

At each number, carefully pierce a hole into which a matchstick marker will fit tightly.

With the match compartment of a matchbox separating the two discs, pierce a 25 mm. (or larger) paper fastener through the centres. After cutting a window in one face, glue the envelope part of the matchbox beneath the centre of the larger disc.

In order to find the product of 41 and 19, first set the binary equivalent of 41 in the Multiplicand Register. Matchstick markers placed in the correct columns will show 101001. Secondly, set the binary equivalent of 19 in the Multiplier Register.

Now, rotate the Product Register until the arrow indicates decimal "1" in the Multiplier Register. In this position, you will notice that the Multiplicand and Product registers correspond. A marker in this "1" represents the first stage of the operation (1×41): wherever a marker is present in the Multiplicand Register, add another below it in the Product Register (thus recording the binary equivalent of 41).

To represent the second stage of the operation (2×41), rotate the Product Register one place clockwise so that the arrow indicates the marker in decimal "2" of the Multiplier Register. As before, add the Multiplicand Register to the Product Register.

Rotating the Product Register successively two places clockwise, you will see that the "4" and "8" columns contain no markers. Subsequently, nothing is added to the Product Register ($0 \times 41 = 0$) until the arrow indicates the "16" column marker (16×41) when the final addition is made.

The Product Register records binary 1100001011 which is then converted into the decimal equivalent 779.

D.I.B.

A FAT SWISS ROLL

submitted by David Moore, Freshwater, Isle of Wight

On to a piece of sponge cake, 8 mm. thick and 450 mm. wide, a layer of cream 1 mm. deep is spread. The sponge is then rolled into a Swiss-roll. What will the "diameter" of the cake be at its thickest part?

CHARLIE COOK AGAIN AND AGAIN

Question: Calculate the distances between (a) the points (43, 49) and (-17, -31), (b) the points (50, 25) and (80, 65).

Solutions given by Charlie.

$$\begin{aligned} \text{(a) } d^2 &= (43 - (-17))^2 + (49 - (-31))^2 \\ &= 26^2 + 80^2 \\ &= 1000 \end{aligned}$$

$$d = 100$$

$$\begin{aligned} \text{(b) } d^2 &= (50 - 80)^2 + (25 - 65)^2 \\ &= 130^2 + 90^2 \\ &= 25000 \end{aligned}$$

$$d = 50$$

The same mistake was made each time but the answer is correct in each case. When will this working give the correct answer?

