

# THE TITHE OF WAR

from an old puzzle book

Old General Host  
A battle lost,  
And reckoned on a bissing,  
When he saw plain  
What men were slain,  
And prisoners and missing.

To his dismay  
He learned next day  
What havoc war has wrought  
He had, at most,  
But half his host,  
Plus ten times three, six, ought.

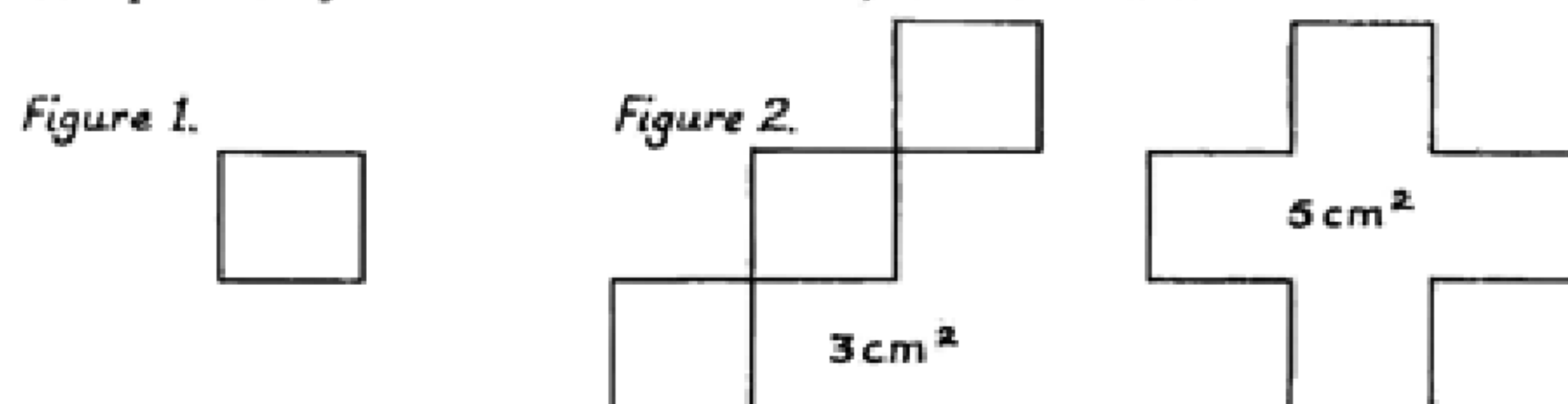
One-eighth were lain  
On beds of pain,  
With hundreds six besides;  
One-fifth were dead,  
Captives or fled,  
Lost in grim warfare's tide.

Now, if you can,  
Tell me, my man,  
What troops the general numbered  
When on that night  
Before the fight  
The deadly cannon slumbered?

## HOW FAR ROUND?

Using a square grid, we can make various shapes of different areas and/or different perimeters. Let us now fix the perimeter to 12 units.

How many different shapes can you make? All the sides must lie on the lines of the grid and all the lines must enclose an area. Fig. 2 gives two examples. Can you make areas of 3 units<sup>2</sup>, to 9 units<sup>2</sup> inclusive?



Now suppose that the area is fixed, at say 9cm<sup>2</sup>. What different length of perimeter can you obtain using the same restrictions?

C.B.A.

## DO WE KNOW ENOUGH?

I have a cuboid with the faces with the largest areas are squares. The diagonals of the rectangular faces are each 20 cm and when drawn form two equilateral triangles with the shorter side of the face. What is the volume of the cuboid?

C.B.A.

## DOUBLE TROUBLE

Trudy and Leah are identical twins. Trudy always tells the truth but Leah always tells lies. How can I distinguish between them by asking one question of one of the twins?

D.I.B.



# MATHEMATICAL PIE

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## BOMBS AWAY

In the fifth Royal Institute's Christmas Lecture on 1st January 1982, Professor R. V. Jones F.R.S. showed how simple geometry could be employed in two unusual ways. In 1943, No.617 Squadron of the Royal Air Force was given the task of dropping spherical bombs designed by Sir Barnes Wallis, F.R.S. from aircraft in order to demolish some German dams. The bombs were made to bounce off the surface of the water and so jump over strong nets put up to prevent torpedoes from being used. These bombs had to be released at exactly 60 feet (18.288 m) above the water and at an exact distance from the dam walls. Search-lights were fitted at the ends of the wings of the Lancaster aircraft used. (P and S in Diagram 1). The wing span of a Lancaster was 102 feet. When the aircraft was made all measurements were in Imperial Units. 1 Foot = 12 inches = 0.3048m = 30.48cm. For con-

