

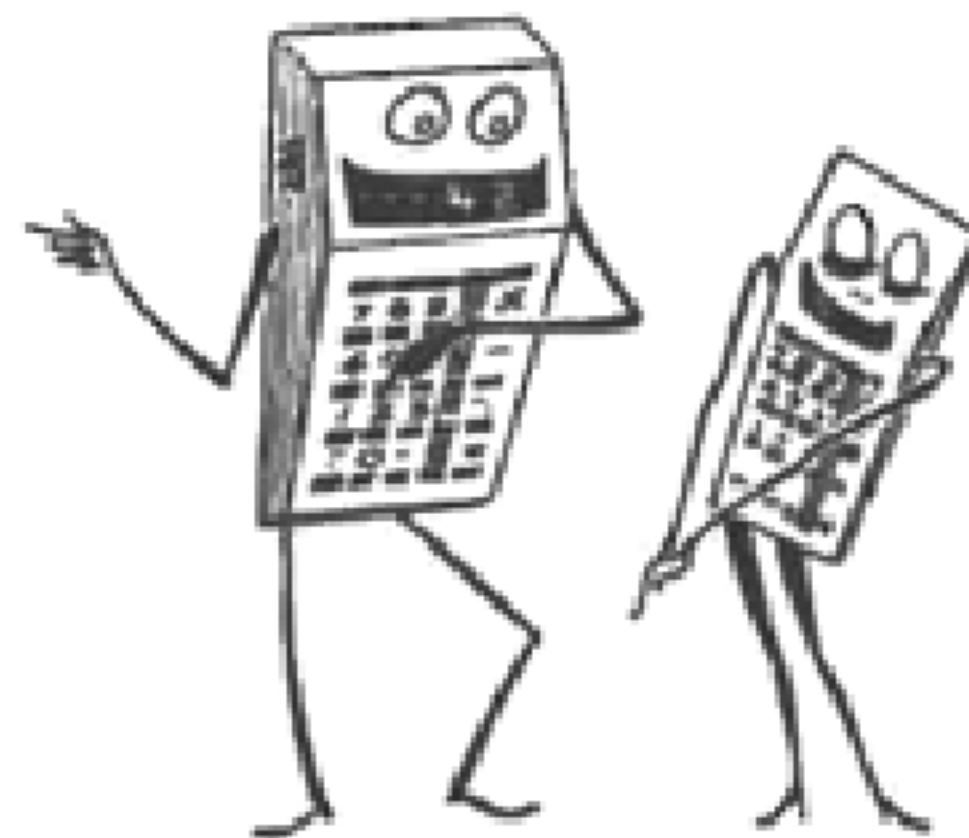
ENVELOPE PATTERNS

Curves can often be represented quite simply as attractive envelope patterns. On each of two sheets of paper, draw a circle and divide the circumference into ten-degree divisions. Inside the circle on the first sheet, mark a point P about one-fifth of a diameter from the circumference. On the second sheet, mark a point P about one-fifth of a diameter outside the circle.

Call one of the points on the circumference of the circle Q. Draw angle PQR equal to 90° , using a set-square or folded paper. Repeat this for each of the thirty six points on the circumference. The envelope of the pattern on sheet one is an ellipse and the envelope of the pattern on sheet two is a hyperbola.

The parabola can be produced by a similar method using equally spaced points on a straight line instead of a circle.

D.I.B.



Computer dating!

COMPUTER DATING

It's alright provided they use the same language.

DULL NUMBERS

The great mathematician, G.M. Hardy went to visit an Indian mathematician whom he had befriended, his name was Shrinavara Ramanujan and he was dying. When Hardy reached the bedside of his friend, he was shy and did not know what to say. He began, after they had greeted each other, "The number of my cab was not very interesting". "What was it Hardy?" said the dying man. "Oh, it was 1729" replied Hardy! At once Ramanujan cried out; "No Hardy, it is not a dull number at all. It is the least number that is the sum of two cubes expressed in two ways".

No number can be dull, for if there were any dull numbers, one of them would be the least and that would make it interesting. Then, by induction, all numbers are interesting.

S.H.F.



