ALIGNMENT

Two points, O and A are given. The problem is to construct points B,C . . . . in line with OA such that OA = AB = BC = ... If you had a ruler it would be easy, but you have no ruler nor straight-edge, only a pair of compasses. The positions of the points is to be given by the intersection of two arcs.

C.V.G.

DATUM LINES

What is revealed if the following sets of points are represented as straight lines on a grid?

$\{(x,y): x = 1, 1 \leq y \leq 3\}$, $\{(x,y): x = 2, 2 \leq y \leq 3\}$,
$\{(x,y): x = 3, 1 \leq y \leq 3\}$, $\{(x,y): x = 4, 1 \leq y \leq 3\}$,
$\{(x,y): x = 5, 1 \leq y \leq 3\}$, $\{(x,y): x = 6, 1 \leq y \leq 3\}$,
$\{(x,y): y = 3, 2 \leq x \leq 3, \ 4 \leq x \leq 5\}$,
$\{(x,y): y = 2, 2 \leq x \leq 3, \ 4 \leq x \leq 5\}$,
$\{(x,y): y = 1, 4 \leq x \leq 5\}$

D.I.B.

A LEWIS CARROLL PROBLEM

Some hikers started a walk at 3 p.m. and returned along the same route after resting for ½ hour at the furthermost point, arriving at 9.30 p.m. If their speed on the level is 4 m.p.h., uphill 3 m.p.h. and downhill 6 m.p.h., what is the total distance walked?

For experienced mathematicians, if the downhill rate is x m.p.h. and the uphill rate is y m.p.h., what must be the rate in m.p.h. on the level for the problem to be solved?

R.H.C.

RANGE FINDER

From a hidden position on flat ground, a gun fired 3 shells which landed exactly the same distance from the gun given by the points (5,10), (12,11) and (9,2). Ignoring the effects of weather conditions, what were the coordinates of the gun's position?

D.I.B.

NUMERICAL VALUES

Can you find any rectangles whose area in square units is numerically equal to the perimeter in linear units? The answers are single digit numbers.

R.H.C.

NOT FOR THE FAINT HEARTED

$2^5 = 32$ and $3 + 2 = 5$. The sum of the digits of the answer equals the power. What is the next power of 2 that has this same property?

C.B.A.

HEIGH HO!

The points A and O are one vertex and the centre of a square ABCD. The problem is to find the other vertices B, C and D using a pair of compasses only. No ruler nor straight edge may be used.

C.V.G.

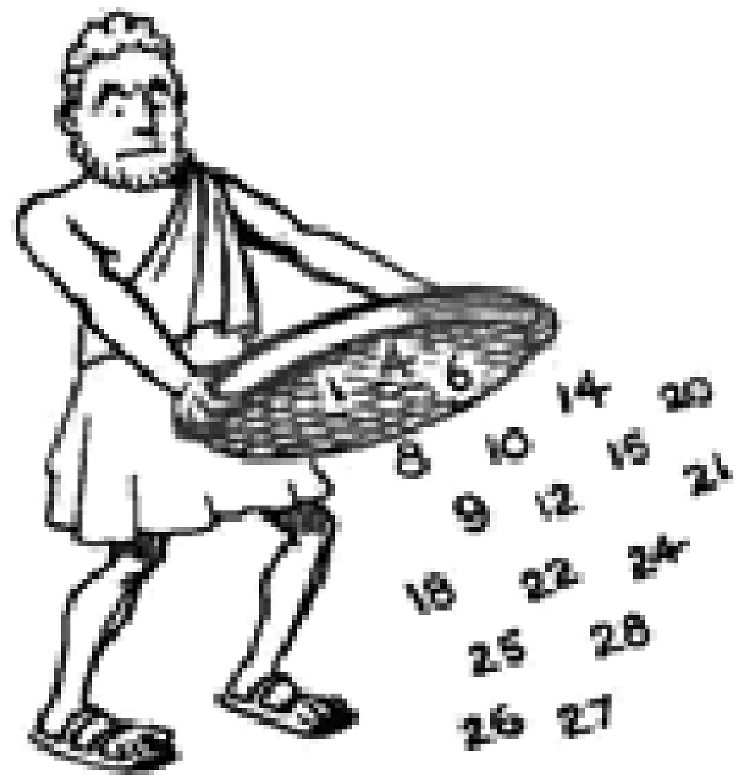
MATHEMATICAL PIE

No. 92 — *Editorial Address :* **West View, Fiveways, Nr. Warwick** — SPRING, 1981

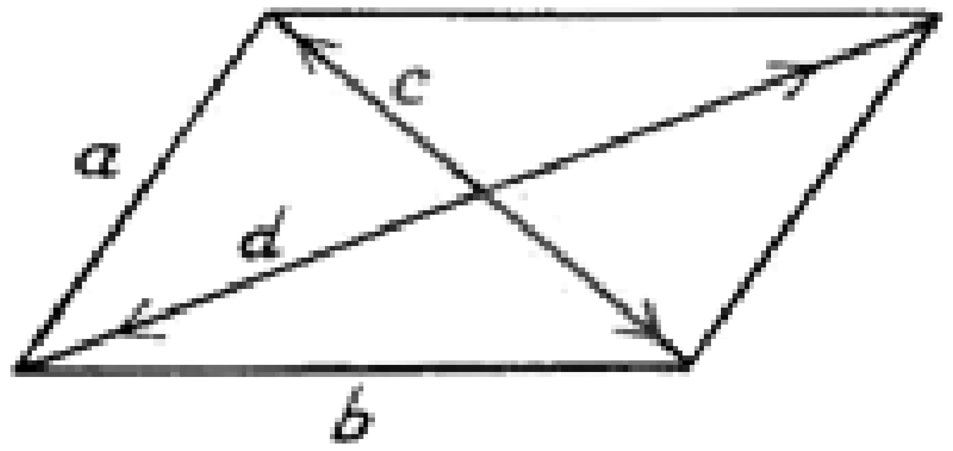
MORE RHYME, EVEN LESS REASON!