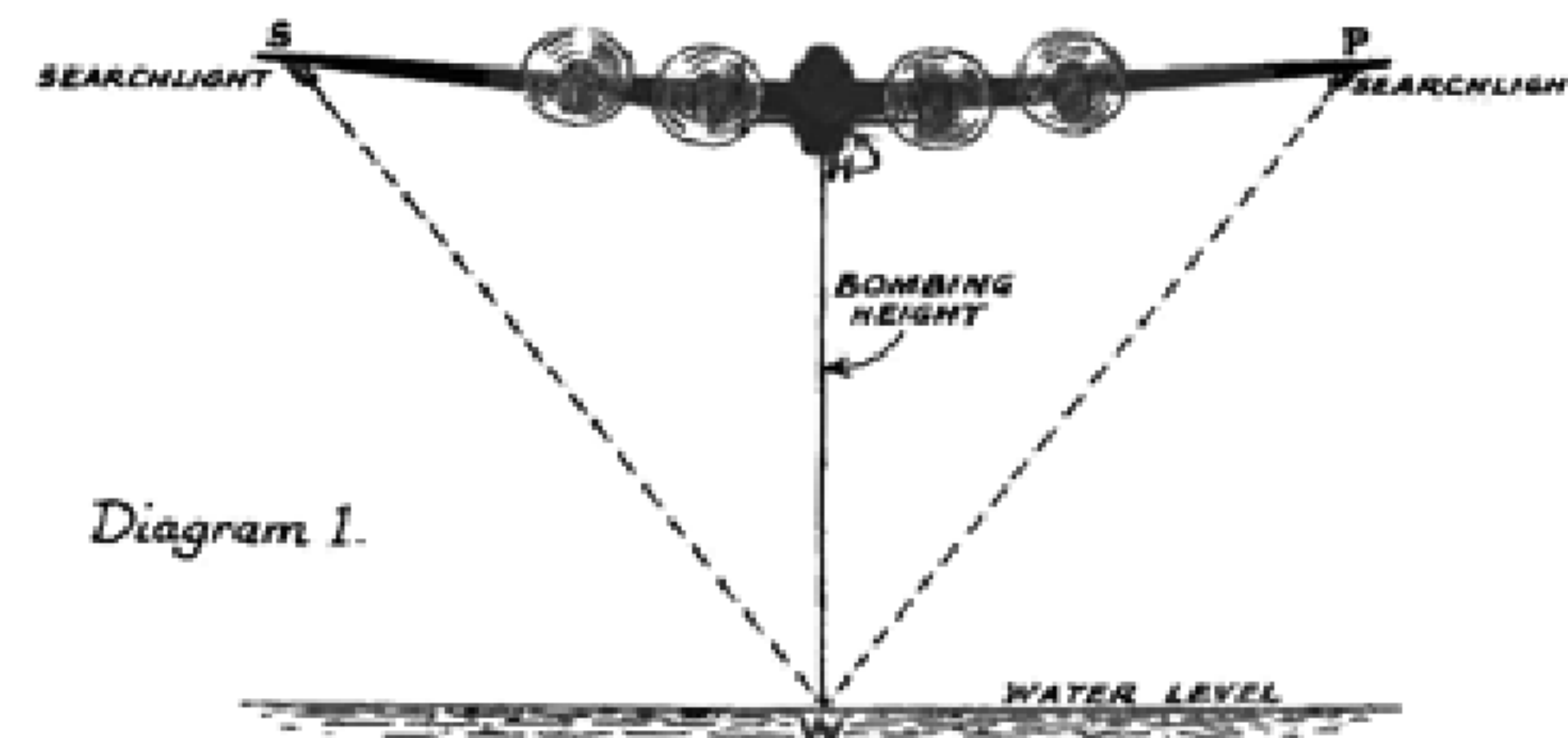


Diagram 1.

venience, let us assume that the lights were fitted so that they were 100ft = 30.48 metres apart. The pilot in his run-up to attack the dam switched the lights on and adjusted his height so that the two rays SW and PW converged on the water surface to give a single spot of light, the height of the aircraft was then exactly 60 feet (18.288 metres) above the water. At about what angle to the main axis of the wings (SWP) had the lights to be set? Angles SPW and PSW are equal. Suppose that the water surface was flat, or almost so, how could the pilot know that his height was still 60ft. (18.288 metres) above the water surface if one of the lights went out? There was another problem, this time for the bomb-aimer who had to know at what distance

from the dam he had to release the bomb. To find this distance, an instrument like that shown in Diagram 2 was invented. The dam had two towers placed symmetrically about a quarter of the width from each bank. These are represented by D and E in Diagram 3. The aircraft attacked on a course at right angles to the line DE, that is AFH. The bomb aimer looked through the hole in the upright at A and waited until the tops of the two nails at B and C coincided with D and E. When this moment arrived the bouncing bomb was released.