MATHEMATICAL PIE

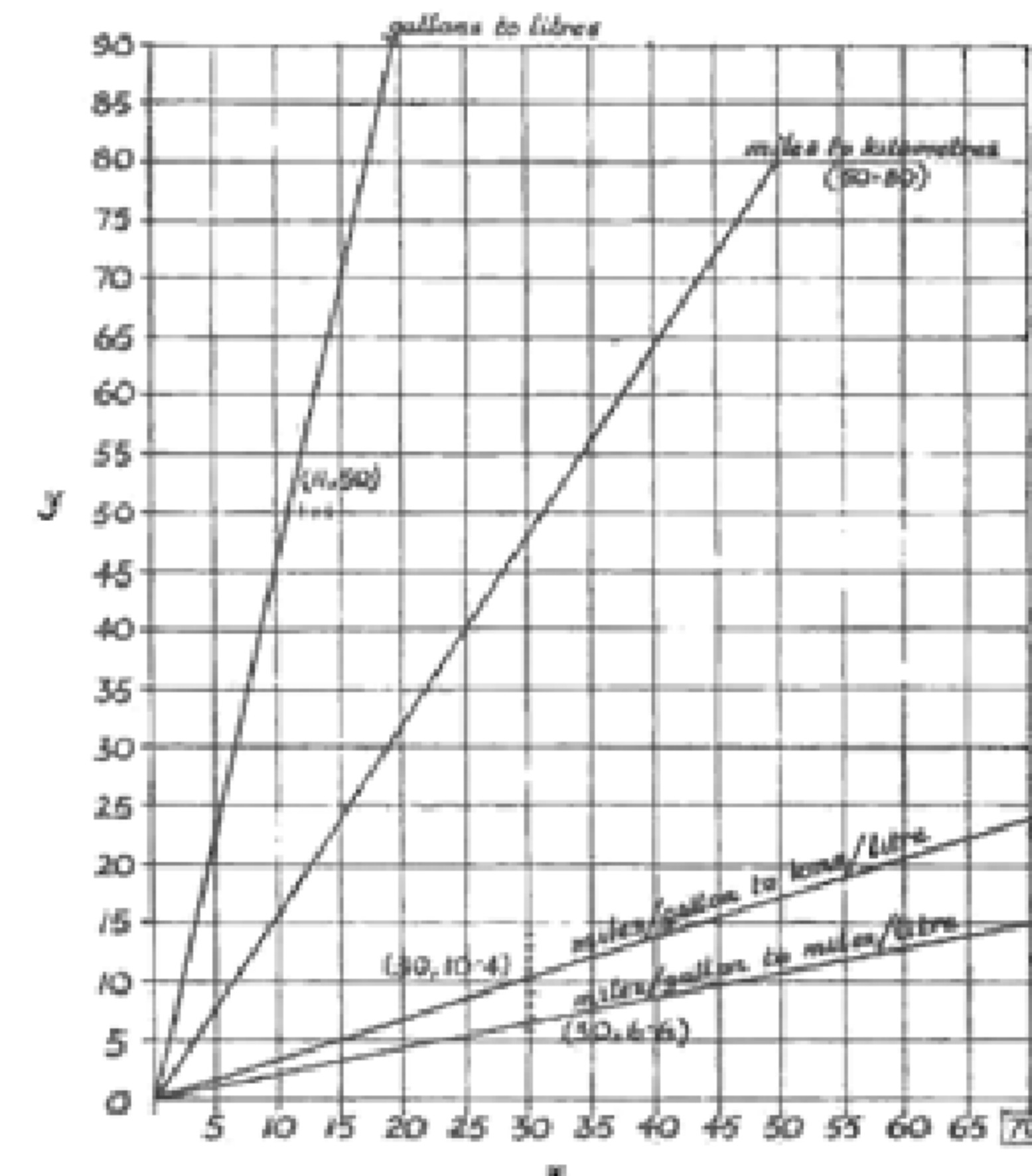
No.95

Editorial Address: West View,
Fiveways, Nr. Warwick

SPRING, 1982

CONVERSIONS

As petrol is now being sold by the litre instead of by the gallon and the possibility of road signs being converted to kilometres from miles, a conversion graph may be not only useful but also interesting. Most of the conversion charts that are available are of a linear form and give only one piece of information per chart. The diagram shows four conversions, read on the x-axis the value of the first quantity in each case and the corresponding value on the y-axis will give the equivalent in the second units.



N.B. If you are getting more than 70 miles per gallon on your vehicle, then halve the number look up the equivalent and then double the answer. Your parents may be interested in these conversions.

C.B.A.

Using the signs of +, - and x, rearrange the digits of 1982 to make up 100. Then rearrange them to make 99, 98, 97, etc. How far can you go before you break down?

What numbers can you not make?

What are the factors of 1982?

What are the largest, and the smallest, numbers you can make with the digits 1982 without using powers or factorials?

R.H.C.

IT'S MAGIC

Arrange the numbers 1 to 13 on a board or piece of paper in a haphazard way. Ask a friend to think of one of the numbers on the board. Tell him you are going to tap the board and he must count on from the number he thought of up to 20. e.g. If he thought of 8, on your first tap he will think to himself "9", the second "10" and so on. When he reaches 20 he must call "Stop". You will then be pointing to the number he thought of!

Here is how to do it. As you tap, count silently to yourself and point to any number you like at first. When you reach 7, make sure that you are pointing to the number 13. Then point to the numbers in descending order, i.e. 12, 11, 10, etc. When your friend says "Stop" you will be pointing to his number.

You will have to practise beforehand to be able to find the numbers to point to in order and quickly.

A.M.A.

AN AWKWARD ANSWER

Write all the one and two digit numbers in words.
Which vowel does not appear?

C.B.A.

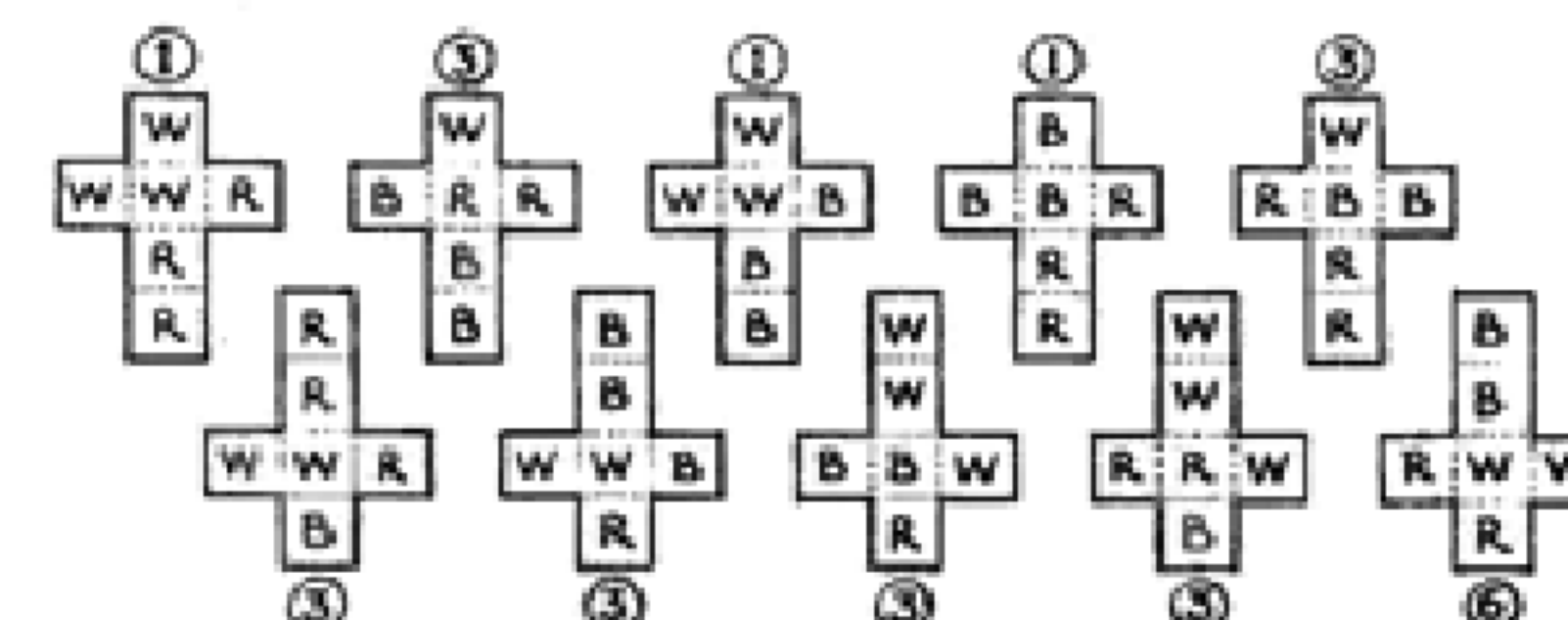
PYTHAGORAS AGAIN AND AGAIN

A right-angled triangle has the two shortest sides of 16 and 63 units. A second triangle has the same length of hypotenuse and one of its sides is of length 33 units. What is the length of the third side?

R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 94

Patriotic cubes



N.B. the "nets" may be coloured in different ways but the resulting cubes must be the same as the ones made from the nets shown.

In the balance The extra profit is 25%p.

For mature mathematicians With the usual notation $ac = b^2 - a^2$.

