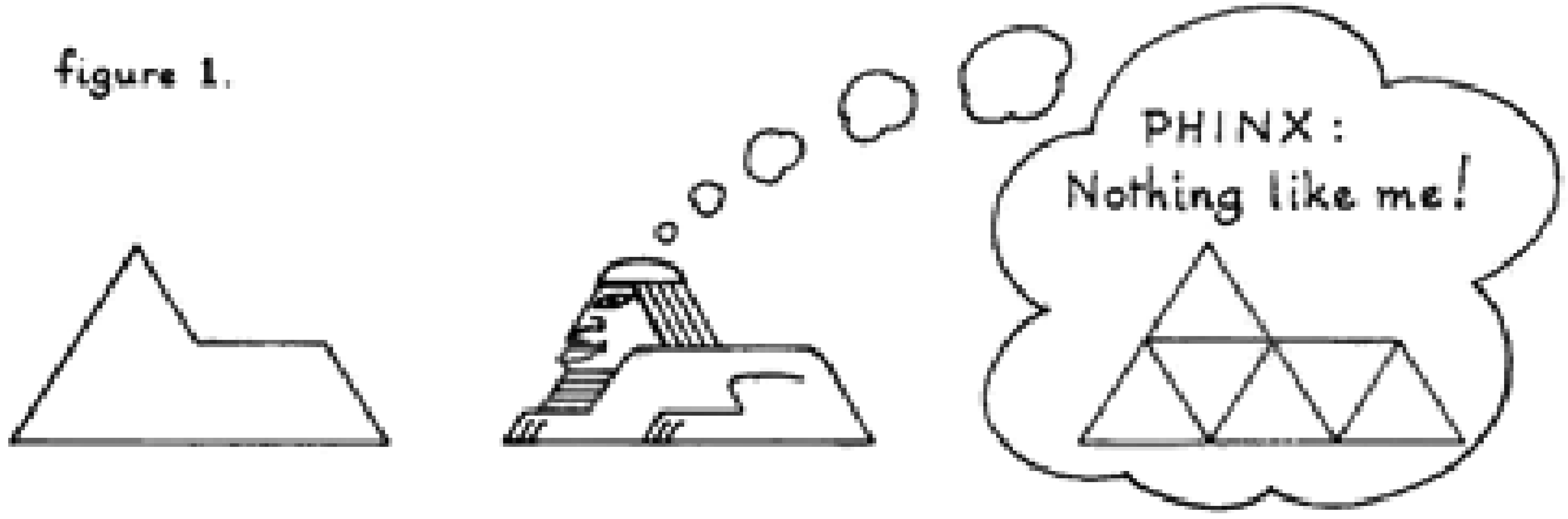
THE SPHINX

The pentagon shown in figure 1 is sometimes known as the sphinx because of its resemblance to that well-known Egyptian monument. For these puzzles, you will need a set of sphinxes cut from card or paper. (You can manage with just a few at first but you will need more for the later problem). Notice that a sphinx is made up of six equilateral triangles, so it is useful to start with a triangular lattice of 1 centimetre triangles. Each sphinx is then 3 cm along its base.

figure 1.



Here are three simple "jig-saw" puzzles to get you started:

1. It is very easy to make a parallelogram whose edges are 3 cm and 2 cm by using two sphinxes. So, of course, you can make "3 by 4" or "6 by 2" parallelograms from 4 sphinxes, or a "6 by 4" parallelogram from 8 sphinxes, and so on. Can you make a "6 by 5" parallelogram using 10 sphinxes?
2. Make a "honeycomb", figure 2, from 6 sphinxes.
3. Make a "bird", figure 3, from 10 sphinxes.