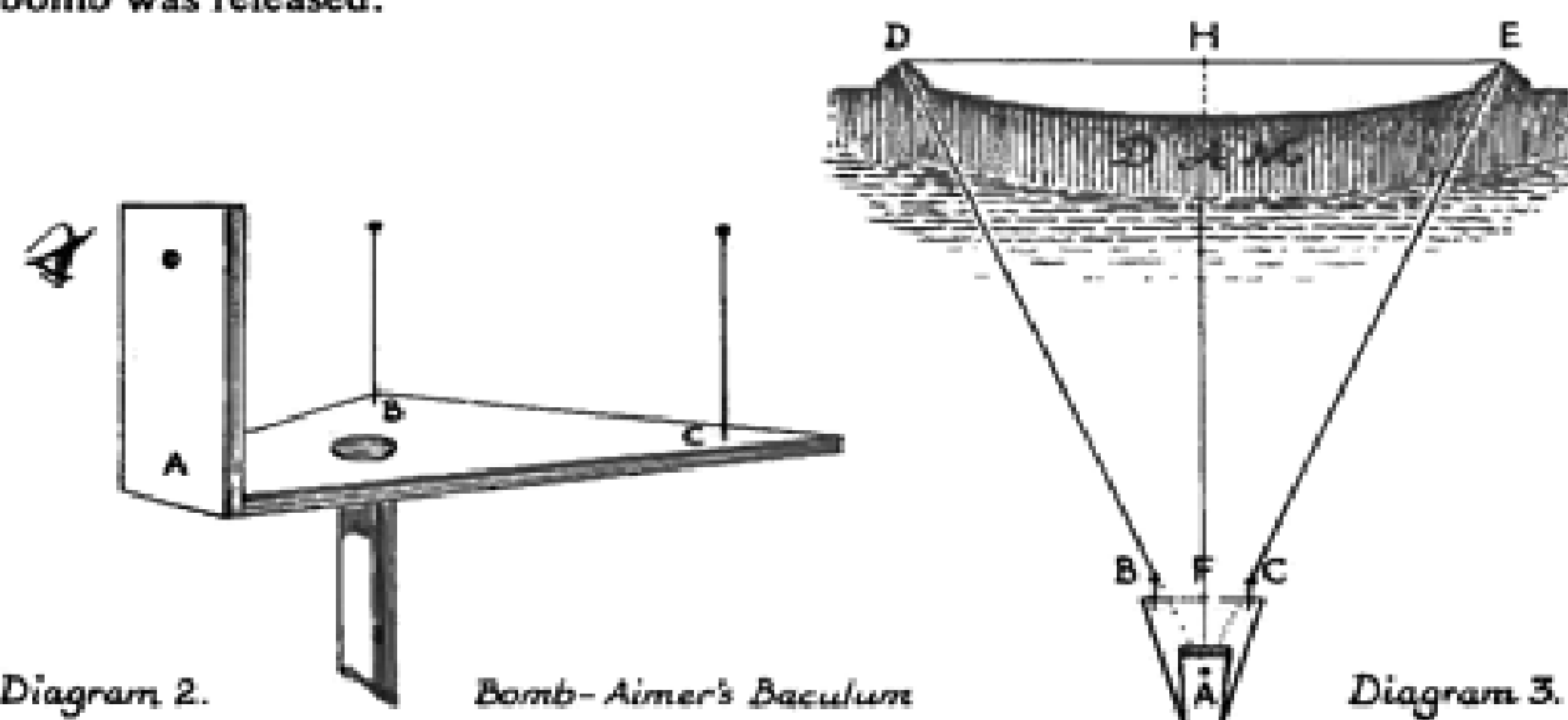


Diagram 2.

Bomb-Aimer's Baculum

Diagram 3.

This use of the properties of similar triangles was employed in an instrument called a Baculum invented for use in the sixteenth century to find the width of distant fortifications. Find the relationship between DE, BC and AF, AH. What do you know about AD and AE also AB and AC? Would it matter if these distances were measured in inches, yards, metres or centimetres? What angles are the same as BAF?

S.H.F.

### FACTORIALS!

6! means  $6 \times 5 \times 4 \times 3 \times 2 \times 1$  and similarly 5! means  $5 \times 4 \times 3 \times 2 \times 1$  and are read factorial six and factorial five.

What do you make of  $1! + 4! + 5!$ ?

R.H.C.

### A CALCULATING PROBLEM – OR IS IT?

Which of the following has the largest value?

$2^{39}$ ,  $8^{12}$ ,  $32^7$ ,  $4^{19}$  or  $16^9$ .

C.B.A.

### PLAYING WITH A CALCULATOR

Dear Editor,

I was playing with my father's calculator and noticed the two interesting answers.

$$100 \div 9 = 11.111111 \text{ and } 11.111111 \div 9 = 1.2345678$$

Ian Atkin (aged 7 years)

Hawaii these holes were made at such a distance that a man looking through a hole over the opposite side was doing so at an angle of  $19^\circ$ . So that he could be sure that the bowl was level, no water must spill from it when in use. Can you say how you could find out if the angle is  $19^\circ$ ?