Eratosthenes, in sunny climes,
Mused, while growing lemons and limes:
"I could have greater stocks
Of good soil without rocks
If my sieve wasn't clogged up with primes".

E.G.



PARALLELOGRAM PROBLEM



The sides of a parallelogram are a units and b units long. The two diagonals have lengths c units and d units.

Show that $\dfrac{c^2 + d^2}{a^2 + b^2} = 2$

R.H.C.

1981

Every number is interesting, and 1981 is no exception. As we start a new year, we can consider some of the interesting dates that will occur.

(a) 18th January, (b) 18th June, (c) 1st August, (d) 18th September and (e) 18th November.

Four of the dates are palindromic and the fifth can be written so that it has an axis of rotational symmetry of order two. Which is the odd one?

Using the digits of 1981 and keeping them in the same order, can you make the numbers 0 to 20 using ordinary arithmetical operations.

$0 = (1 \times 9) - (8 + 1)$, $1 = 1 - 9 + 8 + 1$, $2 = 1 + 9 - 8/1$.

C.B.A.

A POINT TO POINT!

By finding the values of the letters a to k, express the ordered pairs numerically. For example, $k = 2\frac{1}{2} \times 2 = 5$. Therefore, $(3k, k+2) = (15, 7)$. Then, in the order given in each set, join the points with straight lines.

$a = 2\frac{1}{2} + 3\frac{1}{5} - 5\frac{7}{10}$: $b = 3\frac{3}{4} \times 1\frac{3}{5} \times \frac{1}{4}$: $c = 3\frac{3}{8} \div 6\frac{3}{4}$: $d = \frac{7}{20}$ of 10:

$e = 19 \div 7\frac{3}{5}$: $f = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} - \frac{11}{20}$: $g = 3\frac{3}{4} \times 1\frac{2}{3} \div 1\frac{9}{16}$:

$h = 2\frac{2}{5}(4\frac{2}{3} - 2\frac{3}{4} - 1\frac{1}{2})$: $i = (7\frac{3}{10} + 2\frac{3}{4} - 3\frac{4}{5}) \div 3\frac{1}{8}$:

$j = (14 \div 2\frac{5}{8}) - (1\frac{2}{3} \times \frac{1}{2})$: $k = 2\frac{1}{2} \times 2$.

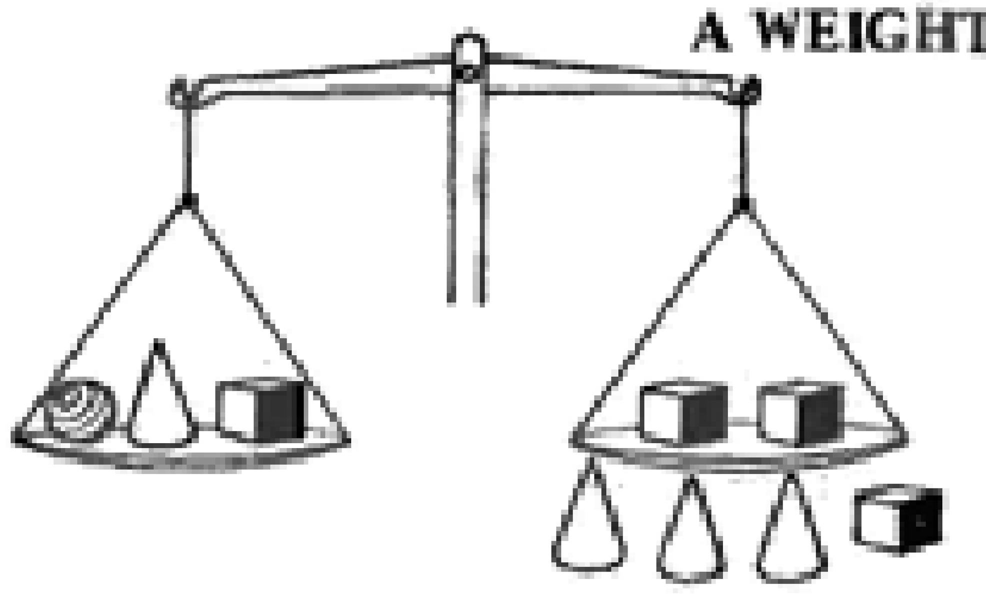
{(a,g), (h,k+1), (k,2g+c), (2g,2g), (2k,2g), (4d−h,2k), (4d−c,2k), (4d,2k+c), (4d,2k), (3k,2d), (4d+c,i+j), (4d,i+j), (4d,2d), (3j−c,2g), (4f,2f), (3d+h,j), (4f−h,g), (2k,d), (2j,i), (3e,c), (3e,h), (2g+c,i), (2j,g), (2d,k), (g,d), (j,i), (j,h), (g,c), (d,c), (g,h), (g,i), (f,g), (g,k), (f,2f), (h,k), (a,g).}

{(k,g), (j,d), (4b+c,i), (4b+c,b), (2d,h), (2d,i), (c+k,d), (2d,k).}

{(3g−h,g), (3d+h,e), (3g−h,i), (7b,e), (2k,i), (3g−h,b), (4f,e), (4f,f), (3d+h,j).}

D.I.B.