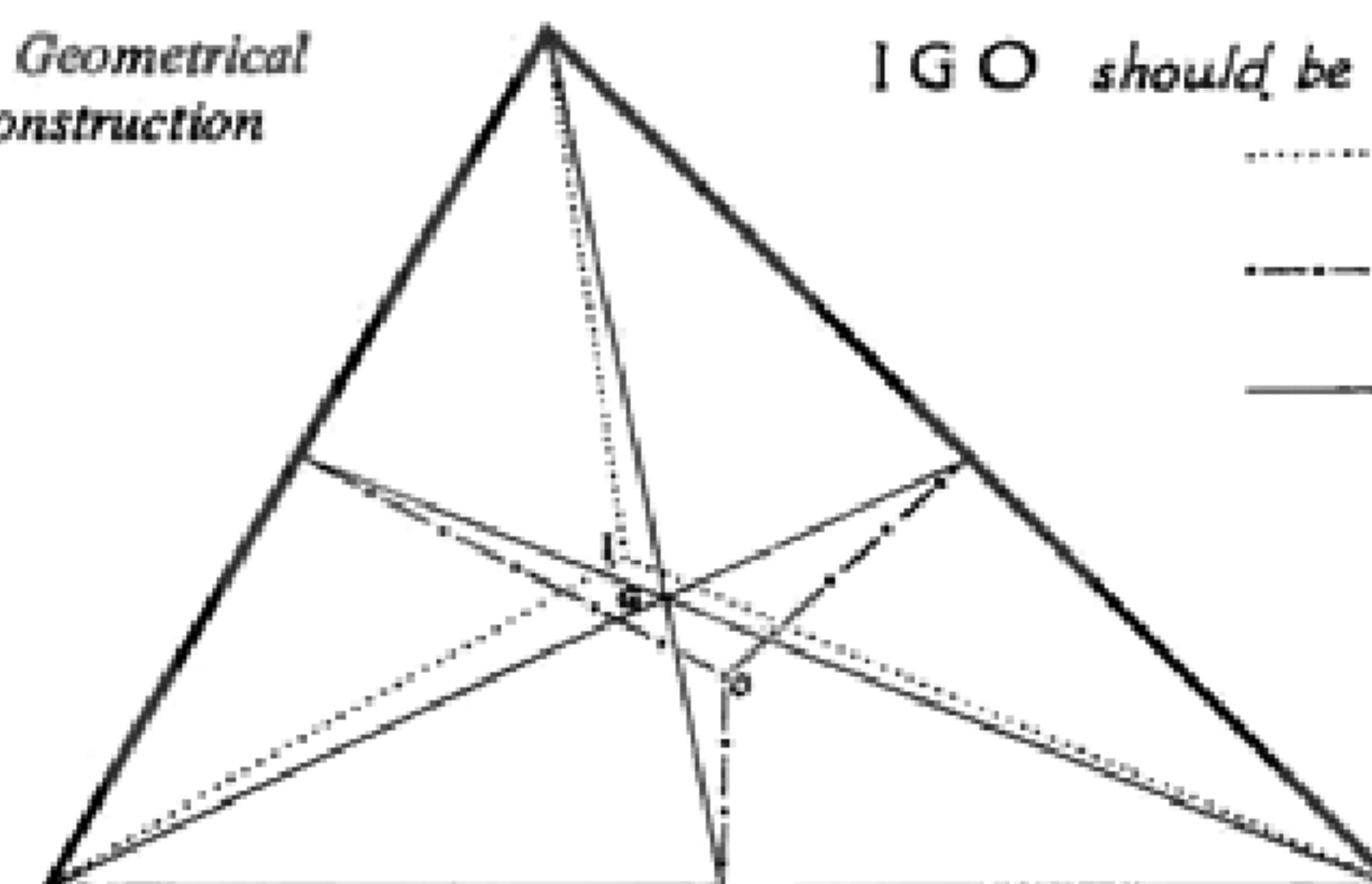
Hard Graft The time would be 20 hours.

Christmas Presents $x = 14$, $y = 158$, $z = 913$, $? = 914$ and $!! = 835\ 396$.

Left Overs Find the lowest common multiple of 8, 7 and 6, then subtract 1 to give 335.

Decimods It was implied in the article that the cycle of repeating digits must have an even number of digits. There are some restrictions on the denominators but the simplest is that it should be prime.

A Geometrical Construction



I G O should be a straight line.

..... Bisectors of angles.

----- Perpendicular bisectors of sides.

—— Medians from a vertex to the midpoint of the opposite side.

Black triangle 13, 12, 10 units cms. or $\frac{1}{2}$ inches.

O is the centre of a circle passing through the three vertices, the circumcircle. I is the centre of a circle touching the three sides of the triangle, the in-circle. G is the centroid of the triangle. A fourth set of three lines is also concurrent, at H called the ortho-centre. The set consists of the altitudes of the triangle.

Inverted Calculator Cross Figure Words Clues across: 1. HOLES; 4. LOSEL; 5. SELLS. Clues down: 1. HILLS; 2. LOSEL; 3. SOLES.

A Fishy Problem The area of the fish scales is 8 cm^2 .

More or LESS - A Space Problem The percentage is $50\pi/3$, approximately 52%. The cones take up exactly half the volume of the sphere. B.A.

Using the golden triangle construct the square shown in Diagram 1. The line PL divides the square into two equal parts so that NL equals LK. Now place one end of another cord at L and stretch it across to M. With centre L and radius LM draw a quadrant of a circle so that LNJ is a straight line. Complete the rectangle JKRS, this is the "Golden Rectangle". Other "Golden Rectangles" can be formed by adding squares to or subtracting squares from the original rectangle as in Diagram 3.

S.H.F.

ACHILLES AND THE TORTOISE

Zeno of Elea (495-435 B.C.) was a country boy with no formal education, however, he succeeded in puzzling the philosophers of Athens by inventing four paradoxes. Here is one of them — the other three can be found in Bell's "Men of Mathematics" Vol. One. Imagine the great athlete Achilles is trying to overtake a tortoise, crawling ahead of him. In an instant he reaches the place where he first saw the tortoise, but by now the tortoise has crawled ahead a little, by the time he reaches the next position of the tortoise, it has moved still further, and so on. In this way the tortoise must always be a little ahead, therefore Achilles can never overtake the tortoise!

Zeno unfortunately lost his head for treason, but there are many who lost their wits and tempers trying to sort out his paradox!

AMA

SOME VERY ODD NUMBERS

The number 1105 is a very odd number because

$$24^2 + 23^2 = 31^2 + 12^2 = 32^2 + 9^2 = 33^2 + 4^2 = 1105.$$

This is the smallest number that can be expressed in four different ways as the sum of two squares which have no common factor.

The next number with this property is 1885. Hint the square of the larger number of each pair is between 942 and 1885.