S.H.F.

### SOLUTIONS TO PROBLEMS IN ISSUE No.95

*An awkward answer* The letter A does not appear.

*Pythagoras again and again* The third side is 56 units.

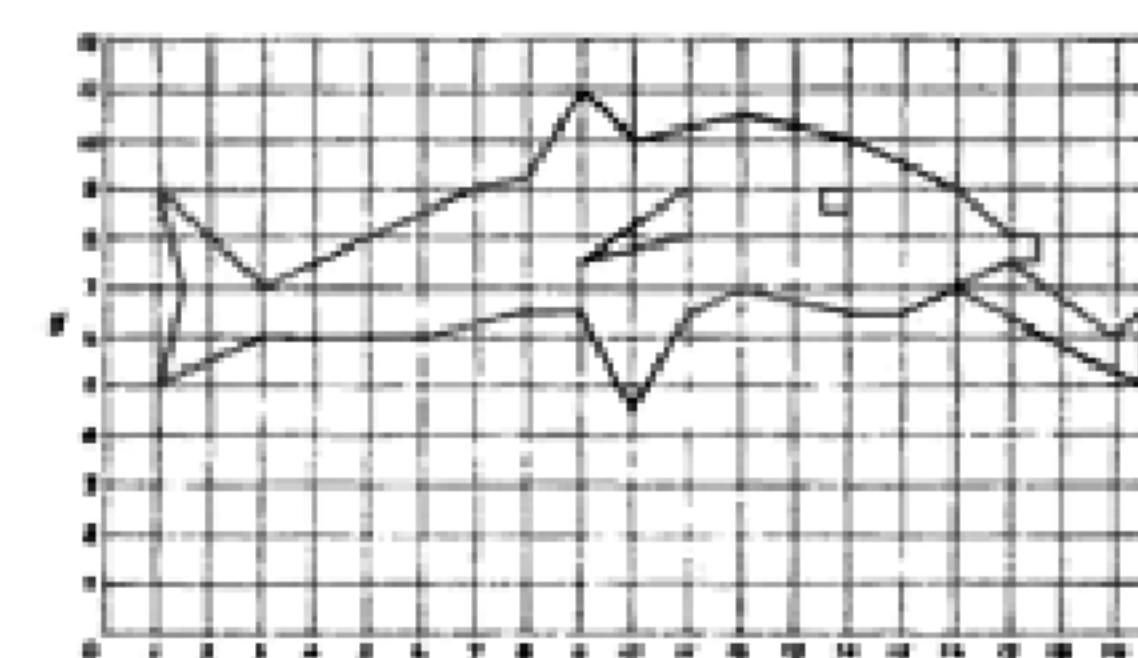
*Simple anagrams* Cone, Triangle, Base, Face and Rectangle.

*Half a minute* The two diagonals of a rectangle are equal in length. EC equals the radius of the circle, 7 cm.

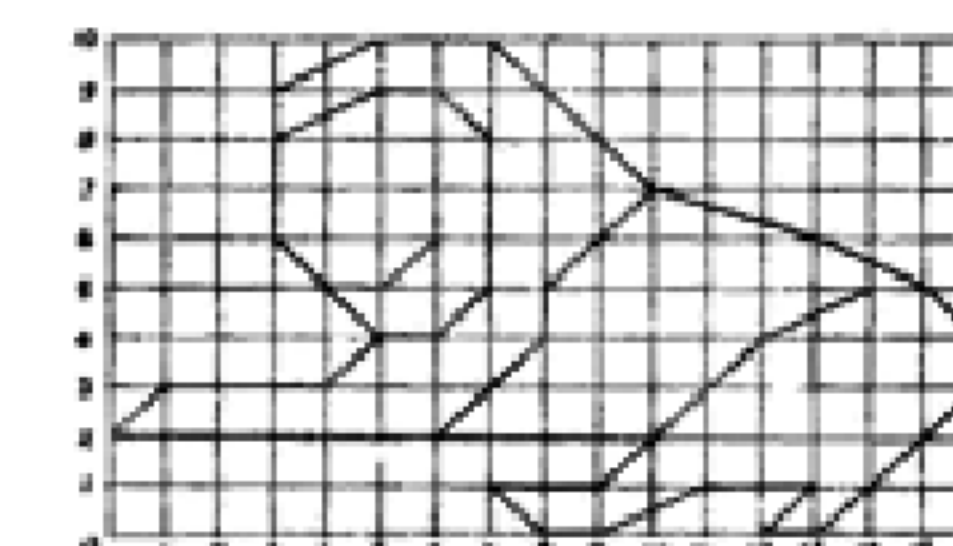
*Next base*  $322_{\text{five}} = 223_{\text{six}} = 87_{\text{ten}}$ .



