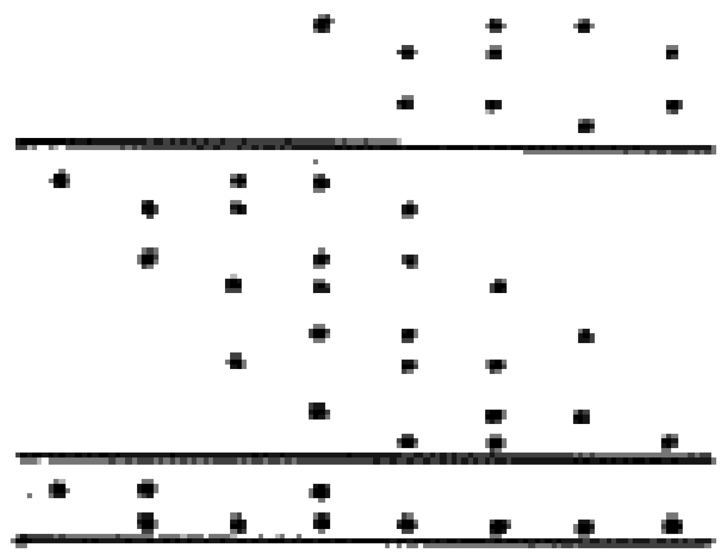
Ah ! why, ye Gods, should two and two make four ?

ALEXANDER POPE

LETTERS TO THE EDITOR from GENERAL TAIKOFF

Dear Comrade,

Everyone knows of the brilliant achievement of Colonel Blastov in successfully landing his space craft on the chief planet of Alpha Centauri. Unfortunately some damage was sustained and a return journey is not possible. Suitable conjunctions for the despatch of a recovery expedition will not occur for another 37 years. In the meantime, Blastov is transmitting observations of considerable interest. The dominant creatures, who call themselves Yaffels, are not men but equal men in intelligence, although they are 3,000 years behind us in technology. They have a highly developed language and have considerable skill in arithmetic. To represent numbers, they use the three symbols ·, ., and : which they make by striking their beaks on tablets of wax or clay. They have never developed an algebra because they have no alphabet. In writing, they represent the 25 sounds used in their language by the numbers from −12 to +12. Blastov has sent the following examples of computation :—



which appear to be an addition and a long multiplication in Yaffel notation. Unfortunately, because of interference, the remainder of the transmission explaining this notation was not received and it is impossible to ask for a repetition as Blastov's receiver is not functioning. Perhaps one of your readers could interpret the notation and provide me with addition and multiplication tables in Yaffel notation. Honorary membership of the Order of the Red Rocket* will be conferred on the solver.

Yours co-fraternally,

A. TAIKOFF, General.

**and also a book token for the best effort.*—ED., Mathematical Pie.

60896 32080 68222 46801 22482 61177 18589 63814

ABCD IS A SQUARE

ABCD is a square cloth that has to be cut into 16 squares of equal size. What is the least number of cuts that are necessary to do this ?

What is the least number of cuts necessary to produce 64 equal squares ? Folding of the cloth is permitted. J.F.H.

Do you ever say things like " Two minuses make a plus " when it is obvious that −2 −2 = −4 ? Or do you think that eight 8's always make 64 ? If so, try filling in the necessary mathematical symbols in this skeleton equation to show that eight 8's can make 1,000.

$$8 \quad 8 \quad 8 \quad 8 \quad 8 \quad 8 \quad 8 \quad 8 \quad = \quad 1{,}000$$

(And be careful that you say precisely what you mean next time !). S.T.P.

If * represents one of the signs + or — and †† represents the other, decide which is which if $a*(b\dagger\dagger c) = (a*b)\dagger\dagger(a*c)$ but $a\dagger\dagger(b*c) \neq (a\dagger\dagger b)*(s\dagger\dagger c)$. C.V.G.