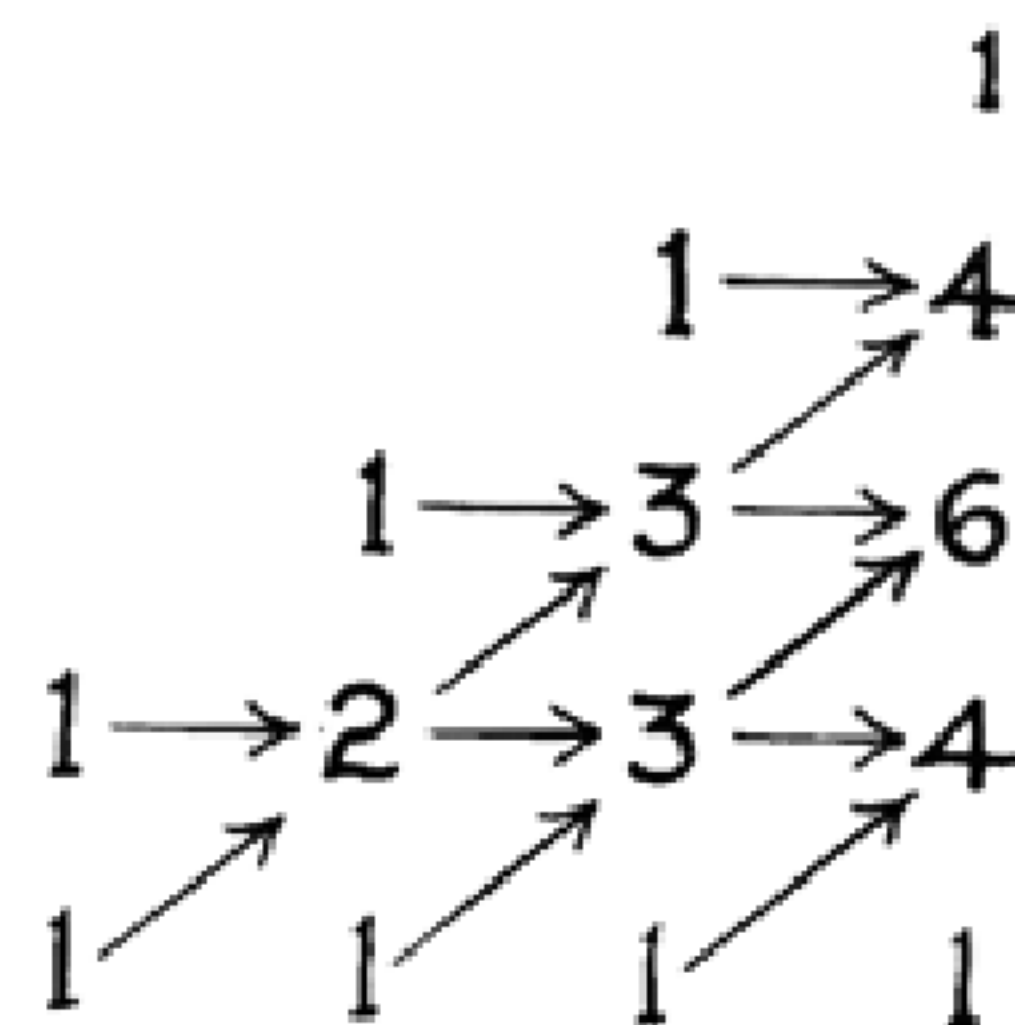
C.V.G.

ORDERED PAIRS

It is well known that $\binom{1}{0} + \binom{1}{1} = \binom{2}{1}$ is correct using the notation of ordered pairs. You may be surprised to know that the following are also true.

$$\binom{2}{0} + \binom{2}{1} = \binom{3}{1} \quad \binom{2}{1} + \binom{2}{2} = \binom{3}{2} \quad \binom{3}{0} + \binom{3}{1} = \binom{4}{1} \quad \binom{3}{2} + \binom{3}{3} = \binom{4}{3}$$

Perhaps you think that my + sign is not the "usual" addition, but I assure you it is *normal* addition. It is the interpretation of the ordered pairs which may be new to you. Plot the ordered pairs above, and show by arrows when two points add together to give a third. Can you suggest answers to the following?



$$\binom{3}{1} + \binom{3}{2} \quad \binom{4}{1} + \binom{4}{2}$$

If you have studied Pascal's triangle the pattern we are using should be familiar, see block. In fact, the notation $\binom{n}{r}$ is an alternative often used for nC_r , the number of ways in which a set containing r elements can be chosen from a set containing n . Check that $\binom{3}{2} = {}^3C_2 = 3$ and $\binom{4}{2} = {}^4C_2 = 6$.

A possible formula for $\binom{n}{r}$ in this interpretation is $\frac{n!}{r!(n-r)!}$ where $n!$, called factorial n , is the product of $n(n-1)(n-2) \dots 3 \times 2 \times 1$. So, for example, $\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$.

Our observations about Pascal's Triangle leads us to believe that $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$. Can you prove this from the formula? E.G.



SOLUTIONS TO PROBLEMS IN ISSUE No. 73

AN OPEN AND SHUT CASE

The butler was guilty as pages 133 and 134 are on opposite sides of the same LEAF.

A TRIDDLE

The triangle is one with sides 5, 12, 13.

MAP READING

The area is 3 km².