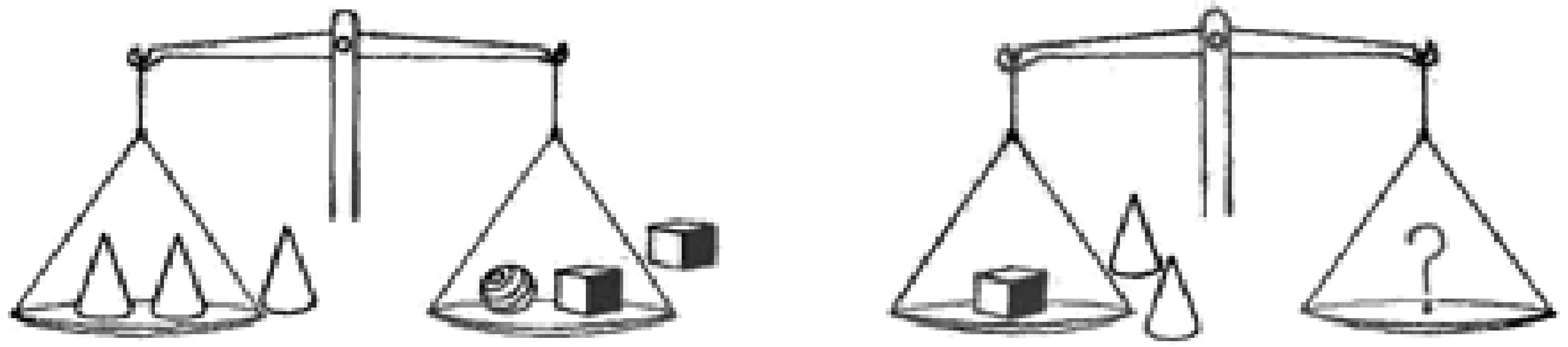
A WEIGHTY PROBLEM



The diagrams show two sets of conditions when the balance is in equilibrium.

What should be placed on the right-hand scale pan in the third part to maintain the balance?

C.B.A.

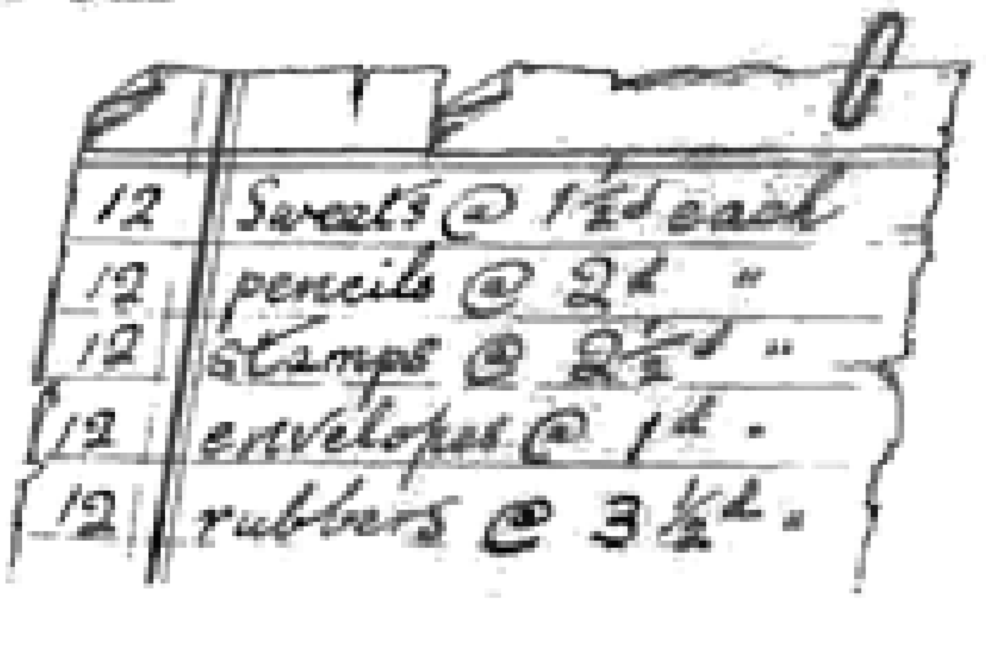


IN DAYS OF OLD

I was looking through some old bills and found one shown on the right.

How much change would I have received from a guinea?

C.B.A.



SOLUTIONS TO PROBLEMS IN ISSUE No. 91

Codes The mathematicians were GAUSS, PASCAL, ARCHIMEDES, BOOLE, LEIBNIZ and NEWTON.

Spell it out C on ten T's = Contents.

Make it unique 1089 if the first and the last figures are different.