The next fifteen are also multiples of 5. The smallest that is not is 6409.

Before you wear out the x^2 button on your calculator consider what 1105 and 1885 have in common.

C.V.G.

MAIN LINE?

On graph paper, set out a grid with the horizontal axis from 0 to 16 and the vertical axis from 0 to 10. Mark the following points and join them with straight lines in the order given.

(5,4); (3,6); (3,9); (4,10); (7,10); (8,9); (10,7); (13,6); (15,5); (16,4); (16,3); (13,0); (12,0); (13,1); (11,1); (9,0); (8,0); (7,1); (9,1); (10,2); (0,2); (1,3); (4,3); (5,4); (6,4); (7,5); (7,8); (6,9); (5,9); (3,8).

(10,2); (12,4); (14,5).

(6,2); (8,4); (8,5); (10,7).

(4,5); (5,5); (6,6).

EXAMINATION HOWLER

A pupil at MANOR PARK SCHOOL, NEWCASTLE UPON TYNE correctly defined the 'top' of a vulgar fraction but called the 'bottom' a DETONATOR!!!

SIMPLE ANAGRAMS

ONCE, a spiky circle. LEGTRAIN, three lines. ABES, the bottom. CEFA, it may have a nose. GERTLANCE, could be two squares.

C.B.A.

HALF A MINUTE

Draw a quadrant OAB of a circle, centre O and radius 7 cm. In the quadrant draw a rectangle OCDE with C on OA, D on the circle and E on OB. OC is 3 cm.

Find the length of the diagonal EC.

Generalise the result.

C.B.A.

NEXT BASE

Find a three digit number in base five whose digits are reversed when the number is written in base six.

What is the value of the number in base ten?

B.A.

WHAT'S MISSING?

$11(492 + x) = \sqrt{(37a10201)}$, where a is a missing digit. Find the value of x.

R.H.C.

FISHING LINES

submitted by Ruth Nixon, aged 10 years, Heron Brook C.E.(C) Middle School, Gnosall

On graph paper, set out a grid with a horizontal (x) axis from 0 to 20 and a vertical (y) axis from 0 to 12 using the same unit of length on each axis. Mark the following points and join them with straight lines in the order given.