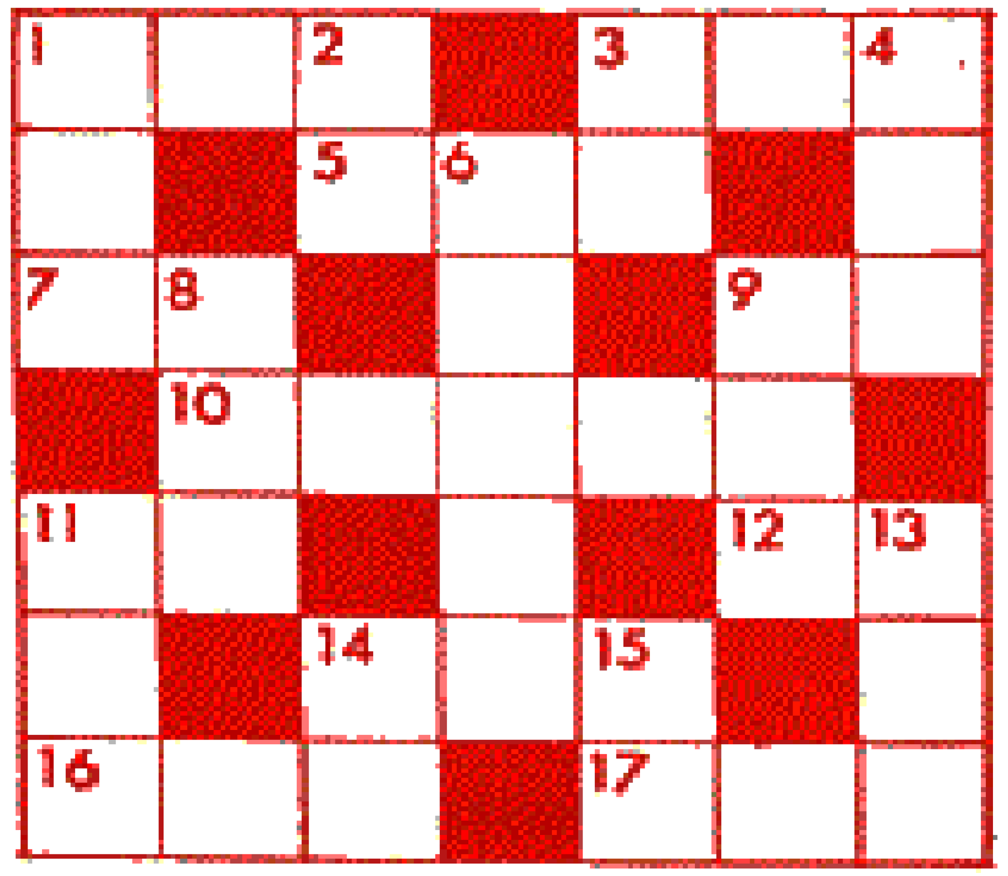
BACKWARDS COUNTERS

We are used to seeing numbers arranged like 4, 3, 2, 1, 0, −1, −2, −3 and make such a scale when we draw a graph. What happens when we go backwards in time from 4 A.D. to 3 B.C. ? How many years were there between 1st January 3 B.C. and 1st January 4 A.D. ? J.F.H.

What is the smallest number which leaves
a remainder of 1 when divided by 2
" " " 2 " " 3
" " " 3 " " 4
" " " 4 " " 5
and " " " 5 " " 6 ? S.T.P.

SENIOR CROSS FIGURE No. 46

Submitted by Mr. T. Noble, Baguley, Manchester.



CLUES DOWN :

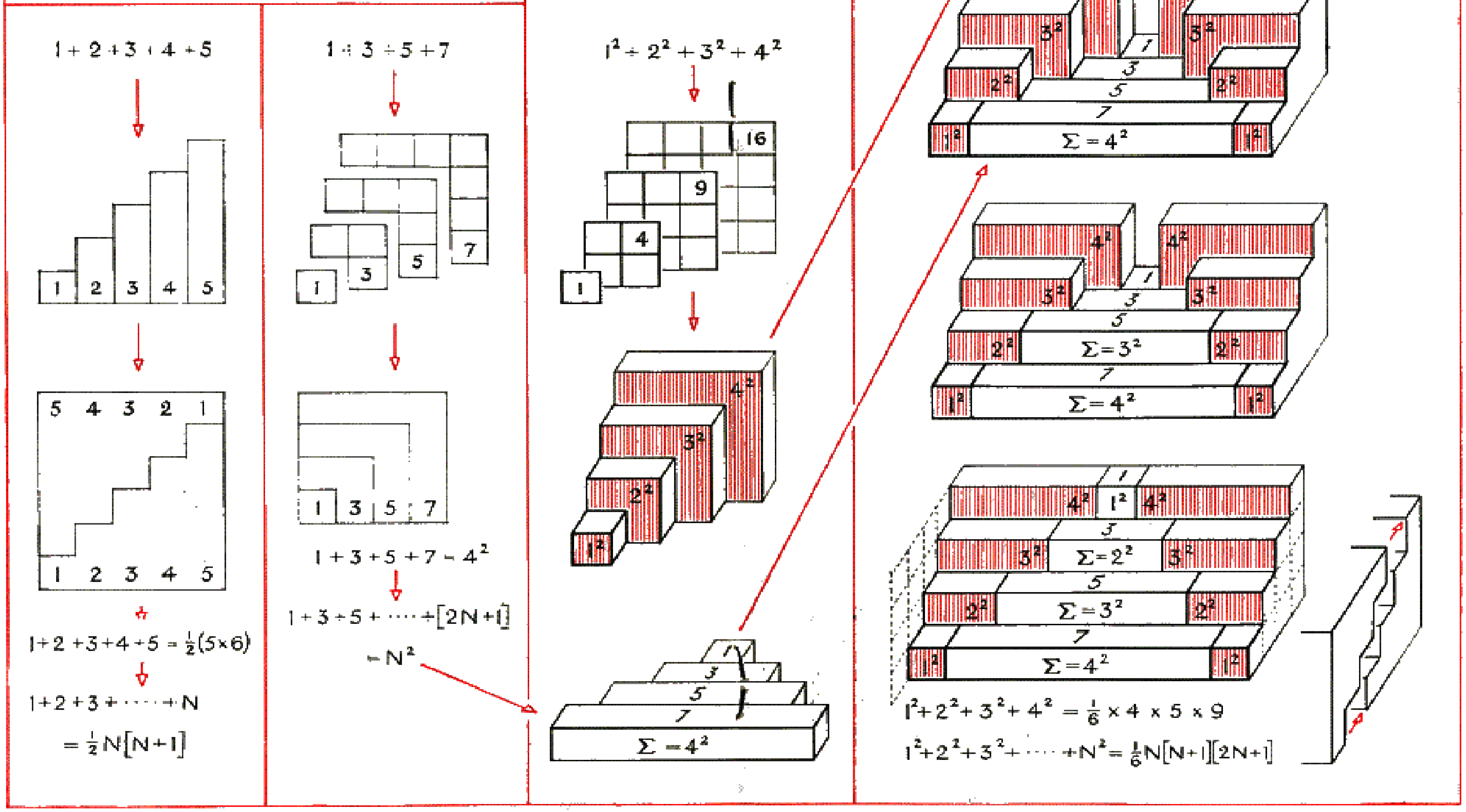
1. A square reversed.
2. A square.
3. A cube reversed.
4. A square.
6. A cube squared.
8. A square.
9. Another square.
11. The square of ' 9 across.'
13. ' 1 down reversed.'
14. A root of ' 6 down.'
15. The square root of ' 2 down cubed.'

CLUES ACROSS :

1. A square.
3. The square of ' 3 down reversed.'
5. A cube reversed.
7. A square.
9. The square root reversed of ' 4 down reversed.'
10. A square cubed.
11. A square reversed.
12. Twice the square root of ' 7 across.'
14. A square squared.
16. A cube.
17. A square.

56576 12186 56196 73378 62362 56125 21632 08628

step by step



Algebraic methods for finding the sum of sequences of numbers are well known. A geometrical representation of the sum of a sequence is often more striking. The results of the illustrations in the left-hand half of the diagram above are used in the right-hand example.