A golden opportunity to shine $X = \frac{1 + \sqrt{5}}{2}$

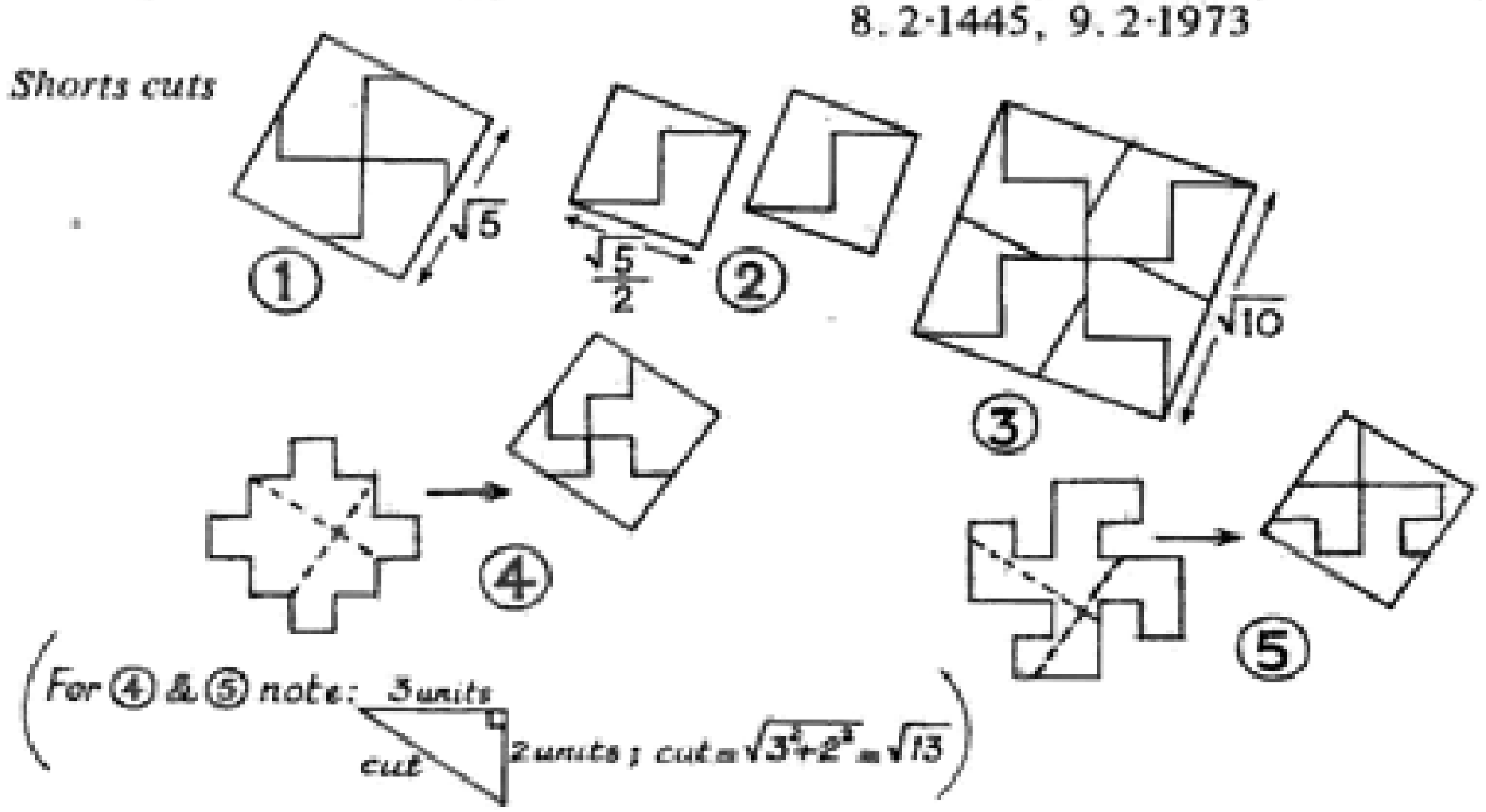
Odd ball Weigh three balls against any other three balls. If they balance, weigh the remaining balls against each other, the heavier will move down. If they do not balance, take two of the balls in the heavier tray and weigh them. If one goes down, it is the heavier. If they balance the third ball is the odd one.

Insomnia cure 1 000 lambs were born and the additional condition was not required.

A geometrical construction The resulting triangle is equilateral.

Four figure tables cross figure Across 1. 1·0212, 6. 1·3265, 7. 1·4142, 8. 2·1445, 9. 2·1973

Shorts cuts



Follow this up! three, five, four are the next numbers in the sequence. Each is the number of letters in the previous number name.

Following Letters SUMS

Happy birthday The retired colleague was 91.

Silver screen cross-figure Across 1. should have been reversed to read 31, 3. 633, 4. 80, 5. 6, 6. 14, 7. 12, 8. 2, 10. 42, 12. 101, 13. 02. Down 1. 38, 2. 10, 3. 65, 9. 11, 10. 40, 11. 22.

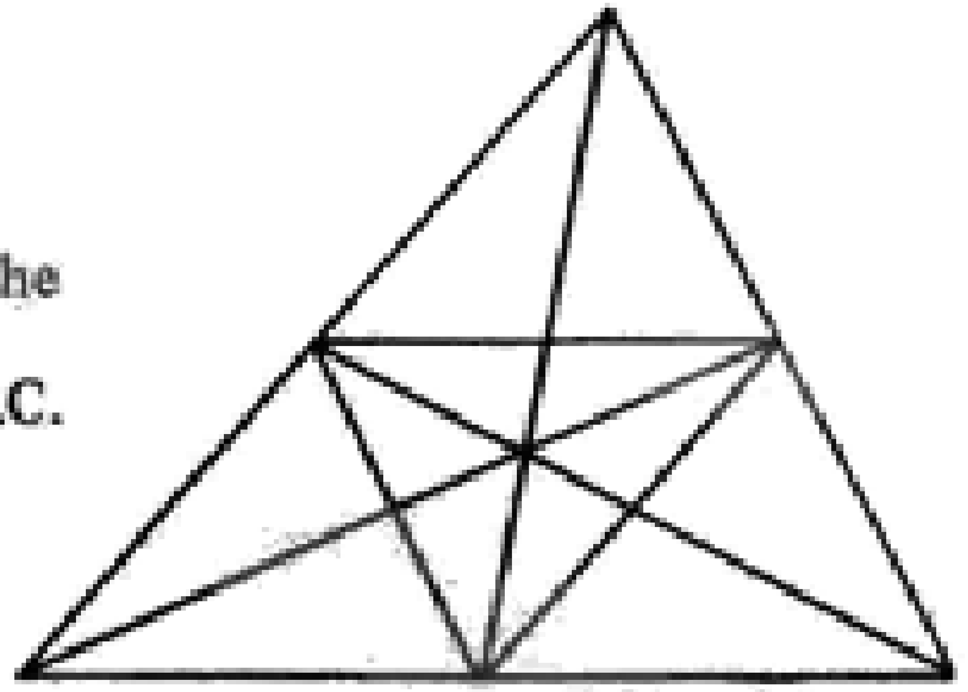
Score line 316, 136, 163, $\frac{3}{6}$, $\frac{6}{3}$, 31^9, number 6 does a hand stand on the cross bar.

B.A.