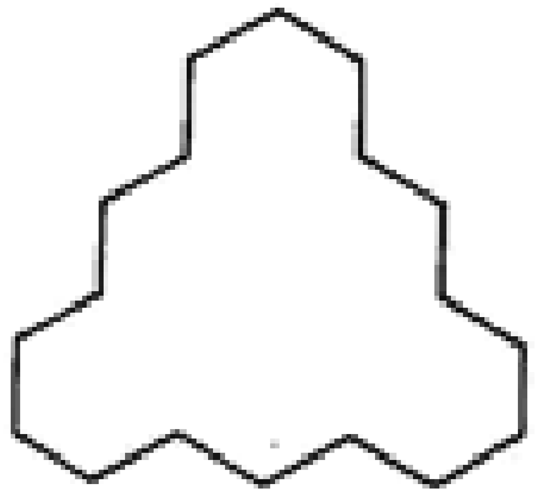


fig. 2

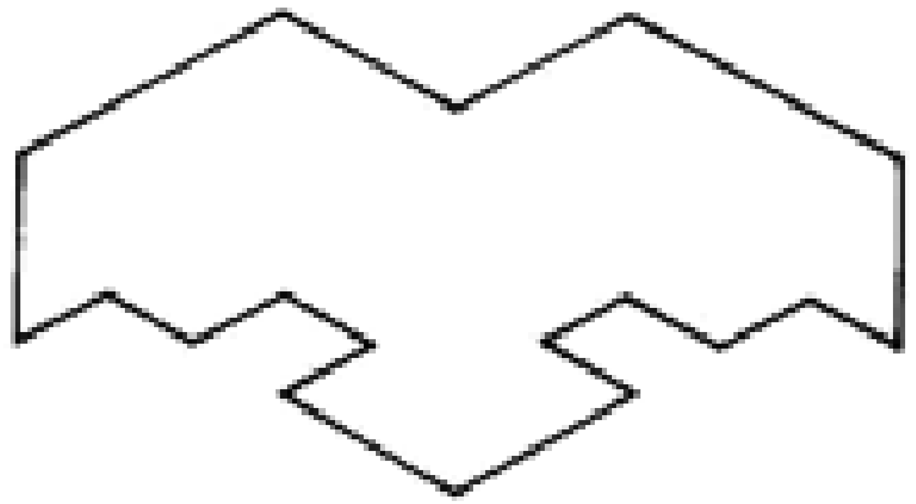


fig. 3

Now – (appropriately for the desert) – for the real "nitty-gritty".

a. Make a big sphinx using 4 sphinxes.
b. Make a (bigger!) sphinx using 9 sphinx using 9 sphinxes.

You shouldn't need very long to realise that it would be easy to make other "super sphinxes" using 16, 36 (or even 64 or 81) unit sphinxes. Observing that 4, 9, 16, 36, 64 and 81 are all perfect squares – (which should not surprise those who have any familiarity with similarity) – can you manufacture these "missing" sphinxes?

c. A 25-unit sphinx and d. A 49-unit sphinx.

E.G.

MATHEMATICAL PIE

No. 86 *Editorial Address :* **West View, Fiveways, Nr. Warwick** SPRING, 1979

CONTACT!

Newspaper reports, 1978: Scientists are considering possible ways of sending 'messages' to beings which may exist elsewhere in space. 'Pictures' could be sent, it is thought, by using binary code to show whether squares on a grid should be black or white – for example (00000) (01110) (00100) (00100) (00000) would indicate 5 rows of squares – the first row all white, the second row having one white square, three black squares and one white square, and so on (see fig.1). Pulse like morse code dots and dashes could stand for o's and 1's respectively.

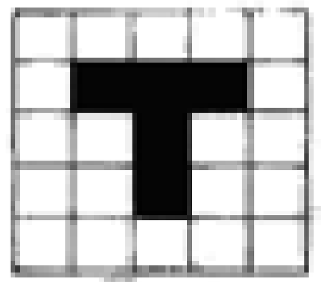


Fig.1

It was just as well that the Commander came in – Alan was just gloomily looking at the print-out from the computer. "Any luck, Alan?" the Commander asked. We were trying to decode the message of blips and bleeps that had come in from the region of Alpha Centauri, and the languages department had nearly gone nuts.

"Not much" Alan replied "I was trying that old idea of binary pictures but the result looks like a high-rise office block with only half the lights on". (See fig.2).

"Just a moment" said the Commander "What are those crosses at the bottom? Surely alien entities aren't sending love and kisses at the end of a message?!" "No" replied Alan "I only had information for 324 squares and those are just to fill the space."