SENIOR CROSS FIGURE No. 69

Clues Across: 1. 210; 3. 49; 5. 63; 6. 2304; 8. 4111; 10. 48; 11. 46; 14. 2520; 16. 5488; 18. 25; 20. 60; 21. 314.
Clues Down: 1. 26; 2. 1344; 3. 40; 4. 945; 6. 21; 7. 3142; 9. 1828; 12. 6021; 13. 256; 15. 58; 17. 40; 19. 54.

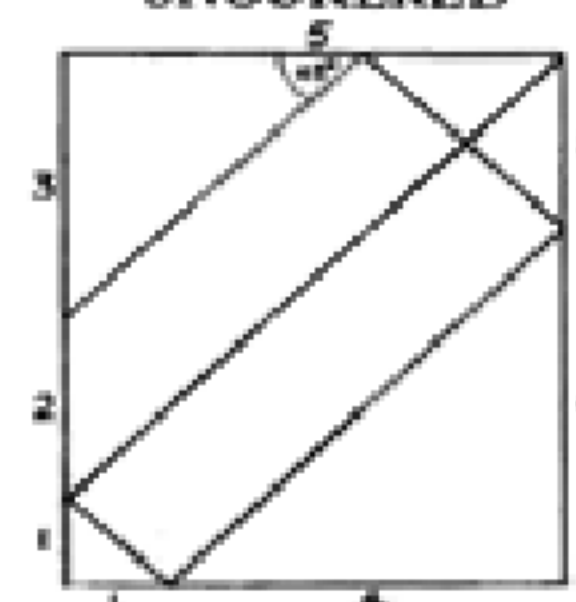
NUMBER PLEASE

The number is 8712.

JUNIOR CROSS FIGURE No. 65

Clues Across: 2. 43; 4. 11; 6. 65; 7. 25; 10. 32; 11. 40; 12. 328; 15. 111; 16. 275; 19. 11.
Clues Down: 1. 2633; 3. 322; 5. 1003; 8. 5; 10. 38; 13. 2472; 14. 1101; 17. 27.

SNOOKERED



CRANKS, CONNECTING-RODS AND LINKAGES

Converting linear motion from a piston into circular motion to drive wheels, a connecting-rod or crank is used in many types of steam and internal combustion engines. The system was first patented by James Pickard in 1780. It is common for a number of rods to be hinged together forming a linkage so that parts may move among themselves.

Using this simple-to-build model, the loci of several linkages may be plotted.

Dimensions given are not critical but suggest the scale for a table-top model.

Cut a piece of hardboard 520 mm. by 400 mm. and bore holes with a bradawl every 40 mm. on the centre line drawn along its length.

From stiff card, cut two circular discs of radius 50 mm. and one of radius 90 mm. Divide and mark each circumference into twelve equally spaced divisions. On each disc, pierce a hole on one division 10 mm. from the circumference.

Again from stiff card, cut two strips 260 mm. by 30 mm. and two strips 260 mm. by 15 mm. Cut a slot of about 2 mm. width and 220 mm. length in each of the wide strips and pierce a hole 10 mm. from each end. Pierce a hole 10 mm. from each end of the narrow strips and bore further holes at 20 mm. intervals between them.

Paper fasteners can be used to form disc pivots and hinged joints.

Fig. 1 shows the model set up to demonstrate Pickard's connecting-rod.