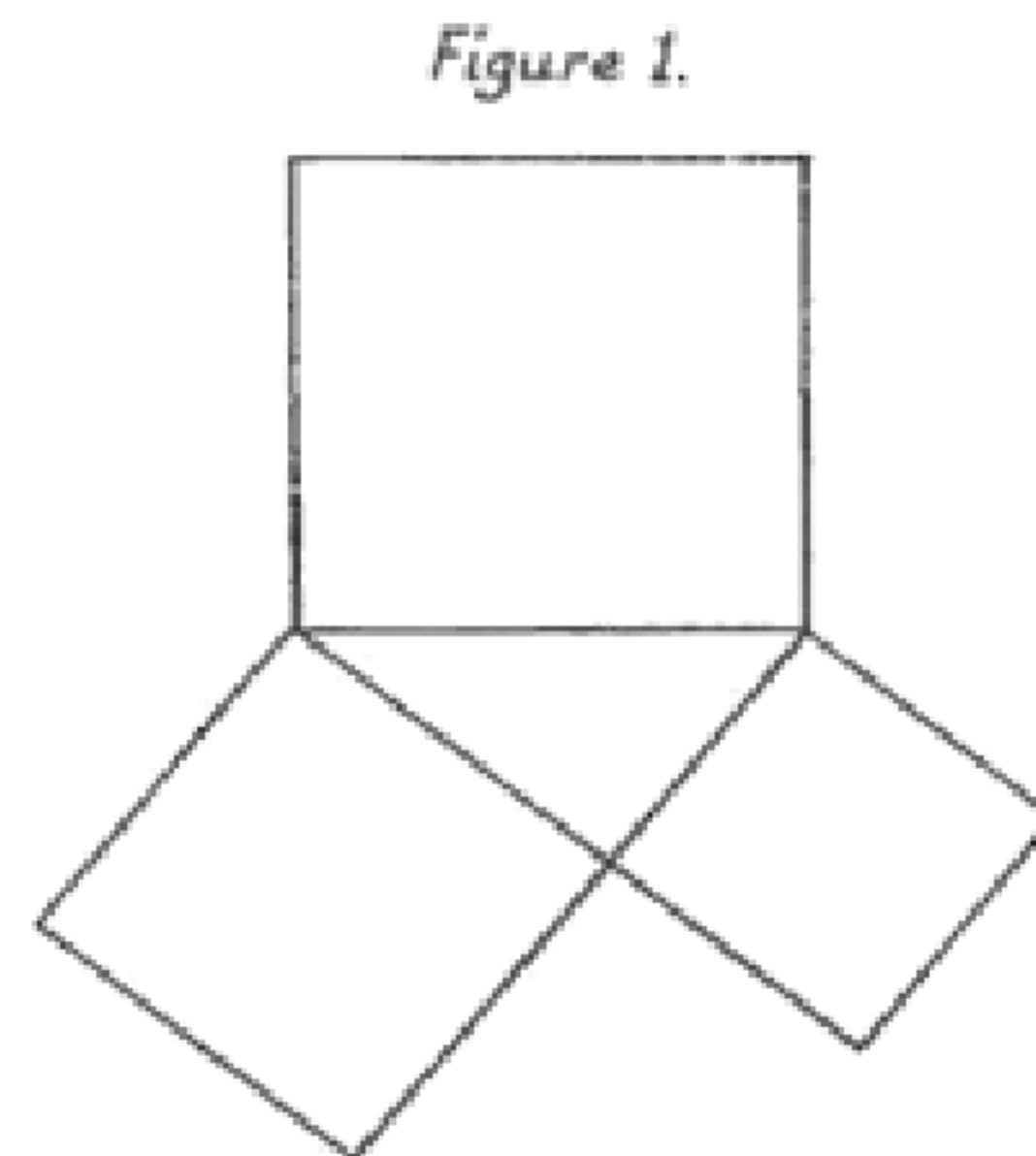
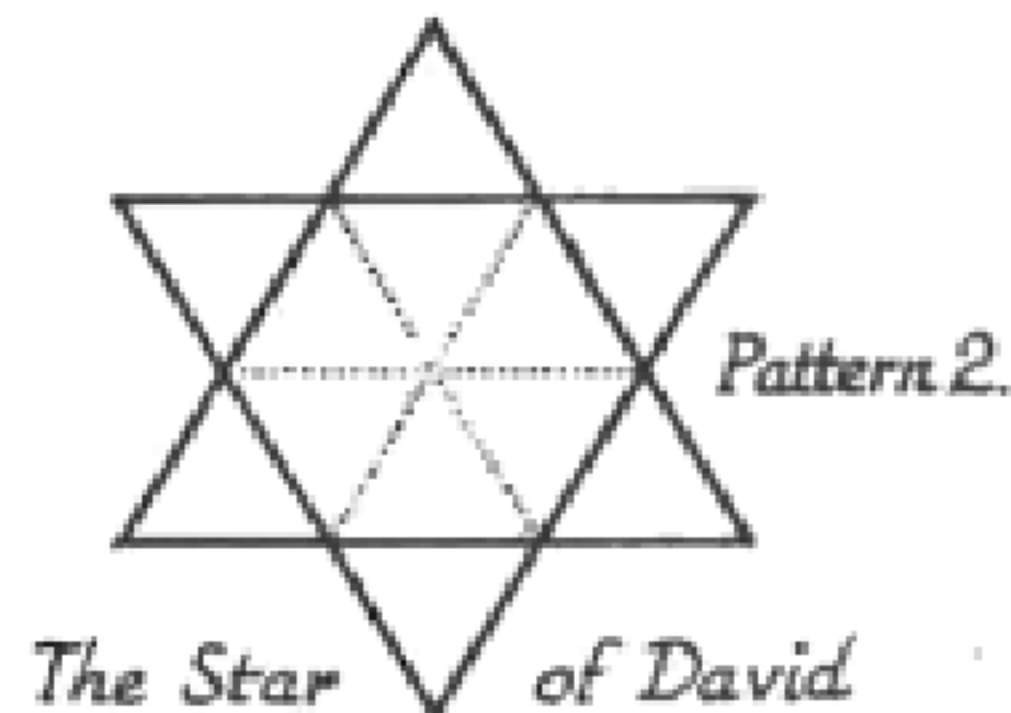
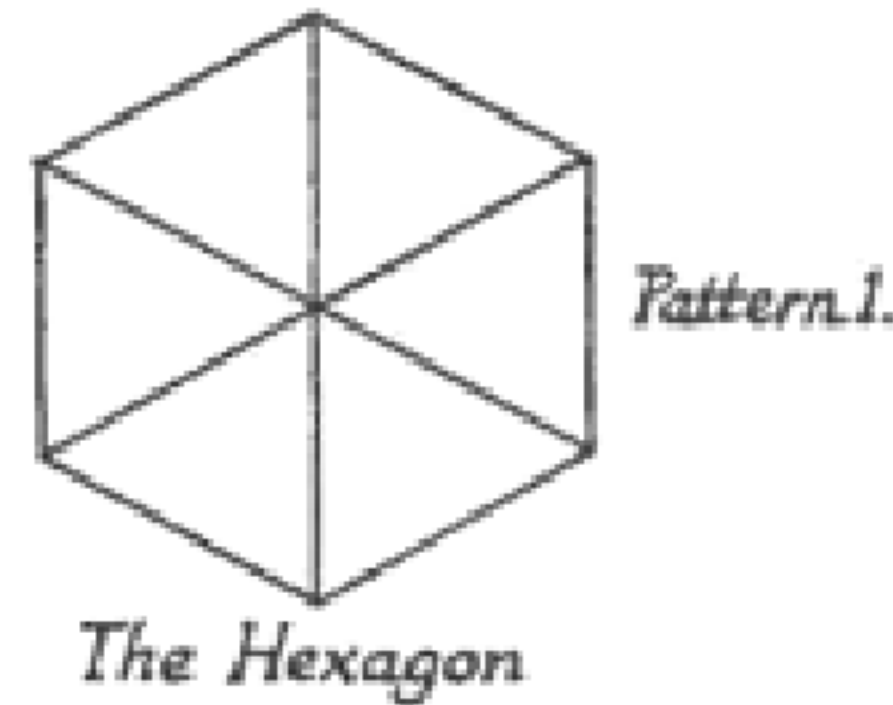
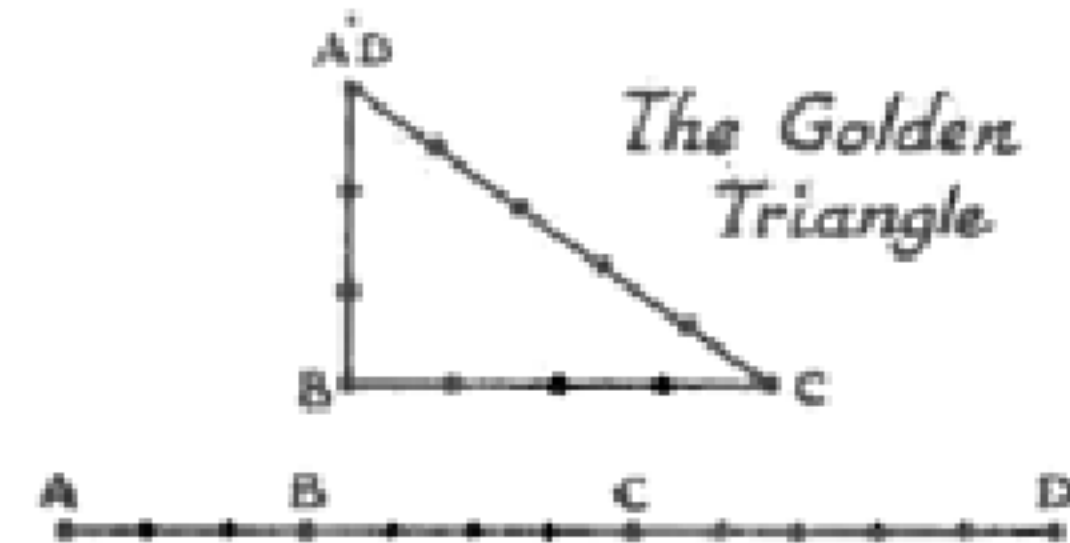
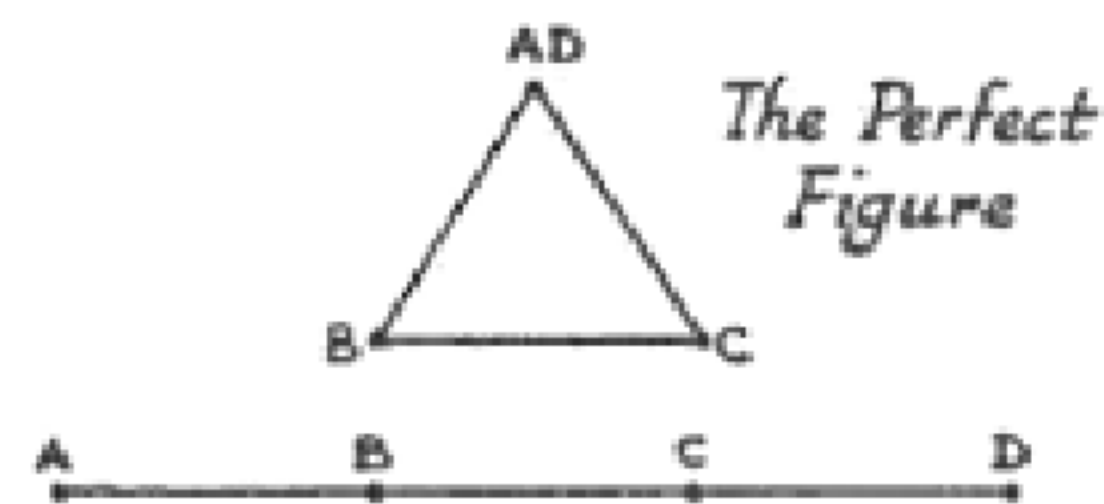
[(1,5); (1½,7); (1,9); (3,7); (7,9); (8,9½); (9,11); (10,10); (12,10½); (14,10); (16,9); (17,8); (17½,8); (17½,7½); (17,7½); (16,7); (15,6½); (14,6½); (12,7); (11,6½); (10,4½); (9,6½); (8,6½); (6,6); (3,6) (1,5)]