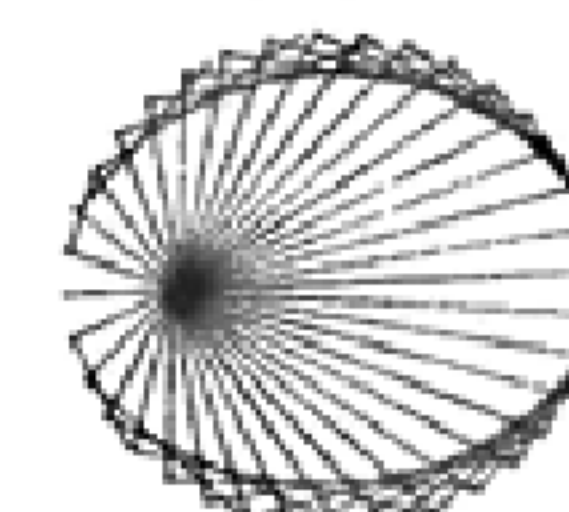
### Fishing lines



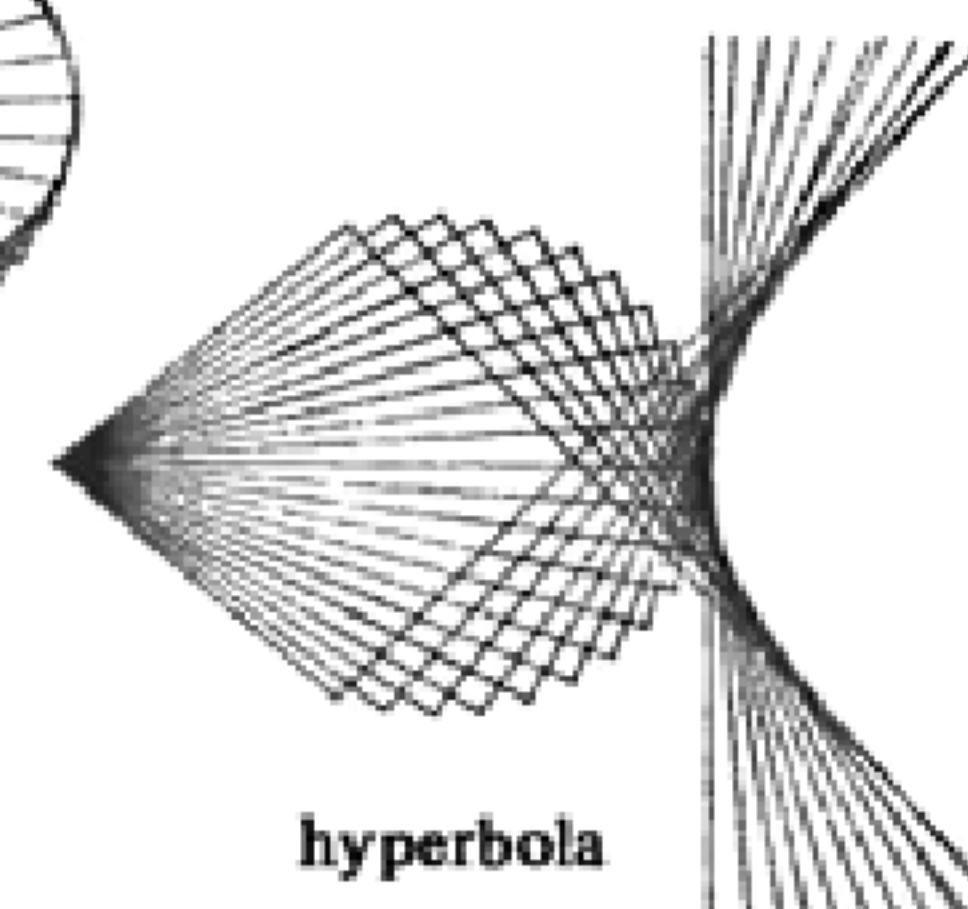
### Main line?