"324 – of course!" yelled the Commander "Why on earth – or on the moon – did you group the data in tens? There's no reason to suppose that an alien entity has ten digits – it's more likely to send something grouped in its own system. 324 is 18 squared, and a square arrangement is far more logical. Maybe this entity has nine 'fingers' and nine 'toes'." "Sounds as though it must be a NON – entity" I grunted. And of course the computer HAD to choose that instant to break down. So the Commander made me regroup the information in eighteens and plot the picture by hand. At least the effort was worth while – he was right! Although we are not certain yet

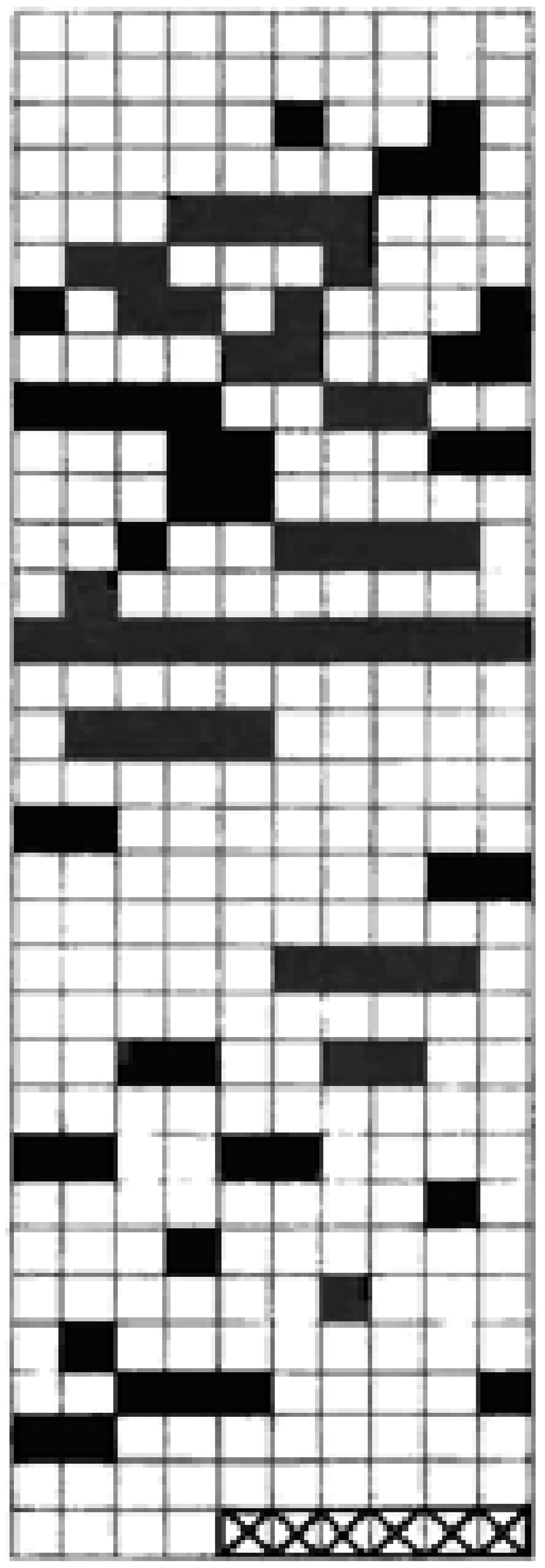


Fig. 2

about the fingers and toes.
To find out what was sent, you will have to do the same. You can tell from the 'Office Block' that the message started with 25 zeros, 1 one, 2 zeros, 1 one, 8 zeros, 2 ones, etc. Regrouping this gives 18 whites for the first row, 7 whites, 1 black, 2 white, 1 black, 7 whites for the second row, 1 white, 2 blacks, etc for the third row ..... the picture has been started for you in fig.3.

E.G.