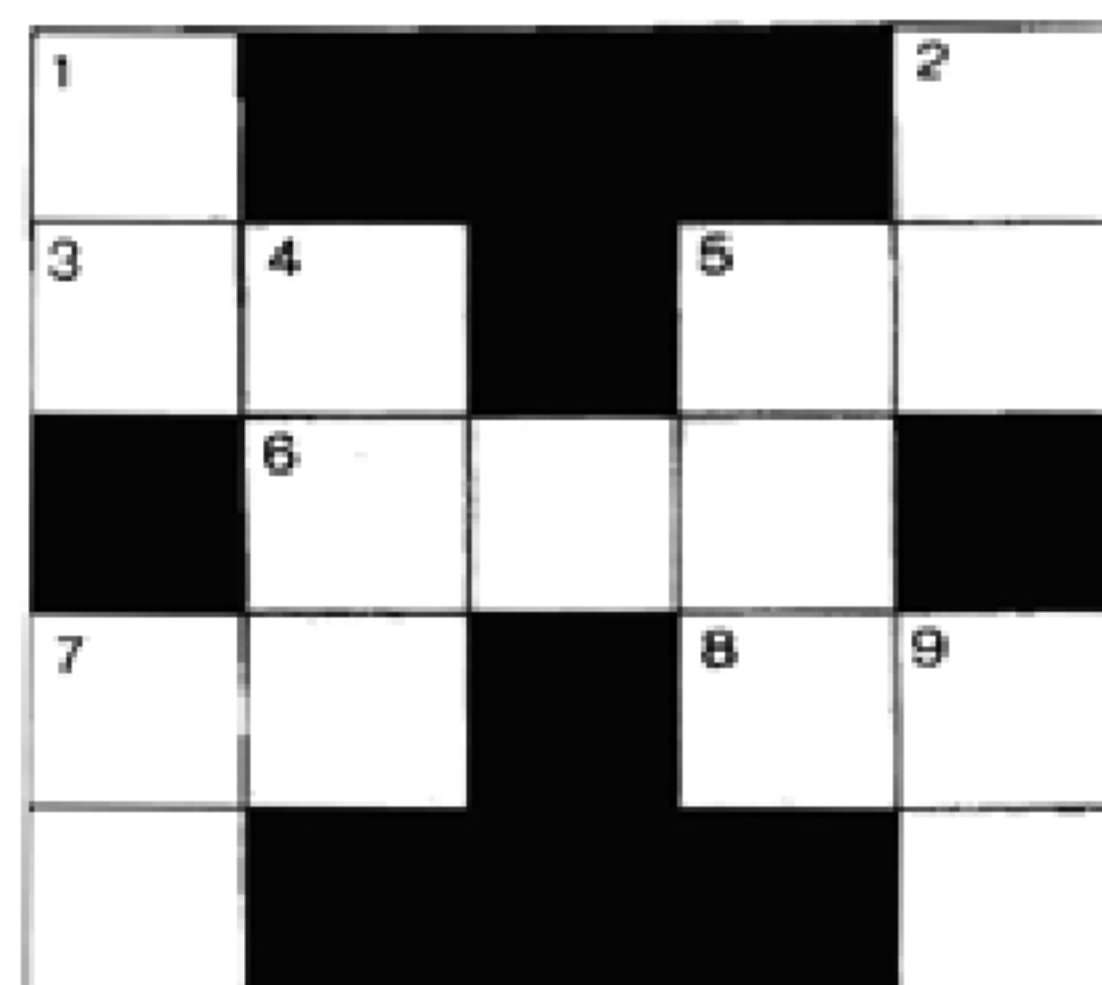
Fig. 2 is an example of another linkage. Place a sheet of paper on the board and then attach the discs, pushing the fasteners through the paper. Draw the centre line on the paper, set the discs in the "1" position and mark a point under "P"; move both discs to the "2" position and mark the new position of "P", etc., until the twelve positions have been recorded. The locus can then be drawn.

(see back page for the diagrams)

D.I.B.

A PRIME CROSSFIGURE

submitted by A. Jackson, Heathfield High School, Leicester



CLUES ACROSS

3. 1 down \times 2.
5. A power of 2.
6. 8 across \times 10.
7. 1 down \times 5.
8. A power of 3.

CLUES DOWN

1. 9 down \div 1.
2. Same as 7 down.
4. Multiple of 3 across.
5. Twice 7 down minus 3 across.
7. Multiple of 11.
9. A prime number.