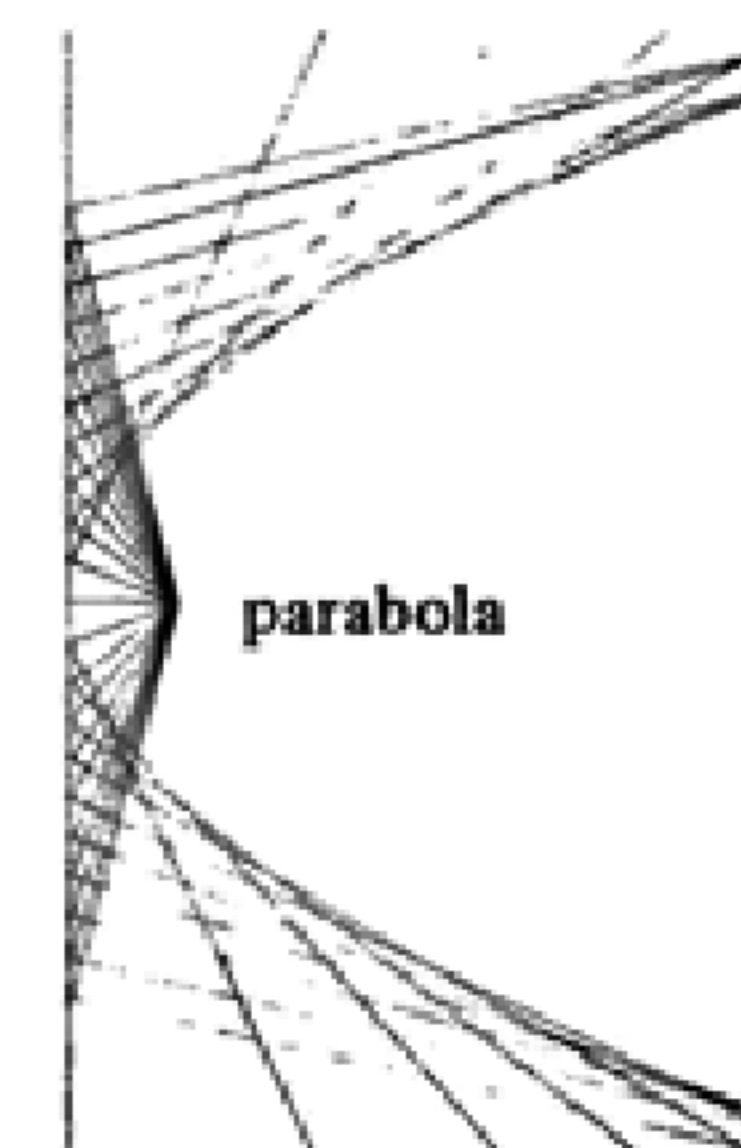
### Envelope patterns



ellipse



hyperbola

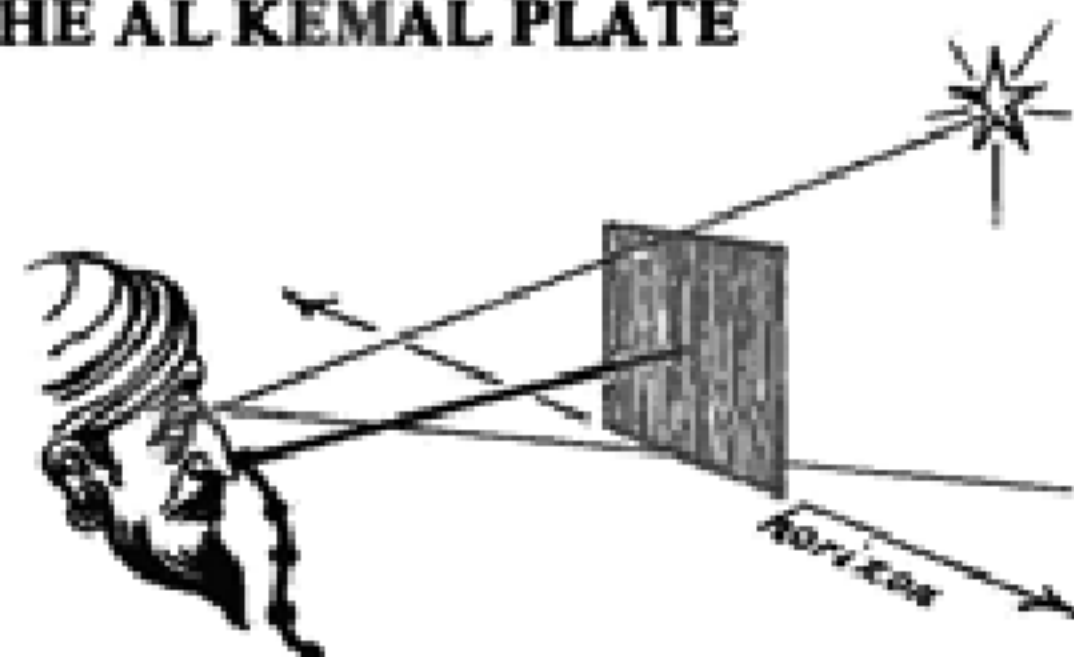


parabola



## KON TIKI, TIGRIS AND THE AL KEMAL PLATE

Thor Heyerdahl is a brave man, who, by making two risky voyages has shown how early sailors were able to find their ways across great distances to foreign ports. To us it seems their journeys were slow, when compared with those we can make today, but then time did not matter. These sailors had several simple ways of finding out whether their course was North or South of the port to which they were sailing. They knew if their latitudes were greater or less than those of their destinations. They could not find their longitudes, or how far East or West they were. But they did have a compass which helped them to sail in the correct direction.