In the right hand diagrams, the space between the two blocks made up of square faced-units is filled with groups of blocks made up of rectangular units which are an odd number of units long and one unit wide.

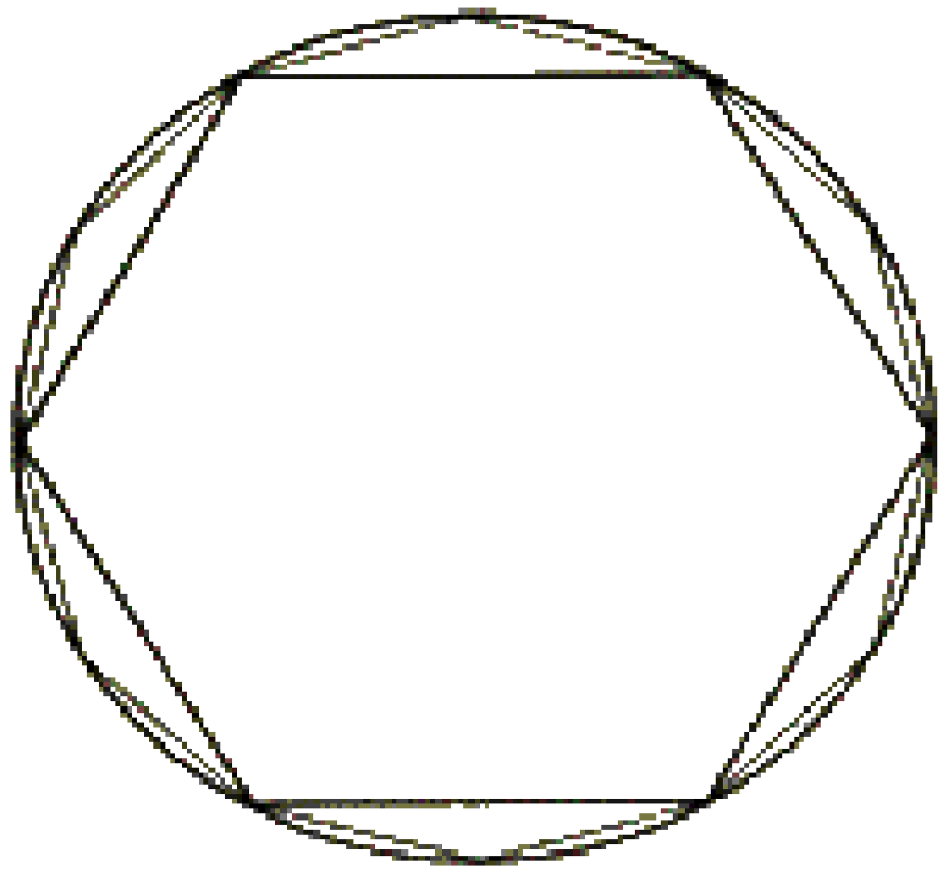
The sum of the odd numbers is equal to the square of the number of terms taken. Hence the complete blocks consist of three parts each made up of the sum of the squares of the natural numbers. Two of these units combine to make a rectangular block whose dimensions are N, N+1, 2N+1, so that the sum of the squares of the natural numbers is one-sixth of the product of these dimensions.

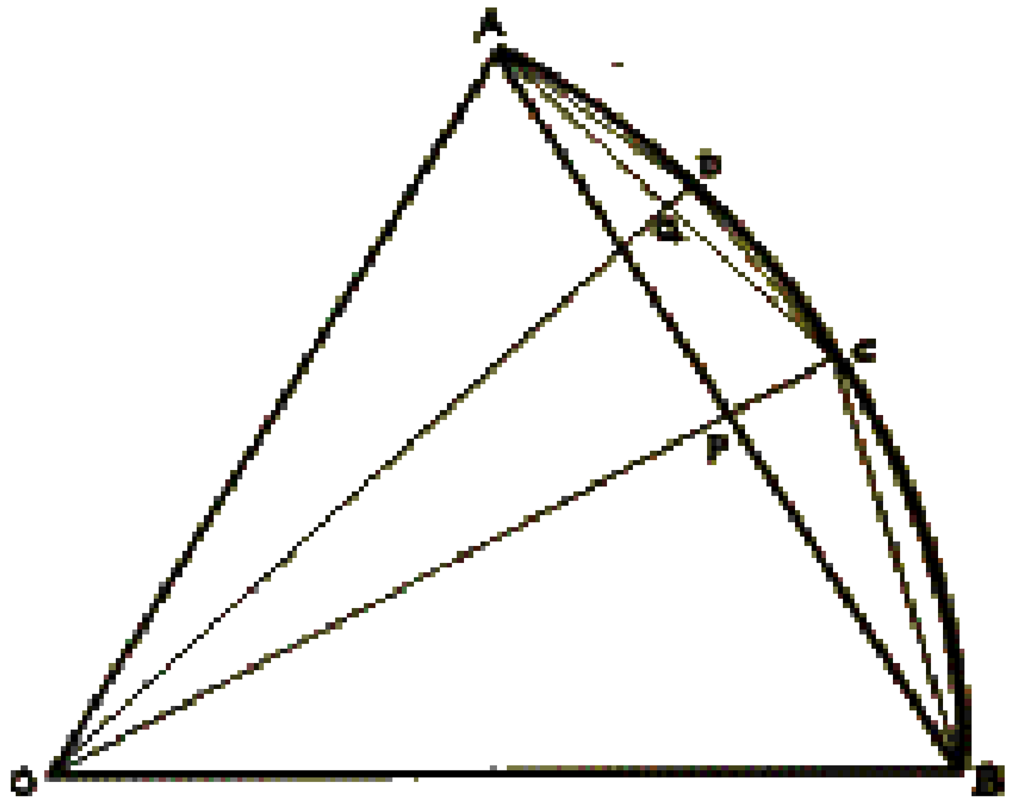
B.A.