PERFECT SQUARES

$$0^2 + 1^2 = 1^2, 3^2 + 4^2 = 5^2, 20^2 + 21^2 = 29^2$$

Under what conditions will the sum of the squares of two consecutive numbers give a perfect square?

R.H.C.

THE FREAK

$$9 \times 1 - 1 = 8$$

$$9 \times 21 - 1 = 188$$

$$9 \times 321 - 1 = 2888$$

$$9 \times 4321 - 1 = 38888$$

Where is the "odd" one out.

R.H.C.

SQUARING UP

What part of $\frac{1}{2}$ square metre is $\frac{1}{2}$ metre square?

R.H.C.

ANAGRAMATH CROSSWORD

CLUES ACROSS

1. A toy graph's turn to be a geometer? (10).
5. It's just a theory that they ship so carelessly (10).
10. This treadle has not led to a rip (4).
11. Small measures of corn—"on ahead", as usual! (4).
12. Names slips who will give two measures—one "handy", and one large (5,5).
14. How we find x in the equation $\frac{x+3c}{4x+2c} = 6$ (2,5,2,1).

CLUES DOWN

1. Of a projectile, or a garden, maybe! (4).
2. The pens you need to get an item in 1 across's theorem (10).
3. and the diet horses use to get to rest! (5,5).
4. Adds up (4).
6. Eureka may have been this if the bathwater was too hot! (4).
7. We could sate ourselves on these meals (4).
8. Damage done in a charming way? (4).
9. Where the treasure lies twisted? (4).
12. It's half when followed by a circle (4).
13. Trigonometry found in South-East? (4).

The answers are all words mainly hidden in the clues.

E.G.

CURVES OF PURSUIT

A rabbit feeding in the middle of a field sees a dog running directly towards it. It runs straight to its hole and the dog runs towards the rabbit at all times. Figure 1a shows what happens. As the rabbit moves along the straight line at the bottom of the diagram the dog adjusts its direction of motion. The dog takes one step towards the rabbit; by the time the step is completed the rabbit has moved. The next step is taken towards the new position of the rabbit, and so on. The same principle applies when a fighter plane attacks a bomber and explains why the fighter always finishes astern of the bomber. The path taken by the fighter, or the dog, is a **TRACTRIX**.

Figure 2a shows what happens when three elements are involved, each moving directly towards the next in an anti-clockwise sense. In practice, the three elements would eventually move in a circle as the force required to produce the acceleration to maintain the path would not be possible. When the force had reached its maximum value, circular motion would result. Figures 3a, 4a and 5a illustrate what happens with four, five and six elements.

Another way of looking at the figures 2a to 5a is that a regular polygon is drawn. Points are marked on the sides of the polygon at equal distances from the vertices to form a new regular polygon and this process is repeated.

All triangles and all quadrilaterals tessellate, i.e., completely cover a flat surface like tiles, and certain other special polygons have the same property. Of the regular polygons, only the equilateral triangle, the square and the regular hexagon form tessellations, so figures 2a, 3a and 5a can be repeated to cover a large area. Figures 1b to 4b show ways in which 2a can be used to cover a surface and so produce different patterns. Figure 2b shows what happens when the curves of pursuit are in the same sense, in this case anti-clockwise, in each of the triangles; this produces the twisted repeated patterns. In figure 1b, the sense of the curve of pursuit is reversed in adjacent triangles. The top one is anti-clockwise and the one directly below it clockwise. The two triangles on either side of this are again anti-clockwise; this produces the shell pattern.

In figure 3b, the top triangle has the curve in an anti-clockwise sense and the one below the same but the curves in the triangles on either side of this one are clockwise. Thus as we move up and down the sense remains the same but as we move right and left the sense is reversed. Other variations are possible which produce both the twisted and the shell patterns. Generally, a simple rule leads to an attractive pattern but a random arrangement, or disorder, produces a less attractive pattern. Figure 4b shows a further variation.

Similar results can be obtained with repeats of figure 3a but not with repeats of 5a. Can you explain why this is so?