[(11,9); (9,7½); (11,8)]

[(13½,9); (14,9); (14,8½); (13½,8½); (13½,9)]

[(17,7½); (19,6); (19½,6½); (19½,5); (18½,5½); (16,7)]

Know the Ropes



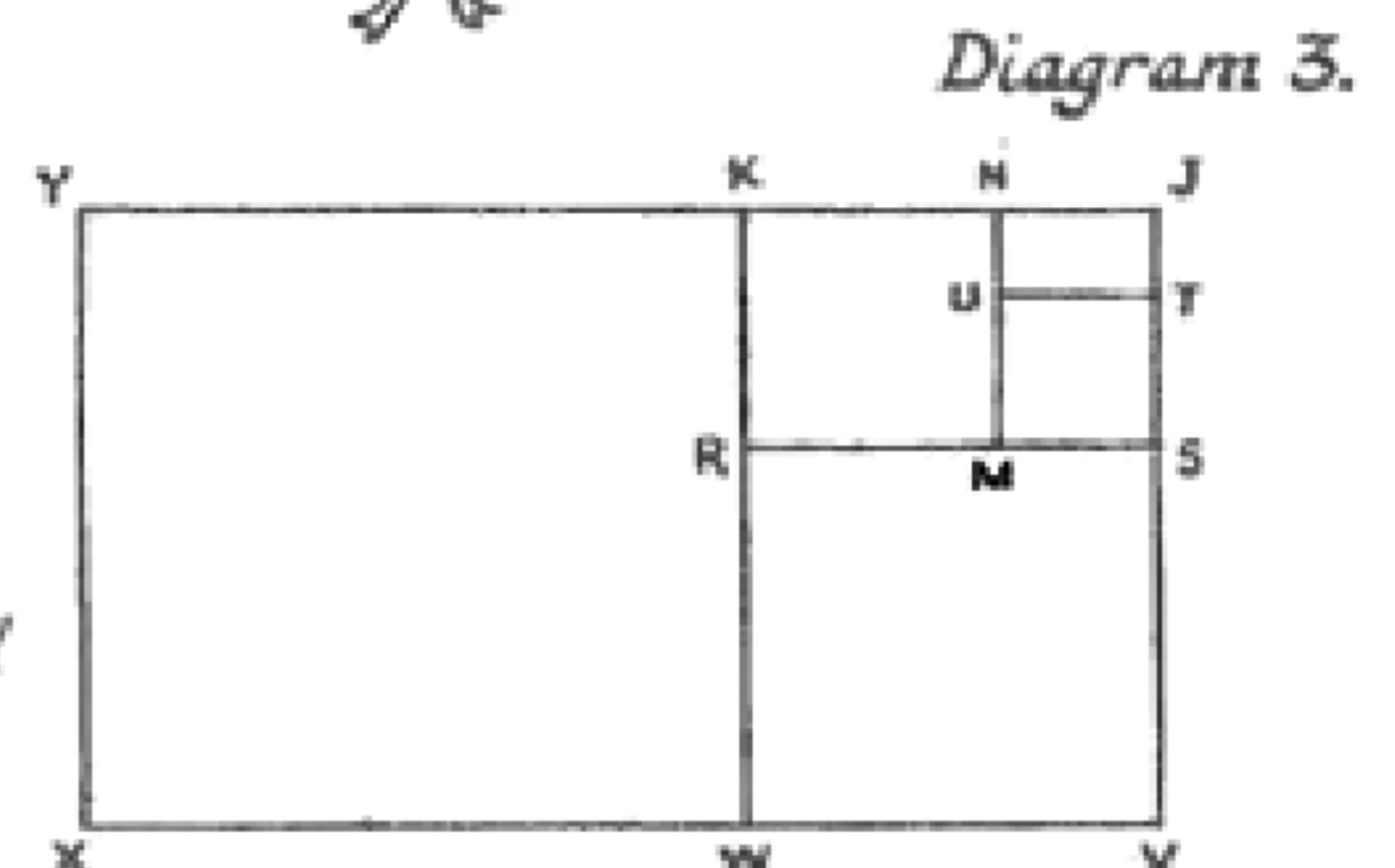