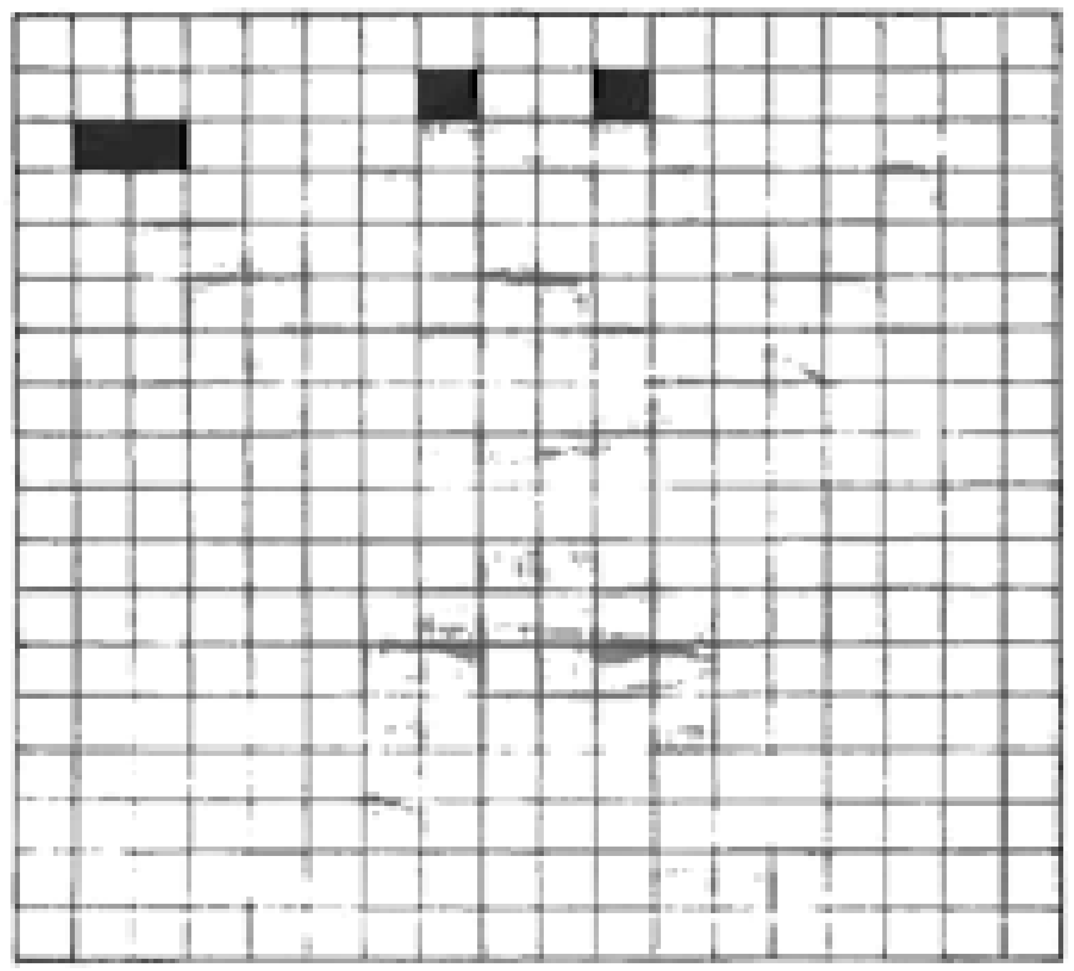


Fig.3

BACK TO FRONT

Work out — (no calculating machines please!) — 23 x 64 = ? Now try the whole thing backwards: 46 x 32 = ?
Can you find ALL the products (two digits times two digits) that can be completely reversed in this way? Of course we don't want products like 11 x 33 = 33 x 11, they are too obvious.

E.G.

TRY THIS

Write down the number of the month of your birthday — (e.g. January = 1) Double it and add 5. Multiply the result by 50 and add your age. Subtract 250. The month of your birthday should appear at the left of your answer and your age should appear on the right.

This method could be used to find the date of birth of a friend. Instead of adding your age, add the day of the month of the birthday and ask for the answer. You then subtract 250 from the answer and the left part of the final number gives the month and the right hand part the day of the month.

A.M.A.

HOW ODD!

A number consists of twenty two digits of which the last is 7. If this digit is moved to the first place, the number is increased to seven times its original value. What is the number?

R.H.C.

FITS

Tiles which are equal quadrilaterals can be fitted together to cover a plane surface.

Can the same be done with a set of tiles which are equal scalene triangles? What is such an arrangement called?

C.V.G.