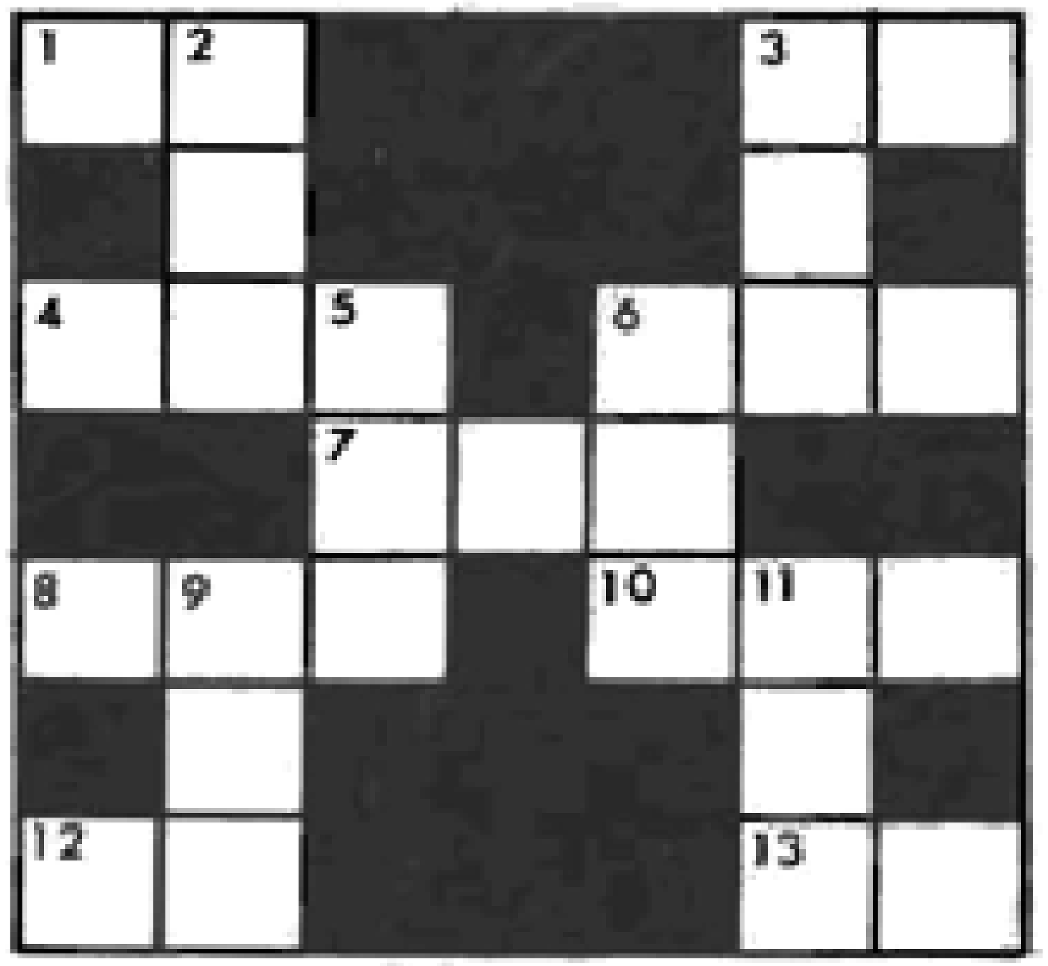
TRIANGULATE

How many triangles are shown in the diagram on the right?

R.H.C.



JUNIOR CROSS-FIGURE No.69



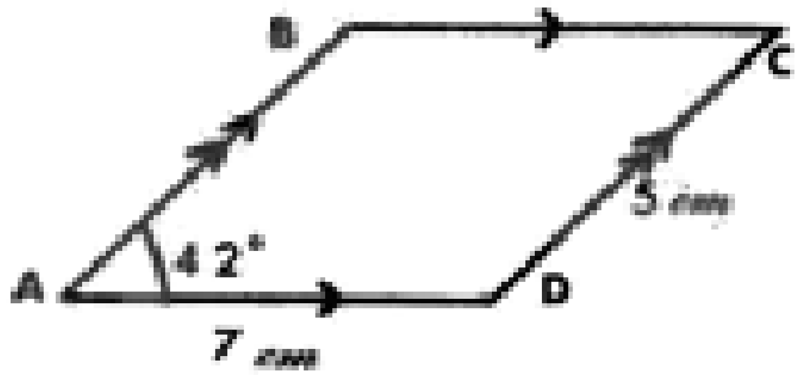
Clues Across

1. Three squared times seven.
3. The number of edges of a cube.
4. Nine dozen
6. Thirty-seven times twenty-four.
7. The number of mm in 17.6 cm
8. Two cubed, all cubed
10. A square number which reads the same back to front.
12. The first two-digit prime.
13. The perimeter of parallelogram ABCD.

Clues Down

2. The total of the angles of a quadrilateral in degrees.
3. The size of angle ABC.
5. A train leaves Birmingham at 6.35 a.m. and takes 97 minutes to get to London. When does it arrive?
6. Half a dozen dozen dozens.
9. One quarter of 10 across, or 12 across squared.
11. 5614 ÷ 7.

E.G.