A simple instrument called an AL KEMAL PLATE was used by Arabs to find the latitude of places they wanted to visit. This was a piece of wood about 15cm square, in the middle was a hole through which passed a cord, or thong, with a knot at the end. Knots were tied along this cord on the other side of the plate by the owner who we will suppose made a journey from SOCOTRA to BOMBAY, then on to GOA and CALICUT. At Socotra he went up on deck at night and holding the plate upright in his left hand held it out towards the Pole Star with the cord in his right hand, he then moved the plate to and fro until the bottom edge appeared to lie along the horizon, with the Pole Star, as it were, sitting upon the top edge. With his right hand he slid his thumb and finger down the cord so that when the plate was in position the cord was tight and his right thumb against his nose. He then gripped the cord in this position and put down the plate carefully tying a knot in the cord at the exact point where his finger and thumb had held the cord. When this was done he checked the position of the knot by repeating the process: Plate in left hand, thumb and finger holding knot, bottom of plate appears to rest on horizon while Pole Star sits on top of Plate. This knot will now be called "SOCOTRA". The ship then sailed East to Bombay then South to Goa and Calicut where the maker of the Plate repeated the routine described. When this was done the owner would have four knots on the cord and so be able to navigate a ship for himself, after many years of practice.

Although Thor Heyerdahl used modern navigational aids on Tigris, he may well have used the Al Kemal Plate, because he travelled over the same course, and in the same way, as that used by sailors many hundreds of years ago.

The Kon Tiki expedition sailed to islands in the Pacific which were visited by primitive sailors. Some of these sailors went to Hawaii which is on Latitude  $19^{\circ}$ . Stretch out your left hand before you with the thumb and little finger extended and the other fingers folded into the palm. Hold up your arm, and place your thumb so that it appears to rest on the horizon, then if the little finger has a star just above it, the elevation of this star is  $19^{\circ}$ . If that star happens to be the Pole Star then you are on the same Latitude as Hawaii.

These primitive sailors also used a coconut shell which had a row of holes bored around and at equal distances down from the rim. To find

## ANOTHER CARD TRICK

*submitted by S. K. Bhandari, Esq. of Francis Combe School, Garston, Watford*

Assuming that the picture cards all take the value 10 and the other cards count their face value, choose a number  $N$ , between 10 and 20. Any number could be chosen but the trick will take longer to set up if the number is outside this range.

Shuffle the cards and begin to deal. Turn over the first card and place it face upwards on the table. If the face value of the card is  $x$ , the next card of whatever face value is counted  $x + 1$  and placed on top of the first card. The next card is counted  $x + 2$  and placed on the same pile. We continue dealing cards onto the first pile until we reach  $N$ . We go on with the process again and again until the remaining cards do not make a complete pile.

Now ask a friend to turn over as many piles as he likes and you will give him the total of the face values of the top cards of these piles. Collect the remaining cards and count them.

## VAT IS DAS

A restaurant bill came to £18.00. On top of this V.A.T. of 15% has to be added as well as a service charge of 10% of this increased total. Does it make any difference if the calculation is carried out by first adding V.A.T. and then the service charge or the other way round, the service charge followed by the V.A.T.

Can you explain your result and say on what mathematical principle it depends?

Would anyone complain about the change in the arrangements?

R.H.C.

## A POWER GAME

The last digit of  $7^1$  is 7 as  $7^1 = 7$ ,  $7^2$  is 9 as  $7^2 = 49$ ,  $7^3$  is 3 as  $7^3 = 343$  and  $7^4$  is 1 as  $7^4 = 2401$ .

If we continue this, can you find a pattern that will enable you to find the last digit of  $7^{55}$ ?

D.H.

## IN THE AIR

*submitted by John Dickinson of Liverpool*

Draw two axes at right-angles to each other, 0 to 7 on the x-axis and 0 to 5 on the y-axis. Plot the following points and join them in the order given.

(1,2,2,4), (1,2,2,3), (4,0,2,9), (3,6,3,2), (1,2,2,4),  
 (4,0,2,9), (5,6,2,2), (5,9,2,2), (6,0,2,5), (5,7,3,0), (3,0,4,1), (3,0,4,0),  
 (2,5,4,1), (2,5,4,3), (2,8,4,3), (2,7,4,4), (2,0,4,3), (2,5,4,3),  
 (2,5,4,4), (2,5,4,9), (2,7,4,9), (2,8,4,3), (3,0,4,1),  
 (2,5,4,1), (2,3,4,0), (2,4,3,8), (2,6,3,8), (2,5,3,6), (2,2,3,7), (2,2,3,8),  
 (2,4,3,8),  
 (2,5,3,6), (3,4,3,1),  
 (2,8,4,3), (3,2,4,3), (2,8,4,4),  
 (6,0,2,5), (5,7,2,5), (5,6,2,4), (5,8,2,2),  
 (5,9,2,8), (5,5,2,6), (2,3,4,0),  
 (5,9,2,7), (5,5,2,5), (2,4,3,9),  
 (4,5,3,5), (6,0,4,2), (6,2,4,2), (4,9,3,3).