VOCABULARY

Two pupils were asked to add two numbers together. The first made the mistake of subtracting them and gave the answer 100, the second said it was 11 900 because he multiplied them. What was the correct total?

R.H.C.

SORT THEM OUT

Arrange the numbers 1 to 9 inclusive in two lines so that each line has the same number of numbers and also adds up to the same total.

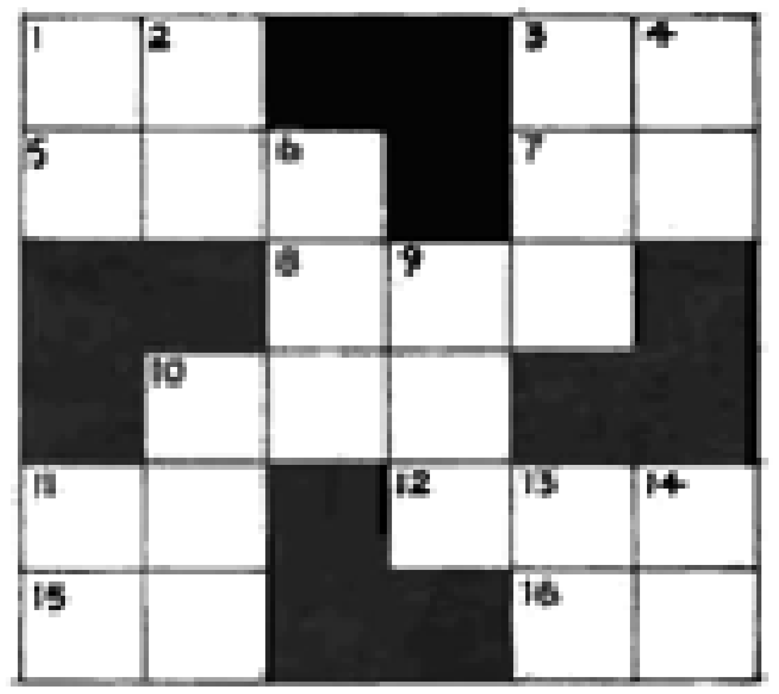
R.H.C.

JUNIOR CROSS FIGURE No.68

Clues Across
1. Number of sides of an OCTAGON, multiplied by the number of faces of a CUBE
3. Next number in the sequence 19, 28, 37, 46, –.
5. 37 x 6.
7. Number of degrees in one right angle.
8. Seven squared followed by two squared.
10. Number of seconds in one-tenth of an hour.
11. One-third of four dozen.
12. An odd multiple of 53.
15. One-quarter written as a percentage.
16. Two soccer teams.

Clues Down
1. Seven-tenths of an hour in minutes.
2. 100 – 18
3. 66 x 9
4. Half a century.
6. The first three even numbers.
9. One more than one hundred less than one thousand.
10. Number of whole days in a normal year.
11. One-fifth of 6 cm in mm.
13. Number of weeks in a year.
14. Four times a prime number.

E.G.



FROM THE PAST

A problem in issue No.74 which is now in the far past for many leads to:
If $a^2 + (a+1)^2 = c^2$, then $(2a+1)^2 = 2c^2 - 1$.
Therefore $2a + 1$ and c are numerator and denominator in the odd terms of the sequence of rational approximations of $\sqrt{2}$.

$$\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}$$

Hence $0^2 + 1^2 = 1^2$, $3^2 + 4^2 = 5^2$, $20^2 + 21^2 = 29^2$, $119^2 + 120^2 = 169^2$, $696^2 + 697^2 = 985^2$, $4059^2 + 4060^2 = 5741^2$, $23660^2 + 23661^2 = 33461^2$.

C.V.G.

TRY SQUARE

Write down a list of the squares of the integers and after each write the sum of the digits. If the sum of the digits is greater than 9, write down the sum of the digits and so on until the sum has only one digit.

Looking at the sequence of numbers in the last column should enable you to see which of the numbers below cannot be perfect squares.

15476 45269 98596

C.V.G.

COMPLETE THE CIRCLE

Arrange the numbers 1 to 10 inclusive equally spaced around a circle in such a way that the sum of two adjacent numbers is equal to the sum of the two numbers opposite. To simplify the problem, we can tell you that 1 and 2 are opposite each other.