HIDDEN WORDS

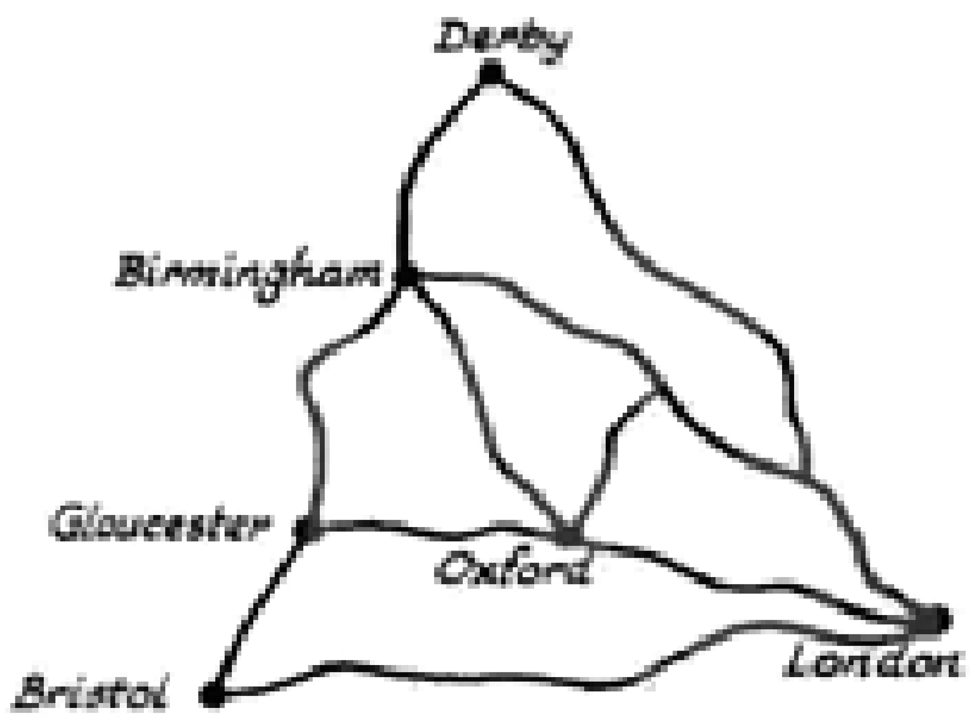
Can you find the following mathematical terms hidden in the block on the right.?

CUBE	CIRCLE	CONE	TON
ROOT	SQUARE	AXES	PIE
LINE	SINE	LOGS	ADD
PRISM	ANGLE	ONE	TWO
SIX	TEN		

C.B.A.

C	U	B	E	F	O	U	E	A	T
T	R	S	I	B	S	N	O	N	A
A	B	O	P	G	O	P	E	G	X
R	E	N	O	C	I	R	C	L	E
T	Q	L	N	T	A	I	Z	E	S
B	W	O	I	U	D	S	O	I	N
U	T	O	Q	N	D	M	N	V	E
S	N	S	I	X	E	E	B	W	T

TOPOLOGICAL EXCURSION



The map, on the left, shows some of the main roads in Central England. Are the routes traversable? (i.e. is it possible to travel along each road only once and without travelling in the opposite direction along a road previously used?)

D.I.B.

FIND YOUR OWN LOGARITHMS, USING A CALCULATOR

John Napier and Henry Briggs based their theory of common logarithms on the number of digits obtained when a number was raised to the ten thousand millionth power. With any desk calculator we can adapt this idea to calculate common logarithms with rather less effort than was required in 1615.

For example, to find the common logarithm of 3 we use the "constant multiplier" facility to obtain $3^{10} = 59049$.

Taking out the factor 10^4, $3^{10} = 10^4 \times 5{\cdot}9049$.

Raising 5·9049 to the 10th power $3^{100} = 10^{40} \times 51537752{\cdot}07$

$$= 10^{47} \times 5{\cdot}153775207$$

If, unlike Napier and Briggs, we are satisfied by logarithms to six figures we can call a halt when we reach

$$3^{1\,000\,000} = 10^{477121} \times 1{\cdot}797710064$$

and therefore $3 = 10^{0{\cdot}477121}$

and $\log 3 = 0{\cdot}477121$.

With a machine which cannot retain a constant multiplier, repeated squaring is easier. $3^2 = 9$, $3^4 = 81 = 8{\cdot}1 \times 10^1$, $3^8 = 65{\cdot}61 \times 10^2$

$$3^8 = 10^3 \times 6{\cdot}561$$

After squaring twenty times we reach

$$3^{1048576} = 10^{500297} \times 7{\cdot}88476901 = 10^{500298}$$

Therefore $\log 3 = 500298 \div 1048576$
$= 0{\cdot}477121$

There is one important difference between modern logarithms and the originals. In 1615 decimals were not in common use, calculations used integers or ratio. Where we might find that $x + y = 0{\cdot}573217$, mathematicians in 1615 would have said that x and y are in the ratio 573,217 to 1,000,000.

Napier invented the decimal point, but used it only in his own calculations. The first tables gave the logarithms of integers from 2 to 20,000 without decimal points. Tables base on the 10,000,000,000 power of the number would have given log 3 = 477,121, log 10 = 1,000,000 and log 30 = 1,477,121.