## CALCULATION Through the Ages

17<sup>th</sup> Century  
— English Maritime —  
— Expansion —

SAN FRANCISCO  Late 20<sup>th</sup> Century

SALAMIS  
ABACUS

of Ancient  
Greece.  
Small pebbles  
were used to  
calculate on an  
engraved  
marble slab.

### ROMAN HAND ABACUS

in which the counting beads moved in slots, the upper row 'Fives' and the lower 'Units'. 954-1 is shown.

## MODERN JAPANESE ABACUS or 'SOROBAN'

A skilled operator can compete very favourably with sophisticated calculators. 987654321 is shown.

After reading about Charles Babbage's Difference Engine, GEORGE SCHULTZ, & son EDWARD of Stockholm spent 20 years designing the Schultz Difference Engine No. 2, completed in 1859, used mainly in calculation of Life Insurance Tables.

*With a sliding section that moved into the columns for units, tens, hundreds etc, the machine built in 1694 by GOTTFRIED LEIBNIZ of Germany was the first that could multiply by rapidly repeated addition.*

Having no written language, the Incas of S. America kept records of accounts on QUIPO CORDS. Knots represented numbers in the decimal system. The sum in a group of cords was recorded on the top cord.

In 1820 —  
**CHARLES XAVIER THOMAS** of  
*Alsace* invented first successful  
 calculating machine to be  
 manufactured commercially.  
 Its basic design was adopted  
 by many manufacturers for  
 over one hundred years.

The First known attempt to construct a CALCULATING MACHINE was by Wilhelm SCHICKARD of Germany in 1623

**CHARLES BABBAGE'S**  
*Difference Engine 1835*  
*Designed to calculate*  
*& print mathematical*  
*tables by the method of*  
*differences. Abandoned*  
*owing to the high cost*  
*of construction.*

An 1892 —  
Hand-desk-calculator of a type in common use until the advent of electronic calculators. This machine incorporated "ODHNER" wheels, with a variable number of teeth from 0 to 9 patented by WILLGODT ODHNER of St. Petersburg in 1891. In 1875 FRANK BALDWIN invented a machine based on a similar principle.

Adapting AMBROSE FLEMING's recently invented DIODE in 1904 LEE de FOREST of the U.S.A. derived the TRIODE. Not only did these Valves pave the way for mass-produced radio sets, but they were used in the first electronic computers.

*Microelectronics has resulted in the production of cheap calculators and relatively inexpensive microcomputers surpassing wild expectations of most people only a decade or two ago.*


**THE SECTOR**—(possibly invented by Galileo in about 1600) was the most popular of all calculating instruments for over two hundred years. It was used mainly by architects, surveyors and navigators for calculations involving ratios and sides and angles of triangles.



**A SPIRAL RULE**  
designed by  
**GEORGE FULLER** in  
1878. Arranged  
spirally on  
the surface  
of a cylinder  
the logarithmic  
line gives a  
working length  
of 41 ft. 8 in.

In 1947 the Americans JOHN BARDEEN, WALTER BRATTAIN and WILLIAM SHOCKLEY invented the TRANSISTOR, which would act like a triode but was much smaller and more reliable. By integrating several transistors into one assembly JACK KILBY of the U.S.A. invented the first integrated circuit in 1958. Then, in 1960, the creation of transistors in the surface of a wafer of silicon revolutionised miniaturisation.

Actual size of a silicon chip



5mm

**OTIS KING'S PATENT CALCULATOR.** A pocket-sized cylindrical slide-rule introduced in 1922 it is a modification of Fuller's spiral rule.

By 1966, 30 components could be integrated on a 5 m. square silicon chip. By 1980 that number had again increased to 250,000.

ANCIENT EGYPTIAN (Hieroglyphic)  
c. 3400 B.C.

1	2	3	4	5	6
7	8	9	10	11	12

**BABYLONIAN (Cuneiform)**  
c. 3000 B.C.

1	2	3	4	5	6	7	8
9	10	11	12	13	14		
15	16	17	18	19	20		

**MAYAN**  
c. 3000-2000 B.C.

I	II	III	IIII	V	VI	VII	VIII
1	2	3	4	5	6	7	8
IX	X	XI	L	C	D	M	
9	10	11	50	100	500	1000	

**ETRUSCAN-ROMAN**  
c. 800 B.C.

ח 100  
ז 80  
ה 40  
ד 20  
ג 10  
ב 5  
א 2

HEBREW  
c. 200 B.C.

—	=	≡	≠	≈	∩	∪
1	2	3	4	4	5	5
∫	∫	∫	∫	∫	∫	∫
6	7	8	8	9	10	100

INDIAN  
200-100 B.C.

۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹  
 ۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹ ۱۰  
 ARABIC 900 A.D.  
 ۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹  
 ۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹

SPANISH  
976 A.D.

760

761