69222 10327 48892 18654 36480 22967 80705 76561 51446 32046 92790 68212 07388 37781 42335 62823

EXHAUSTION

The crude idea of the measurement of the area of a figure was to count the number of unit squares which would cover it. It was a great advance when it was shown that the area of a rectangle is found by multiplying the length by the breadth, even if these are fractions. As soon as it is accepted that a square inch need not be a square, it is fairly easy to calculate the areas of parallelograms and triangles, or any figures with straight sides. To find the areas of figures with curved sides is more difficult. In the third and fourth centuries B.C., the Greek Mathematicians invented a method which was still in use in the seventeenth century A.D. Rather surprisingly, they used the theorem of Pythagoras to find the area of a circle. Anyone who can calculate a square root can calculate π accurately to several decimal places. (The Greeks had a harder task as they had to work out their square roots as fractions).





We can start by fitting a regular hexagon inside a circle of unit radius. In the figure ABO is an equilateral triangle and P is the mid-point of AB. Therefore AP is 0.5 inches.

Then $OP^2 = 1^2 - 0.5^2 = 0.75$ and $OP = 0.86603$.

The area of the hexagon is therefore $6 \times \frac{1}{2} \times 1 \times 0.86603 = 2.59809$.

In the spaces between the hexagon and the circle we fit six isosceles triangles, like triangle ABC. $CP = 1 - 0.86603 = 0.13397$.

So the area of the six triangles is $6 \times \frac{1}{2} \times 1 \times 0.13397 = 0.40191$. Into the twelve places that are left we fit twelve more isosceles triangles like ACD.
$AC^2 = AP^2 + CP^2 = 0.5^2 + 0.13397^2 = 0.26794$, and $AC = 0.51763$
$OQ^2 = OA^2 - AQ^2 = 1^2 - 0.258815^2 = 1 - 0.06698 = 0.93302$
$OQ = 0.96592$ and $DQ = 0.03408$ So the area of the twelve triangles is $12 \times \frac{1}{2} \times 0.51763 \times 0.03408 = 0.10584$. The method consists of fitting more and more isosceles triangles into the gaps until all the space is exhausted. Unfortunately, the mathematician is always exhausted first. If we stop now and add together the areas of the hexagon, the first six triangles and the next twelve triangles, we find the area of the 24 sided figure is 3.1058. By making two more steps Archimedes showed that the area of the 96-sided figure inside the circle was more than $3\frac{10}{71}$ and that the area of the 96-sided figure outside the circle was less than $3\frac{10}{70}$ or $3\frac{1}{7}$, and so showed that the area of the circle was between these two values. By A.D. 300, the value of π was known

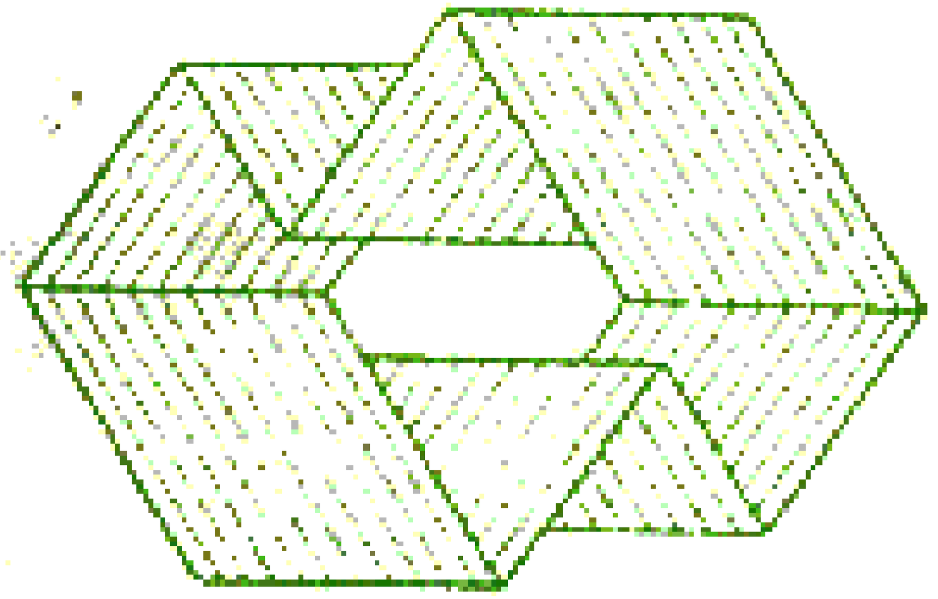
Continued on page 395

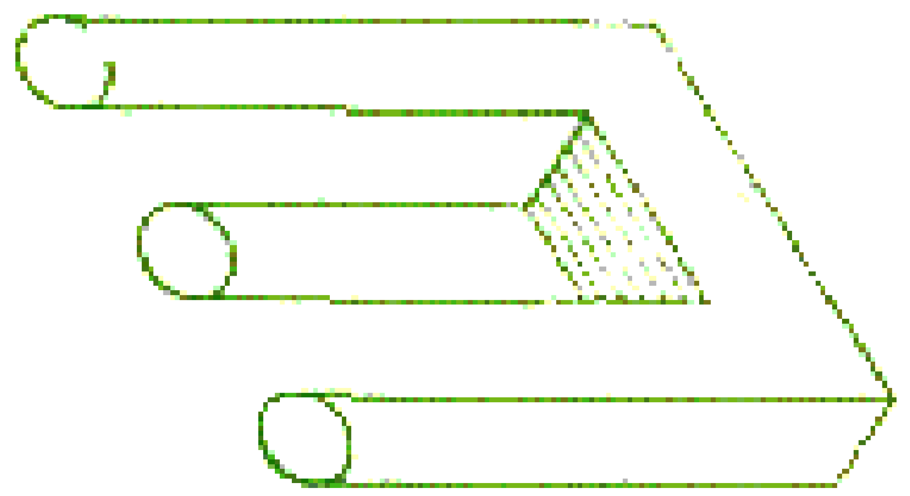
27056 23526 60127 64848 30840 76118 30130 52793

mathematical pie

No. 50 *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England **FEBRUARY, 1967**