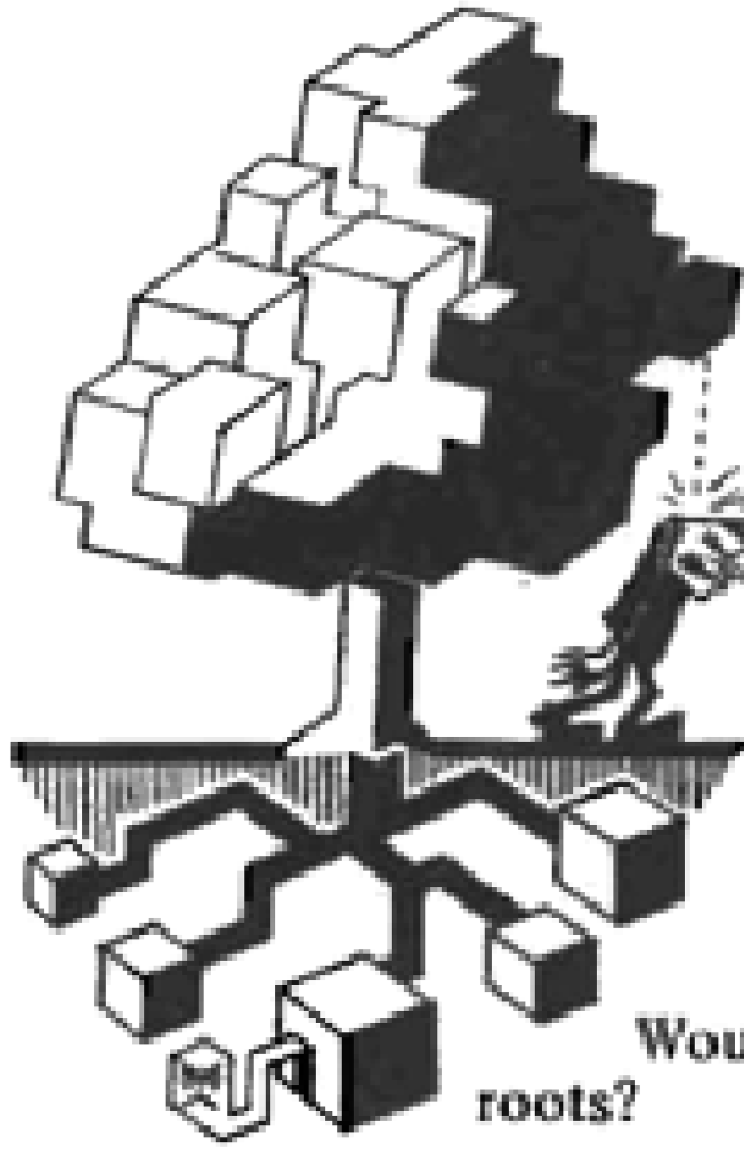
C.V.G.

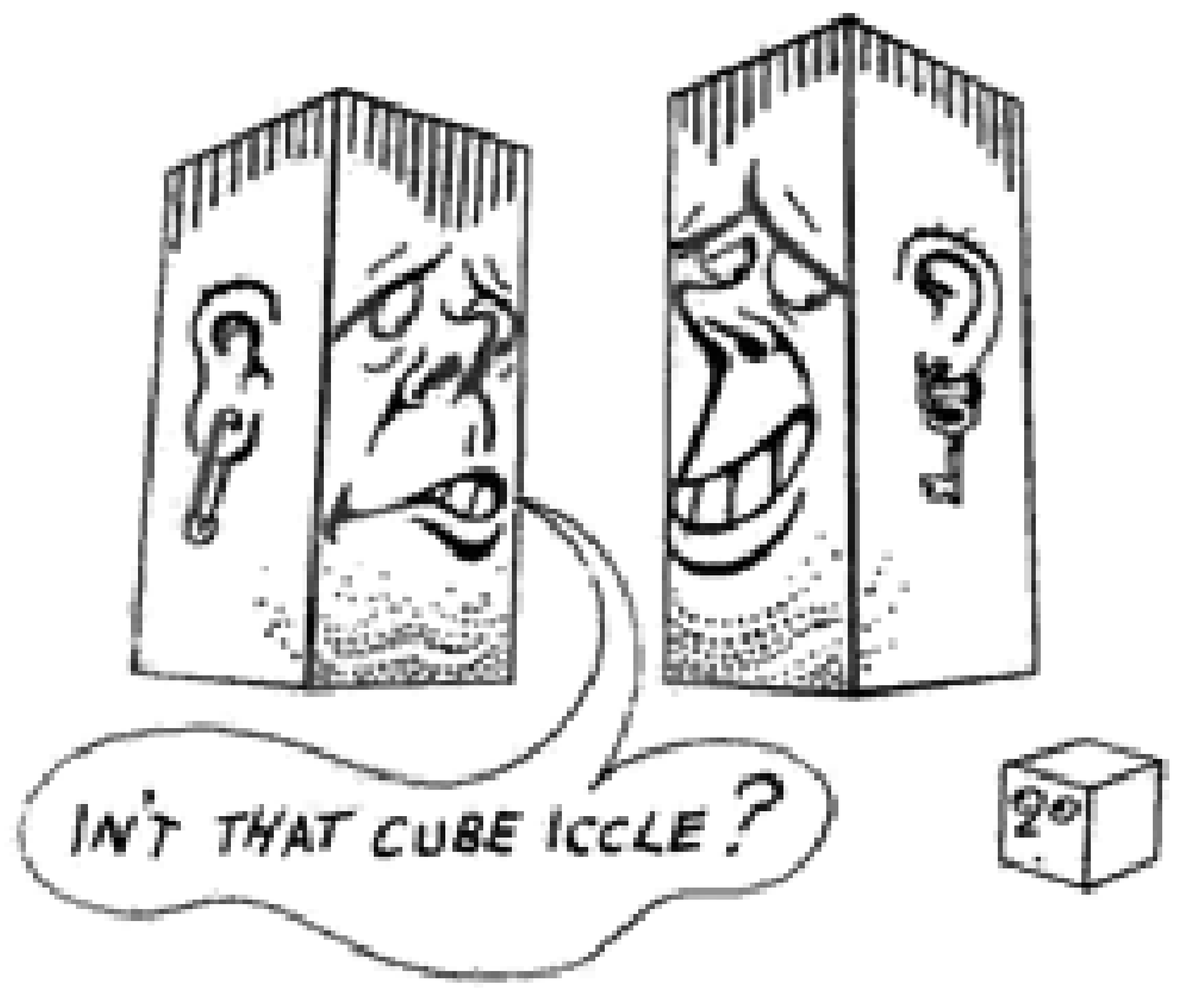
LEGS ELEVEN GOES METRIC

11 pounds equals 5 kilogram, 11 yards equals 10 metres,
11 gallons equals 50 litres, well nearly.

C.V.G.

CUBE TREE





Would a cube tree have cube roots?

C.B.A.