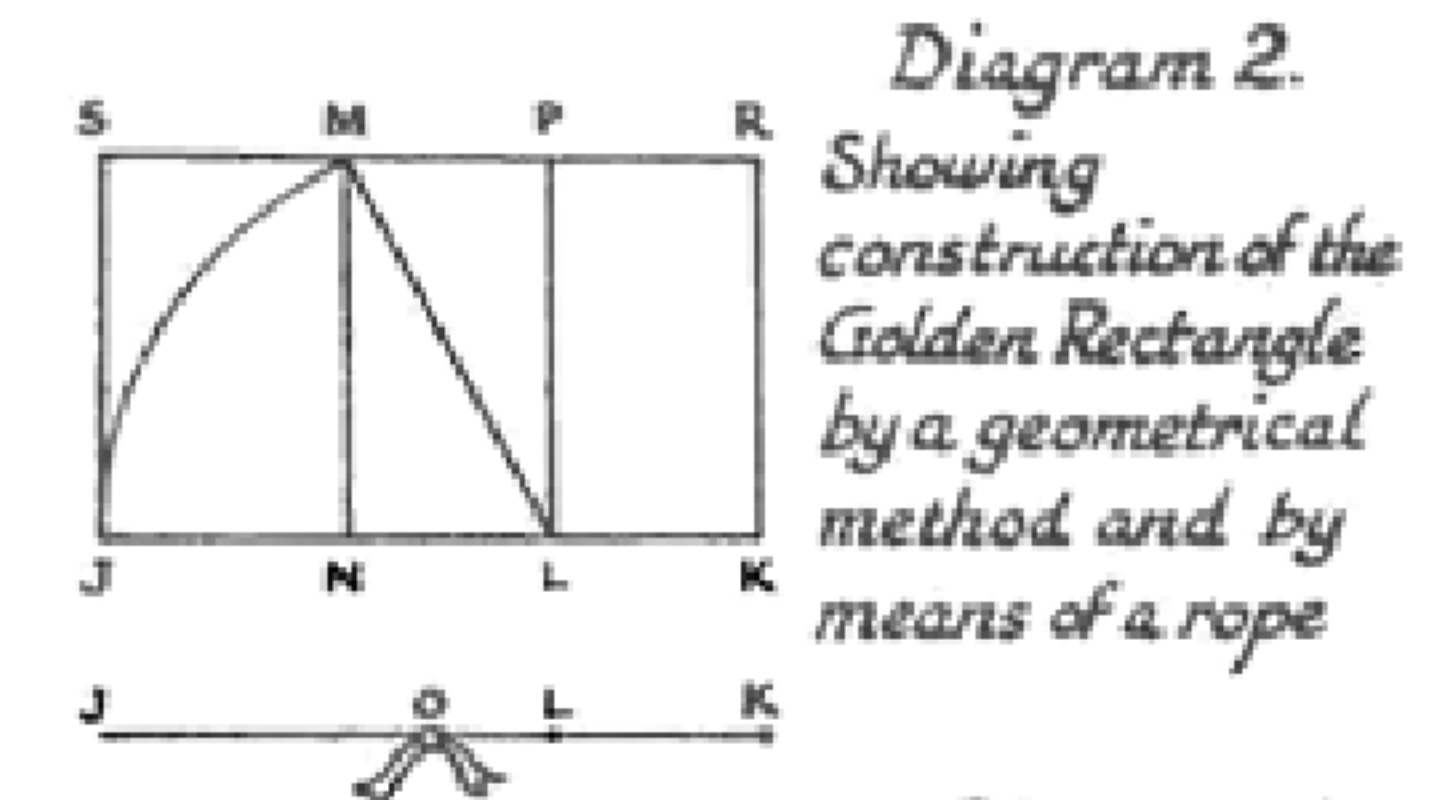
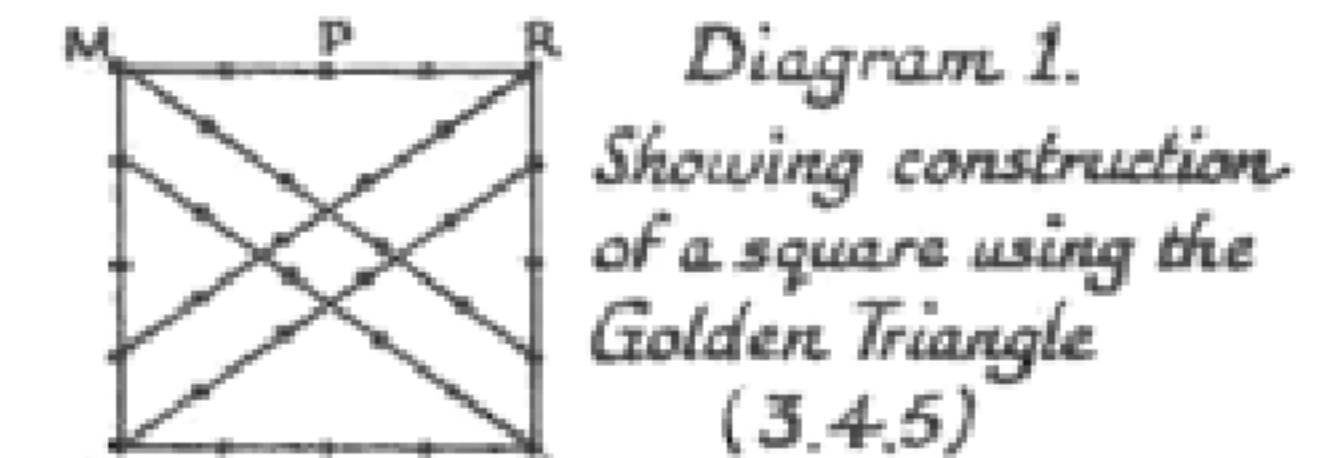
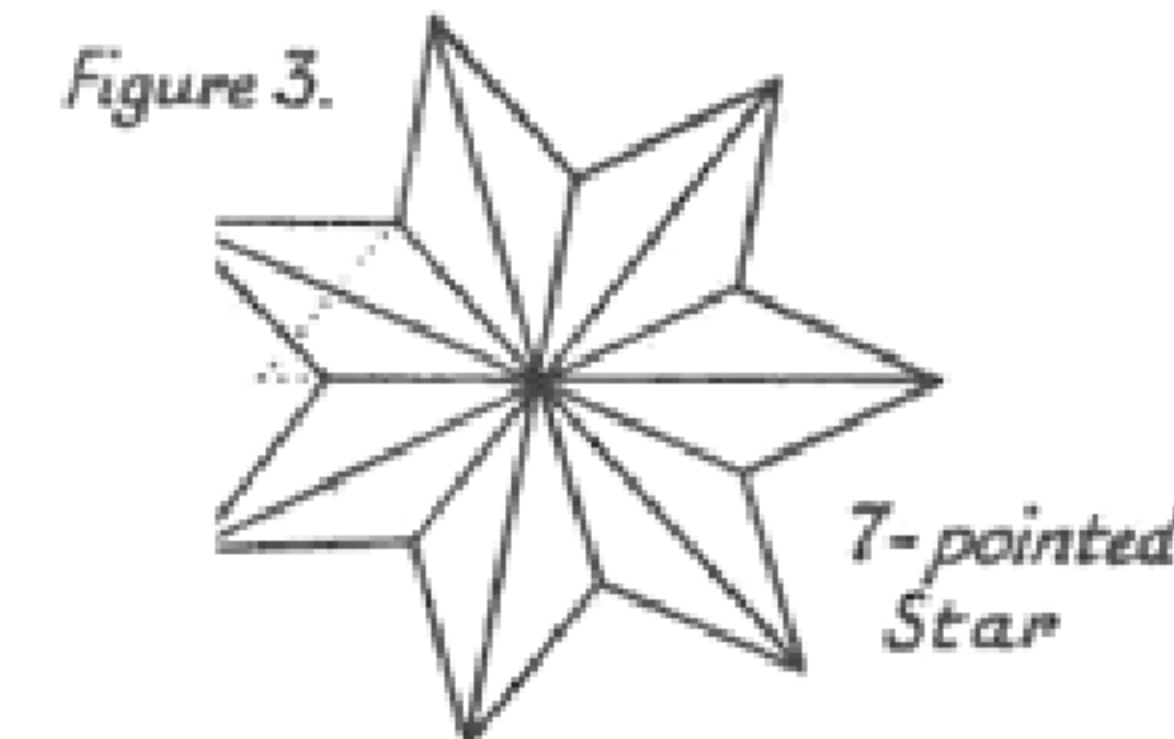
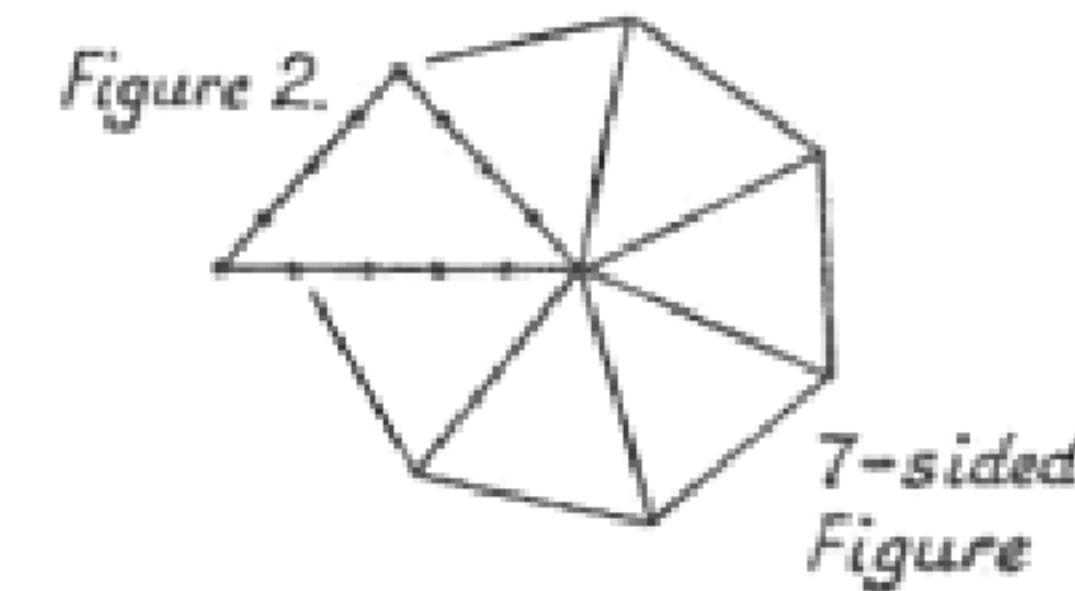
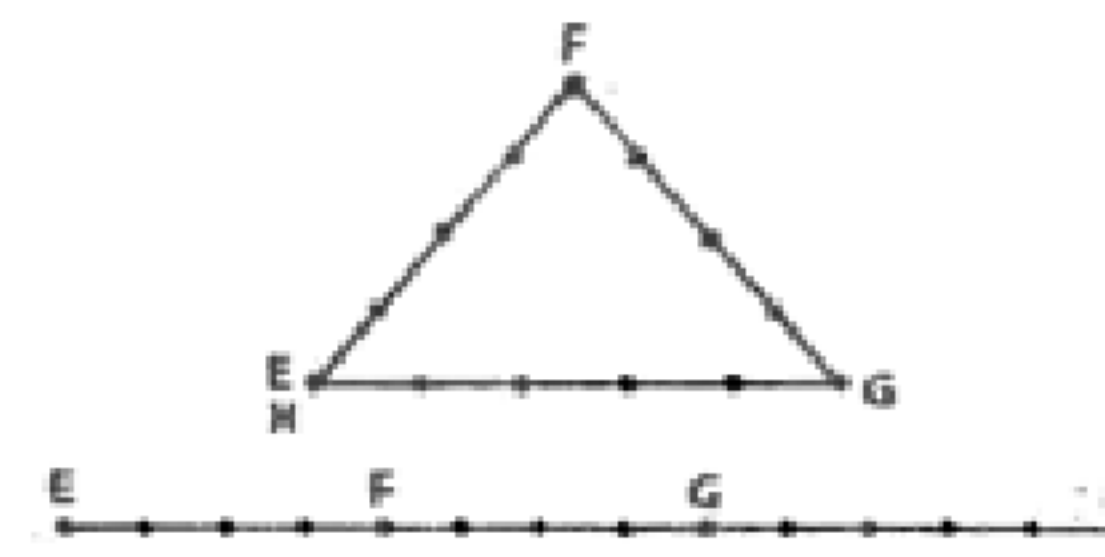
KNOW THE ROPES