R.H.C

CUBES

$17^3 = 4913$. If you add together the digits 4, 9, 1 and 3 you get 17. Can you find any more numbers which when cubed give digits which add up to the original number?

Put another way can you find any other number whose cube root is the sum of the digits? (Hint: you do not need to consider numbers beyond 30 and a calculator would help).

R.H.C.

THE POWER OF A CALCULATOR

Can you find values for n and m so that $m^n - n^m$ has the value 1844 and a different pair to give 1927?

R.H.C.

CARD SHARPERS

This is a game for any number of players.

Take a pack of cards and stand the pile in the middle of the table. The first player takes a card and places it face upwards in front of himself. If it is a seven, he turns it face downwards into his private bank. The second player then takes a card from the pack and places it face upwards on the table. If this card and the previous one give a total of seven or any multiple of seven, then the player picks up the two cards for his private bank. The third player then takes a card and places it face upwards on the table and whichever of the cards make up a multiple of seven he too can claim for his bank. This continues until the pile in the centre of the table is exhausted. The winner is the one with the most cards.

R.H.C.

BODMAS

Some of you may not be aware that, by convention, a calculation involving mixtures of x, ÷, + and − is worked out leaving the + and − signs until the last. If we mean that any + or − signs should be done before x or ÷ we show this by using brackets.

So, 3 + 4 x 2 would ALWAYS mean 3 + 8, which is 11, but (3 + 4) x 2 would mean 7 x 2 which is 14.

By putting in ONE pair of brackets, how many different answers can you get for this calculation? $5 \times 4 + 3 \div 2 - 1$

E.G.

REFLECTIONS

There are seven capital letters which read the same upside down, but after much reflection I can find only two English words which when printed in capitals are the same right way up and upside down. Can you do better?

C.V.G.

THIS & THAT

This and that and $\frac{2}{3}$ of this and that is what part of $\frac{4}{3}$ of this and that?

R.H.C.

PROPERLY PRIMED

A prime number, you should know, has exactly two factors.

Primes = 2, 3, 5, 7, 11, 13, ...

If we add together the digits of "eleven", the first two digit prime, we get 1 + 1 = 2 which is prime. So I think of 11 as "very" prime.

13 is not "very" prime as 1 + 3 = 4 which is not a prime. Can you list all the "very" prime numbers below 100?

E.G.

12345679 IS AN ODD NUMBER

Multiply the number by 3 to give 037 037 037
by 30 to give 370 370 370
by 57 to give 703 703 703
and the sum of the digits of the product is 30 in each case.
Now multiply by 6 to give 074 074 074
by ? to give 407 407 407
and by ? to give 740 740 740
and the sum of the digits is 33.
Now try 12 which gives 148 148 148 with totals of 39
Find the other multipliers.
Now try 15 and then see if you can predict a few more to try.

R.H.C.

An Ordered Pair

MORE CLEVER PLOTS

From the Royal School, Bath, we received several examples of pictures which may be drawn by using co-ordinates of points to be joined in sequences. The original article was printed in issue No.82. Join up the points in the order given but do NOT join points separated by an asterisk *.