OPTICAL ILLUSIONS



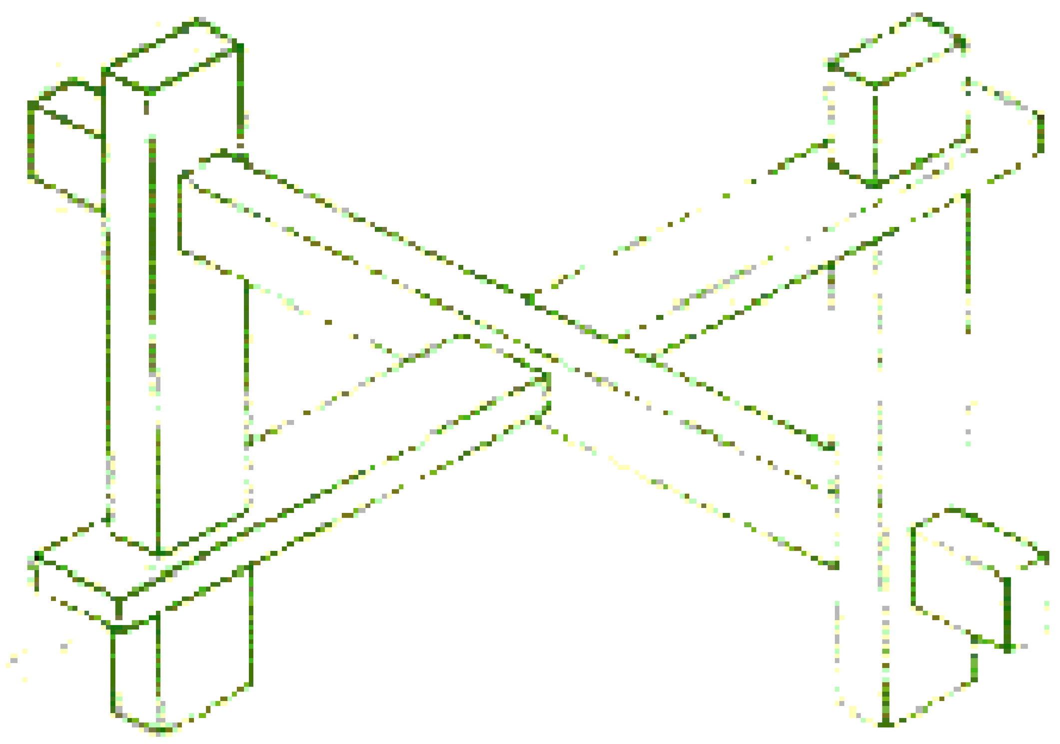


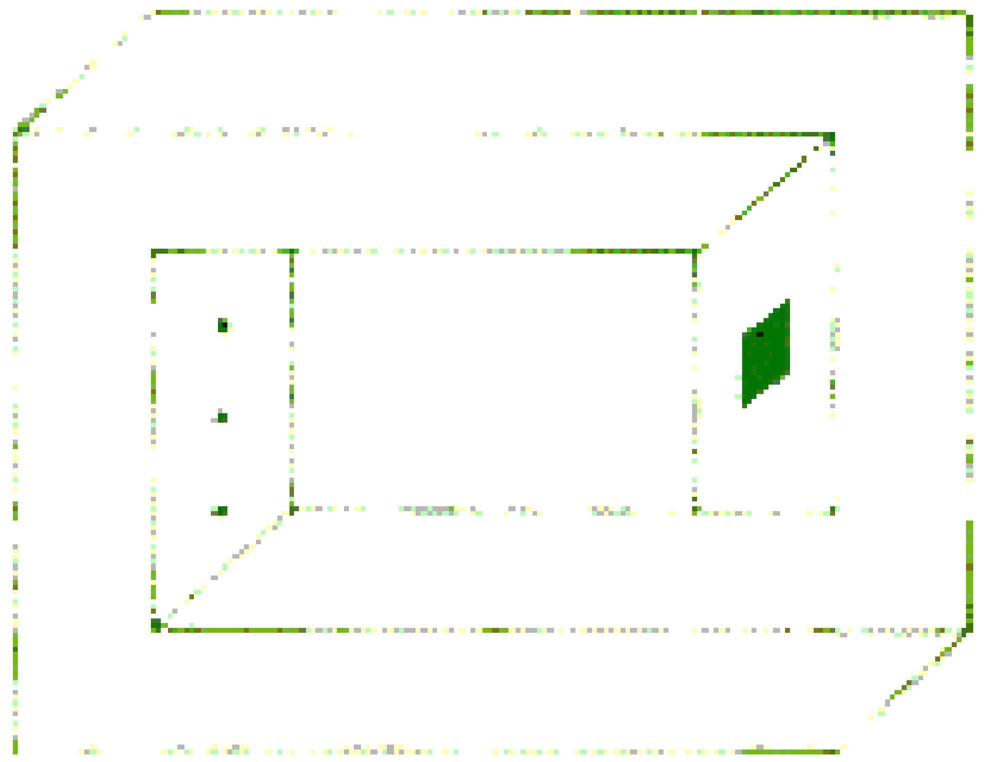
The brain interprets what the eyes see in the light of past experience, taking into account the apparent size, shape and distance of the object from the observer. Hence, the effectiveness of an illusion depends upon the experience of the observer. Serious illusions may occur when there is no framework against which an object can be judged.

The figures show four different illusions in which the brain interprets the picture in one way when concentrating on one part and in another when the concentration is moved to a different part of the figure. It is possible to fit the upper right figure into the lower right figure if the scales are changed, so that the three circular rods fit on to the three dots and the other end fits on to the green patch. Nuts to fit these rods have been designed in various drawing offices, but of course they have not become a commercial proposition.

It is hoped that more care will be exercised when interpreting geometrical figures.

B.A.





25437 09069 79396 12257 14298 94671 54357 84687

MATHEMAGIC

All that you need for this Party piece is one month page from an old calendar. Ask a friend to put a square round any 4×4 block of dates. At this stage you write down a number N on a piece of paper, fold it over so that it is concealed and give it to someone to hold.

Now ask your friend to circle one of the 16 numbers and then strike out the rest of the row and column containing it. Ask another friend to circle one of the remaining 9 numbers and strike out the rest of the row and column leaving 4 numbers. Repeat with one of the remaining four numbers, striking out the rest of the row and column leaving one number which you circle.