Ancient mathematicians and builders did not have the shiny and accurate instruments we use today, but they did wonders with what little they had. Four examples of what can be done with ropes are described here. All that is needed is a length of rope, string or picture cord, about 4 metres in length. One must learn how to tie a number of knots at exact distances apart. A ruler or tape measure could be easier, but it is best done in the way it was done in the past, by using body measures such as hand-widths, spans or cubits.

Begin with the very simple but beautiful shape, the equilateral triangle. Ancient mathematicians called it the "Perfect Triangle" or the "Perfect Figure". Tie knots at A, B, C and D such that $AB = BC = CD$. Hold the rope firmly at B and C, then fold the ends so that A and D come together. Ensure that the three parts of the rope are taut and we have an equilateral triangle. This can be used to produce Pattern 1, the hexagon and pattern 2, the Star of David.

The second triangle was first made by using a knotted rope thousands of years ago, the ancient Egyptians called the men who used it "Rope Stretchers". It is made in the same way as the equilateral triangle but this time AB is 3 units long, BC four units and CD 5 units. Again the ends A and

The Druid's Cord



D are brought together and the three segments stretched out into straight lines. When this is done, the angle ABC is 90° . This rope pattern was used to lay out temples and pyramids, it is still used in marking out pitches and courts for games. It was also used to mark out fields so that each man had a fair share of the land after the flooding of the Nile. Because it was so simple, it was called the "Golden Triangle". Figure 1 is associated with this triangle and is well-known in geometry, whose name is linked with it?

A third triangle can be formed using a knotted rope called the "Druid's Cord". It was used by the Egyptians when planning their pyramids, by the Druids to assist them with their diagrams and it was used by the builders of great cathedrals; probably at Chartres when the present building was begun in 1194. This time there are thirteen units with EF and FG 4 units and GH 5 units. Bring E and H together and stretch the sides of the triangle into straight lines. It is the front face outline of an Egyptian pyramid.

Angles FEG and FGE are equal and each is very nearly the seventh part of a complete revolution. This enables us to produce a near regular heptagon as in Figure 2 and the seven-pointed star in Figure 3.

The "Golden Rectangle" was the name given to an elegant rectangle whose shape was discovered thousands of years ago and has been used by builders of beautiful buildings ever since. Here is one way of constructing it.