From Kate Ball

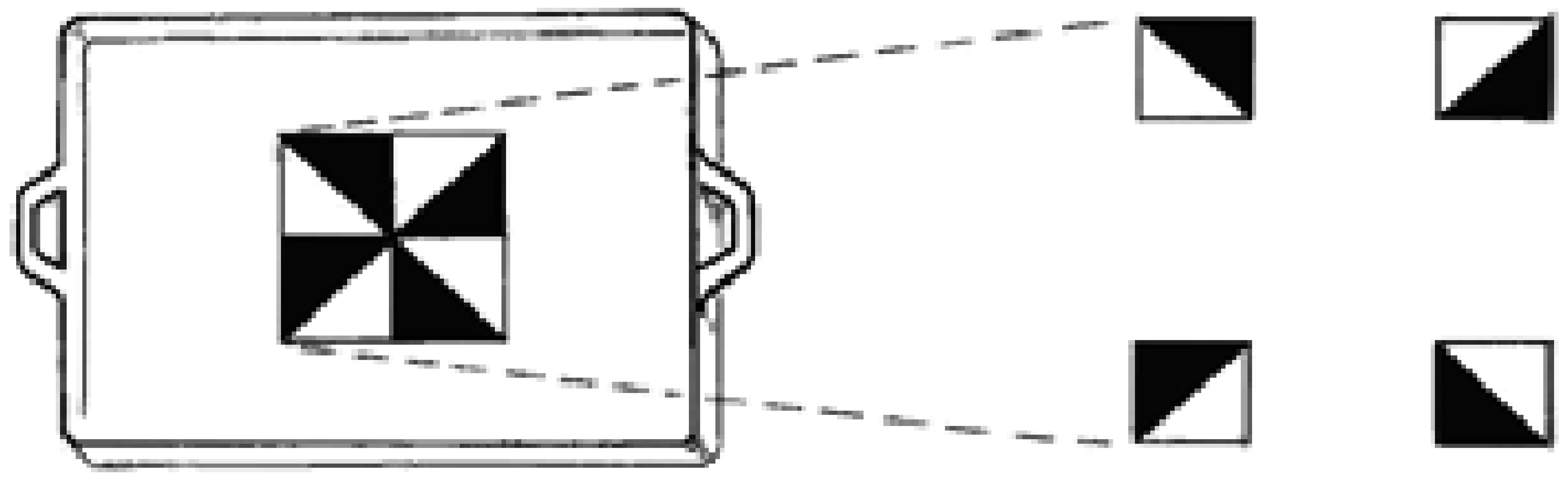
(1.0,2.8)	(0.0,2.8)	(1.1,4.8)	(7.1,4.8)	(8.0,2.8)	(7.0,2.8)	(7.0,0.0)*
(1.5,5.4)	(1.3,5.2)	(0.7,5.0)	(0.5,5.3)	(0.8,5.7)	(1.0,5.8)	(1.2,5.6)
(1.3,5.7)	(1.8,6.3)	(1.8,6.0)	(2.0,6.3)	(4.0,5.8)	(4.4,6.2)	(5.1,6.7)
(5.7,6.7)	(6.0,6.2)	(6.0,6.0)	(6.4,5.9)	(6.7,5.5)	(6.8,5.2)	(6.6,4.8)
(6.0,4.8)	(5.7,4.9)	(5.5,5.0)	(5.4,5.1)	(5.2,4.9)	(4.4,4.8)	(3.5,4.8)
(2.8,4.9)	(1.9,4.8)	(1.5,5.2)	*	(4.4,6.2)	(4.3,5.8)	(4.5,5.6)
(5.3,6.0)	(5.4,6.4)	*	(4.4,5.2)	(4.0,5.1)	(3.6,5.1)	(3.7,5.3)
(3.6,5.5)	(3.3,5.4)	(3.1,5.0)	*	(1.6,4.0)	(7.5,4.0)	*
(1.0,0.0)	(1.0,2.8)	*	(1.0,2.0)	(7.0,2.0)	*	(1.0,1.0)
(7.0,1.0)	*	(6.6,4.9)	(6.8,4.8)	(6.9,5.0)	(6.7,5.1)	*
(5.7,6.0)	(5.5,5.5)					