SEASON'S GREETINGS

If the most likely value of MERRY CHRISTMAS is 189, what is A HAPPY NEW YEAR worth?

R.H.C.

A LET DOWN

A man rides a bike into the country at 12 m.p.h. but has to walk back because of a flat tyre at 4 m.p.h. If he were gone a total of 5 hours, how far did he ride into the country.?

Clever Dick said "That's easy. Take the time divided by the slower rate and multiply this by the faster rate to get the distance".

The answer is correct. I said "What if the time had been 8 hours would your method still work ? ".

Would the method work for 5 hours if the rates were changed to 15 m.p.h. and 3 m.p.h.?

R.H.C.

CHEW ON THIS

The grass in a certain field would feed 11 cows for 5 days or 7 cows for 10 days. For how long would it feed 8 cows?

C.V.G.

TWENTY-ONE

This is a game of skill for two players. If you have a pack of playing cards, you can use nine cards from it – an ace, king, nine, eight, seven, six, five, four and three. (The suits don't matter). Or you could easily make a set by labelling nine pieces of paper or card.

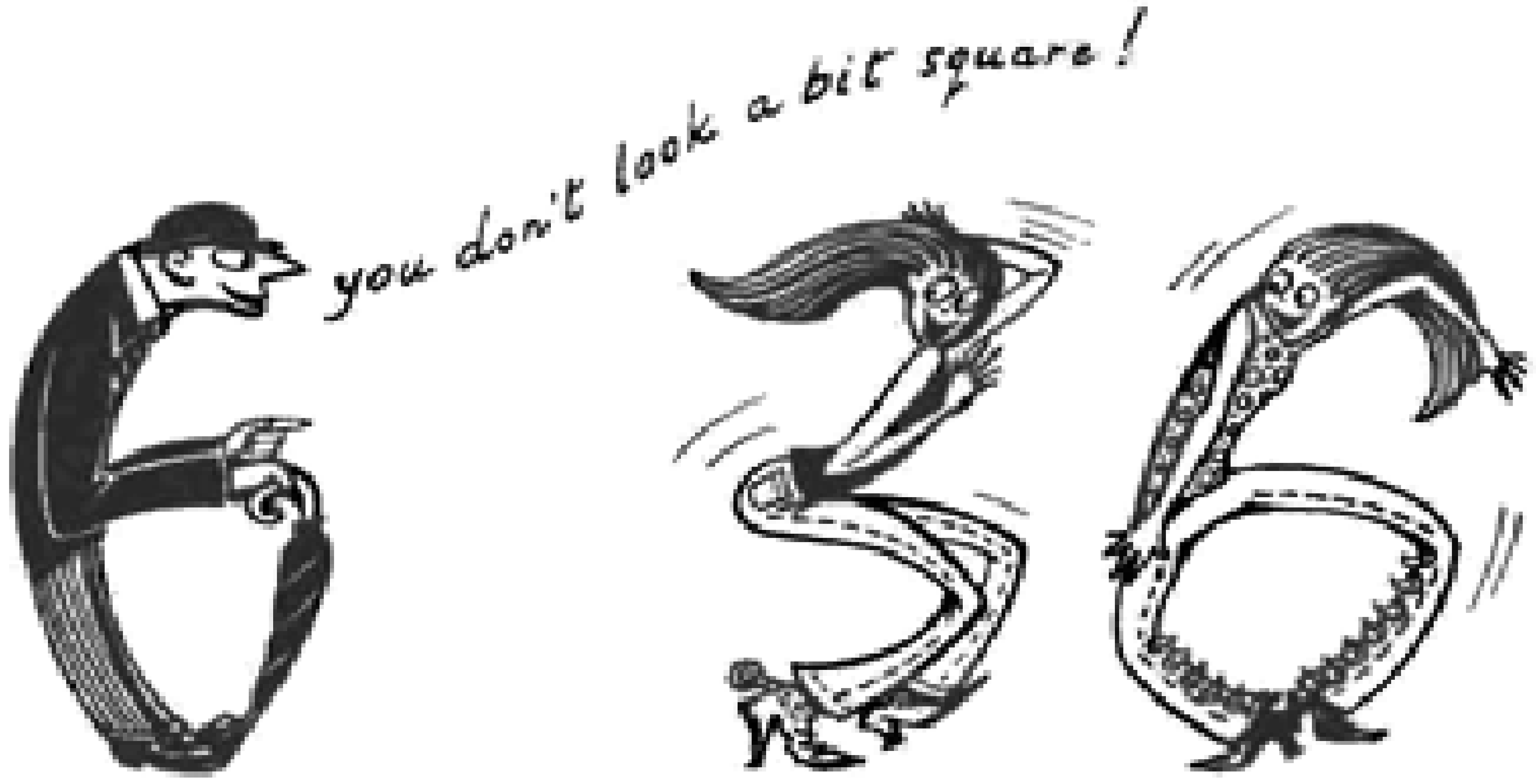
Place all the cards FACE UP spread on a desk or table. The two players take turns choosing a card and keep it, placing it face up in front of them. The aim – like "Pontoon", "Vingt-et-un" or "Blackjack" – is to make a total of twenty-one; but the difference is that there are no "busts" – the players do not have to count ALL their cards but can use a subset of them if they wish. As usual, the King counts as "ten", and the ace can count as "eleven" or "one".

Sometimes all of the available cards may be "picked up" without either player being able to make twenty-one. In this case the player who can make the largest total under twenty-one (by using a subset of his cards) wins the game.

For fairness, the players take turns at having first choice. With practice and thought, you may be able to work out a strategy for winning. We'll tell you more about the game in the next issue. Meanwhile, have fun!

E.G.

EVEN MORE CRAZY NUMBERS



INFLATION AT WORK

In 1934, I bought 576 screws for 6 shillings and have just sold them for 1p each. What was the percentage profit?

C.B.A.