Ask your friends to add the four circled numbers and compare their result with the number N — lo and behold they are the same!

In the solutions you will find how to construct the number N — but before you turn to it, can you think out how to do it? R.M.S.

CUBISM

Each face of a cube is painted with one of six different colours, all six colours being used on each cube. How many distinguishable cubes can be produced in this way? S.T.P.

7	+		−		= 8
+		+		+	
	+		−		= 6
−		−		−	
	−		+		= 7
= 8		= 9		= 2	

WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct.

D.I.B.

AS EASY AS A B C

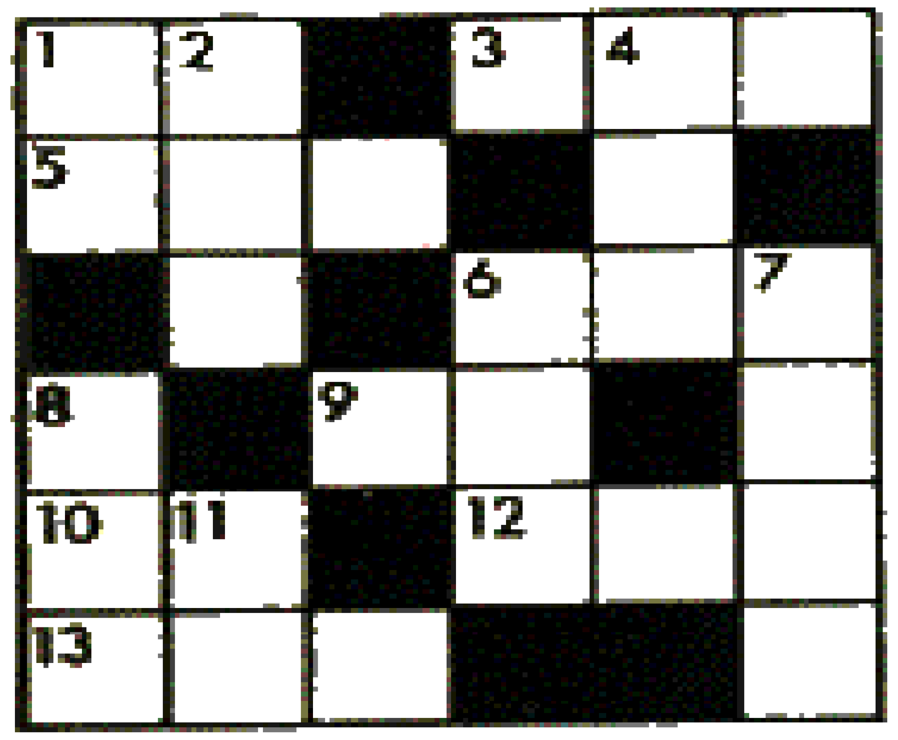
ABC is a triangle in which the bisectors of the angles at B and C meet the opposite sides at D and E respectively. BD is equal to CE. Can you show that the triangle ABC is isosceles? J.F.H.

STAMP COLLECTORS' CORNER No. 23

Pierre Simon, Marquis de Laplace, 1749-1827, who was the son of a peasant farmer became a Professor in the Royal Military School in Paris. He was made a Count by Napoleon and a Marquis by Louis XVIII. To his contemporaries his chief work seemed to be in Astronomy where he solved many problems on the satellites of the planets, but he also worked on probability and algebra where he developed much of the foundations of the modern theory of matrices. C.V.G.

88614 44581 23145 93571 98492 25284 71605 04922

JUNIOR CROSS FIGURE No. 42



Clues Across

1. $a + b + d$.
3. a^3.
5. $(a - b - d)^2$.
6. $(b + d)^2$.
9. $3(a + b)$.
10. Twice 1 across.
12. $a(a^2 - a)$.
13. $d^3 - 1$.

Clues Down

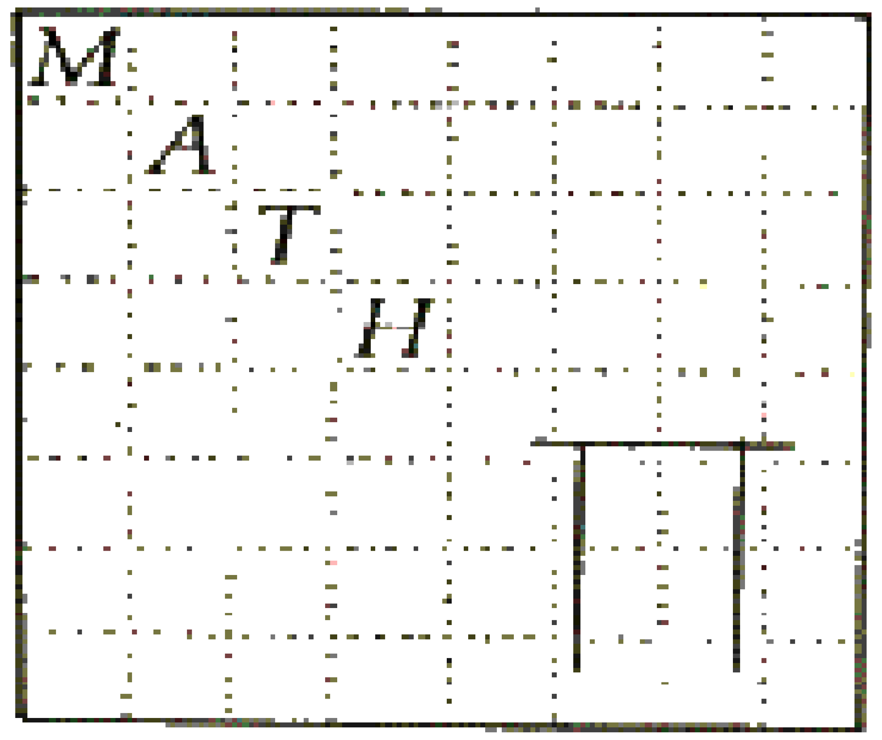
1. $2(a + b)$.
2. $4bd$.
4. $2(a + b)^2$.
6. $9(2b + 3a)$.
7. $9d^3$.
8. $b^2 + d^2$.
11. b^2.