From Sarah Hardy

(1.4,0.6)	(1.2,0.8)	(1.1,1.0)	(1.7,1.5)	(2.1,2.0)	(2.3,2.7)	*
(1.1,1.0)	(1.1,1.2)	(1.5,1.5)	(2.2,2.3)	(2.3,2.7)	(2.0,3.0)	(1.7,3.1)
(1.5,2.8)	(1.0,2.5)	(0.6,2.5)	(0.5,2.6)	(0.5,2.7)	(1.0,2.8)	(1.2,3.0)
(1.4,3.3)	(1.4,3.7)	(1.2,3.9)	(1.5,4.0)	(1.4,4.2)	(1.2,4.3)	(1.4,4.6)
(1.6,4.5)	(1.9,4.3)	(2.1,4.5)	(2.1,4.3)	(2.5,4.3)	(3.0,4.0)	(3.3,3.6)
(4.0,3.5)	(4.5,3.5)	(5.0,3.6)	(5.5,3.5)	(6.0,3.1)	(6.1,2.9)	(6.0,2.3)
(5.7,1.9)	(6.1,1.0)	(6.3,0.5)	(6.1,0.5)	(5.8,1.1)	(5.7,1.5)	(5.6,1.9)
(5.7,1.9)	*	(6.1,0.5)	(6.1,0.4)	(5.9,0.2)	(5.6,0.2)	(5.0,1.5)
*	(5.0,1.5)	(4.0,1.5)	(3.5,1.5)	(2.8,2.1)	(2.0,1.3)	(1.4,0.6)
*	(2.9,4.1)	(4.0,4.5)	(4.6,5.0)	(4.6,5.5)	(4.4,5.8)	(4.4,5.8)
(3.8,5.9)	(3.0,5.5)	(2.5,5.0)	(2.1,4.5)	*	(1.4,3.7)	(1.8,4.0)
(2.1,4.5)	*	(1.4,3.7)	(1.8,4.0)	(2.1,4.3)	*	(1.8,4.0)
(2.0,3.7)	(2.4,3.0)	(3.0,2.1)	(4.0,1.8)	(4.6,2.1)	(4.5,2.5)	(4.3,3.0)
(3.3,3.6)	*	(1.5,4.0)	(1.9,4.3)	*	(6.0,3.1)	
(6.2,3.1)	(6.4,2.5)	(6.4,2.4)	(6.2,2.5)	(6.1,2.4)	(6.2,2.7)	(6.1,2.9)
*	(1.9,3.1)	(1.9,3.2)	*	(1.6,3.5)	*	*

TRAY DIFFICILE

Mary Turner and her French friend Imogen Mireau got a Saturday job helping to make trays in a factory. The trays had a square pattern made from four identical square tiles like this:



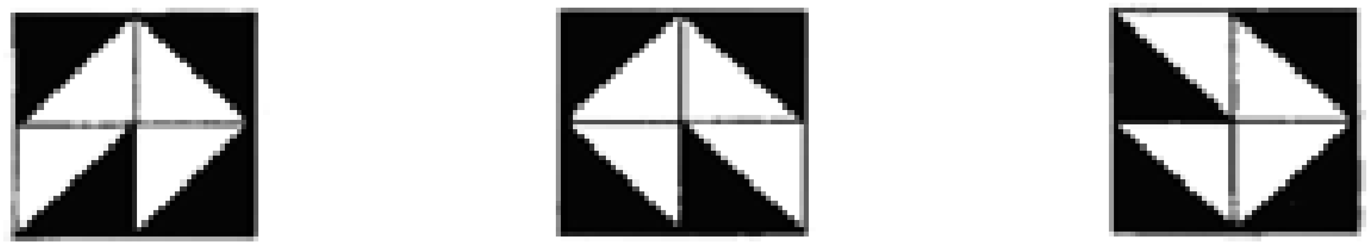
"I do get bored with this "windmill" pattern" said Mary. "Let's see how many **different** trays we can make today".

"No doubt it will be many" replied Imogen. "Each tile can be placed four ways, and I think that makes what you call four to the power four!"

"I don't think we'll get that many if we rule out patterns that are rotations of each other" said Mary. "Let's try".

You could try too, using squared paper. You have to be patient to get the answer, but it's nowhere near as many as four to the power of four. The diagrams below give you a start: numbers 1 and 2 are different because they are mirror images of each other. Number 3 would not count because it is a rotation of pattern number 1.

E.G.



1 2 3

AN ECONOMY DRIVE

If you had to construct motor ways to join four towns situated at the four vertices of a square, what routes should there be to ensure the minimum length of road?

R.H.C.

HOW STRANGE!

63 x 67 = 60 x 70 + 3 x 7 = 4221
52 x 58 = 50 x 60 + 2 x 8 = 3016
76 x 74 = 5624

Can you write an algebraic expression to explain these results?

C.V.G.

OVERTAKING PROBLEMS

A car and caravan combination travelling at 80 km h^{-1} overtook a juggernaut travelling in the same direction at 70 km h^{-1}. If the juggernaut had a length of 18 m, how long did the car and caravan take to completely overtake it if its total length was 10m?

D.I.B.