A.B.

$a = 5$, $b = 7$, $d = 10$.

CUTTING UP THE PIE

Cut the square into four congruent pieces such that each has one letter and a piece of PIE. J.F.H.



SOLUTIONS TO PROBLEMS IN ISSUE No. 49

We are still receiving letters on General Taichoff and the Scalene Triangle. These will be dealt with in the next issue.

SENIOR CROSS FIGURE No. 46

CLUES ACROSS: 1. 121; 3. 729; 5. 612; 7. 81; 9. 31; 10. 46656; 11. 94; 12. 18; 14. 256; 16. 125; 17. 441.
CLUES DOWN: 1. 148; 2. 16; 3. 72; 4. 961; 6. 15625; 8. 144; 9. 361; 11. 961; 13. 841; 14. 25; 15. 64.

JUNIOR CROSS FIGURE No. 41

Clue 2 down should have read 111 ÷ 1 reversed.
CLUES ACROSS: 1. 341; 4. 422; 11. 343; 12. 024; 14. 1011.
CLUES DOWN: 1. 30341; 2. 14301; 3. 32; 10. 224; 13. 21.

SOLUTIONS TO PROBLEMS IN ISSUE No. 50

Two minuses make a plus. Mr. Barker of Hymers Colleges points out that (8888−888) ÷ 8 = 1,000. This makes for any number from 1 to 9 as well as 8.

WITHOUT A WORD: 7+4—3=8 4+6—4=6 3—1+5=7.

CUBISM: There are 30 different cubes. MATHEMAGIC: N = 2 [1st + last dates].

INITIAL ALGEBRA 2

The only possible amounts I have found are 57s. 0d. + 14s. 6d., which represents purchase tax of 25%. This gives a code word C??TIAN?Y is presumably CERTIANLY.

FUN WITH NUMBERS No. 8: 10 = 9 $\frac{99}{99}$ among others.

THE ETERNAL TRIANGLE: The number of triangles is 118.

SEQUENCES

1. The sequence continues 12, 13 and the scale is five.
2. 100 101 111 and the scale is two.
3. 8, 9 @, *, 10, 11, 12, and the scale is twelve where @ means ten and * means eleven.

Another solution suggested by a student was 8, 9, A, B, and the scale is C. B.A.

Continued from page 396

correct to six decimal places. If you have the issues of *Mathematical Pie* from Number 18, you have the decimal equivalent of π to 9,270 places of decimals.

43840 70070 73604 11237 35998 43452 25161 05070

MORE MATHEMATICAL SHORTHAND — PART 1

In issue No. 46 we were introduced to $\int$, the sign of mathematical fiddling, and previously we have met ! These are both mathematical symbols to signify, as briefly as possible, some lengthy process. $\int$ for example, is a shorthand way of writing a particular kind of addition, and ! an important type of multiplication.

Now you are all familiar with the sort of riddle-me-re which starts

My first is in ladder and also in stairs
My second in apples and also in pears

Strange though it may seem, a mathematician may also need to solve problems like this, and he has invented a whole set of symbols to help him. For example, he might write

$$A = \{l,a,d,e,r\}$$

meaning that A is the set of different letters which appear in the word ' ladder.' Similarly he might write

$$B = \{s,t,a,i,r\} \qquad C = \{a,p,l,e,s\} \qquad D = \{p,e,a,r,s\}$$

for the letters used in writing ' stairs,' ' apples ' and ' pears.' Then if he is solving the first part of the riddle-me-re, he wants to say " those letters which are common to the words ' ladder ' and ' stairs ' " and he writes $A \cap B = \{a,r\}$. In other words there are two letters which appear in both words. And if he does the same thing for the second line of the riddle-me-re, he writes $C \cap D = \{a,p,e,s\}$. That is there are four possible letters he can choose.

Suppose, on the other hand, he wishes to indicate which different letters are necessary to write ' ladder ' and ' stairs,' he writes $A \cup B$. In this case $A \cup B = \{l,a,d,e,r,s,t,i\}$ and similarly $C \cup D = \{a,p,l,e,s,r\}$.

Got it ? Then try and decode this message. (You may have to re-arrange the letters in each word to do so). If you find you really have ' got the message ' then code one of your own and send it to another member of your form.

1. $A = \{m,o,u,s,e,y\}$
   $B = \{y,o,u,t,h\}$
   $A \cap B =$?
2. $C = \{h,a,v,e,n\}$
   $D = \{s,h,a,v,e,d\}$
   $C \cap D =$?
3. $E = \{c,r,o,w,n,e,t,h\}$
   $F = \{l,o,w,i,n,g\}$
   $E \cap F =$?
4. $G = \{p,l,o,d\}$
   $H = \{c,o,m,e,t\}$
   $G \cup H + e =$?
5. $I = \{h,e,r,m,i,t\}$
   $J = \{h,e,a,t\}$
   $I \cap J =$?
6. $K = \{f,i,t\}$
   $L = \{s,t,i,r\}$
   $K \cup L =$?
7. $M = \{e,a,t\}$
   $N = \{g,a,s\}$
   $M \cup N =$?

S.T.P.

SEQUENCES

Sequences submitted by Mrs. G. Beard, Oastler College, Huddersfield.

Continue these sequences

1. 1, 2, 3, 4, 10, 11, –, –,
2. 1, 10, 11, –, –, –,
3. 1, 2, 3, 4, 5, 6, 7, –, –, –, –, 10, 11, –,

State the scales of notation.

INITIAL ALGEBRA 2

Some years ago I found the following on a price ticket :

IN/Y + CT/A = 72s. 3d.

The first letter group is the cost and the second group represents the purchase tax, which is a percentage of the cost such as 66⅔, 50, 33⅓ per cent. or a similar fraction.

Can you see why I concluded that someone in this shop is weak at spelling ?

Submitted by Mr. G. Edgcombe, Plymstock Comprehensive School.

FUN WITH NUMBERS No. 8

Express the number 10 by using five 9's and find four different methods of doing this. R.H.C.

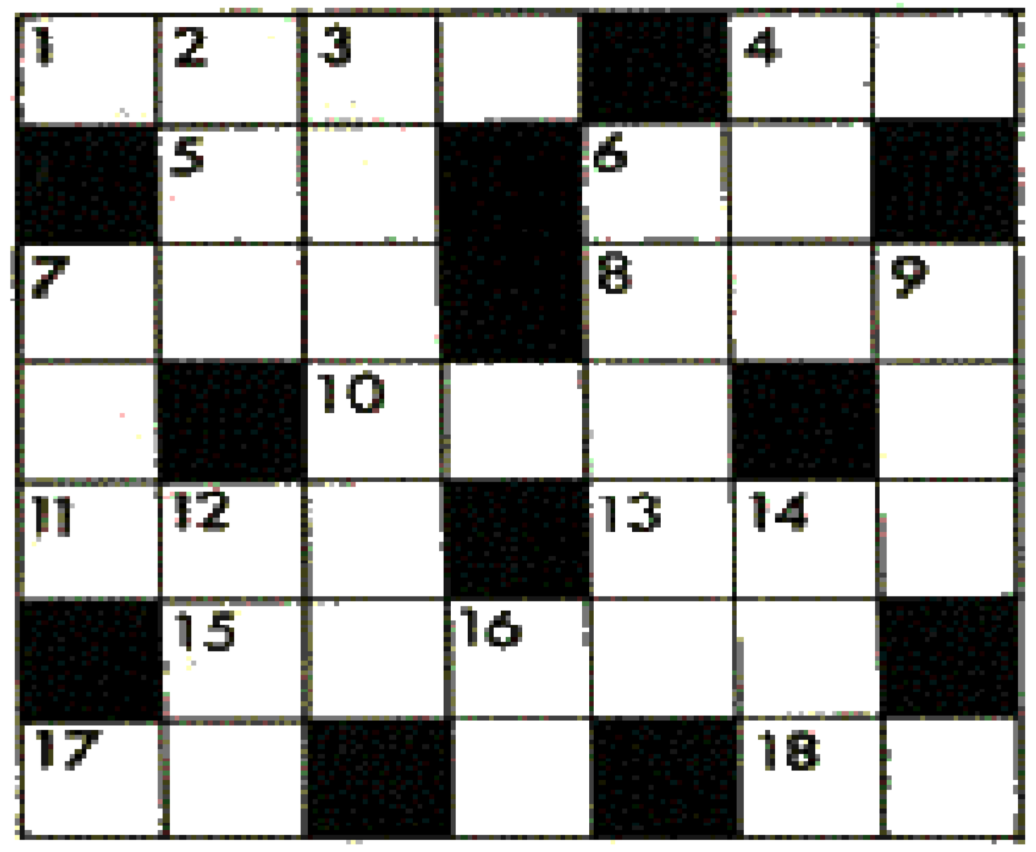
CAN YOU UNDERSTAND THIS ?

A body by describing a kind of spiral might descend towards a revolving globe so that its apparent motion with respect to a point on the surface might be a straight line tending to the centre. (LEONARDO DA VINCI, 1510 A.D.)

THE ETERNAL TRIANGLE

An equilateral triangle of side 7 units is divided into equilateral triangles of unit length. How many triangles are there in the figure ? R.H.C.

SENIOR CROSS FIGURE No. 47



Ignore decimal points and work to the required degree of accuracy.

CLUES ACROSS :

1. The largest angle in degrees and minutes, of a triangle with sides 5.42, 7.63, and 6.21 units.
4. $\sqrt{2209}$.
5. $\sqrt{441}$.
6. The diagonal of a rectangular block 3 by 4 by 12 units.
7. The reciprocal of 14.
8. $\sqrt[3]{120}$.
10. 5 cubed.
11. $\sqrt{273}$.
13. $\sqrt{570}$.
15. $\sqrt[3]{26}$.
17 and 18. The smallest angle of the triangle in 1 across.

CLUES DOWN :

2. 11 squared.
3. π.
4. The longest stick, in inches, that will fit into a box 25, by 30 by 20 inches.
6. Area, in square inches, of a circle of diameter 4.3 inches.
7. The length of the hypotenuse, in cm., of a triangle whose other sides are 5 and 6 cm.
9. $\sqrt[3]{59}$.
12. $\sqrt{3894}$.
14. $\sqrt[3]{44}$.
16. 4^3.

B A.

The sportsman need not be a student of Euclid to be aware of the difference in physical form between the long distance runner and the shot putter; or between the full-back and the outside-right. However, he may not realise the mathematical reasons for these disparities.

The athlete's energy depends on a continuous supply of oxygen which reaches the tissues via the surface area of the lungs. The dissipation of heat similarly depends on a surface area, the skin. In contrast, power is a function of a solid, the muscle bulk. A slight physique has a greater surface area in proportion to body mass than a sturdy physique. Thus the former is better adapted for endurance activities. Over a short distance the bulkier physique has the advantage. Not surprisingly, the heavier crew is generally favoured to win the Boat Race.

Large animals, with a smaller surface area in proportion to body mass, can more easily retain body heat than small animals. It follows that wild life in Polar regions partly depends on its size for survival and is rarely

02763 47870 81081 75450 11930 71412 23390 86639

small. Carl Bergmann recognised the mathematical reasons for animal size variations.

Considering an animal, plant or engineering structure, it is evident that the weight is proportional to the volume. Yet the strength is proportional to the area of cross-section of the supports. Thus, of two geometrically similar structures, the larger is the weaker. Large animals require thicker skeletons for support. Large fruits are often produced at low levels for ground support.

Discovering that stalling-speed varies as the square root of the linear dimension, one can appreciate why large birds require a running take-off and why the ostrich does not take to the skies.

A detailed account on the practical applications of similarity and proportion is given in that ever useful volume 'On Growth and Form' by D'Arcy W. Thompson.
D.I.B.

38339 52942 57869 05076 43100 63835 19834 38934