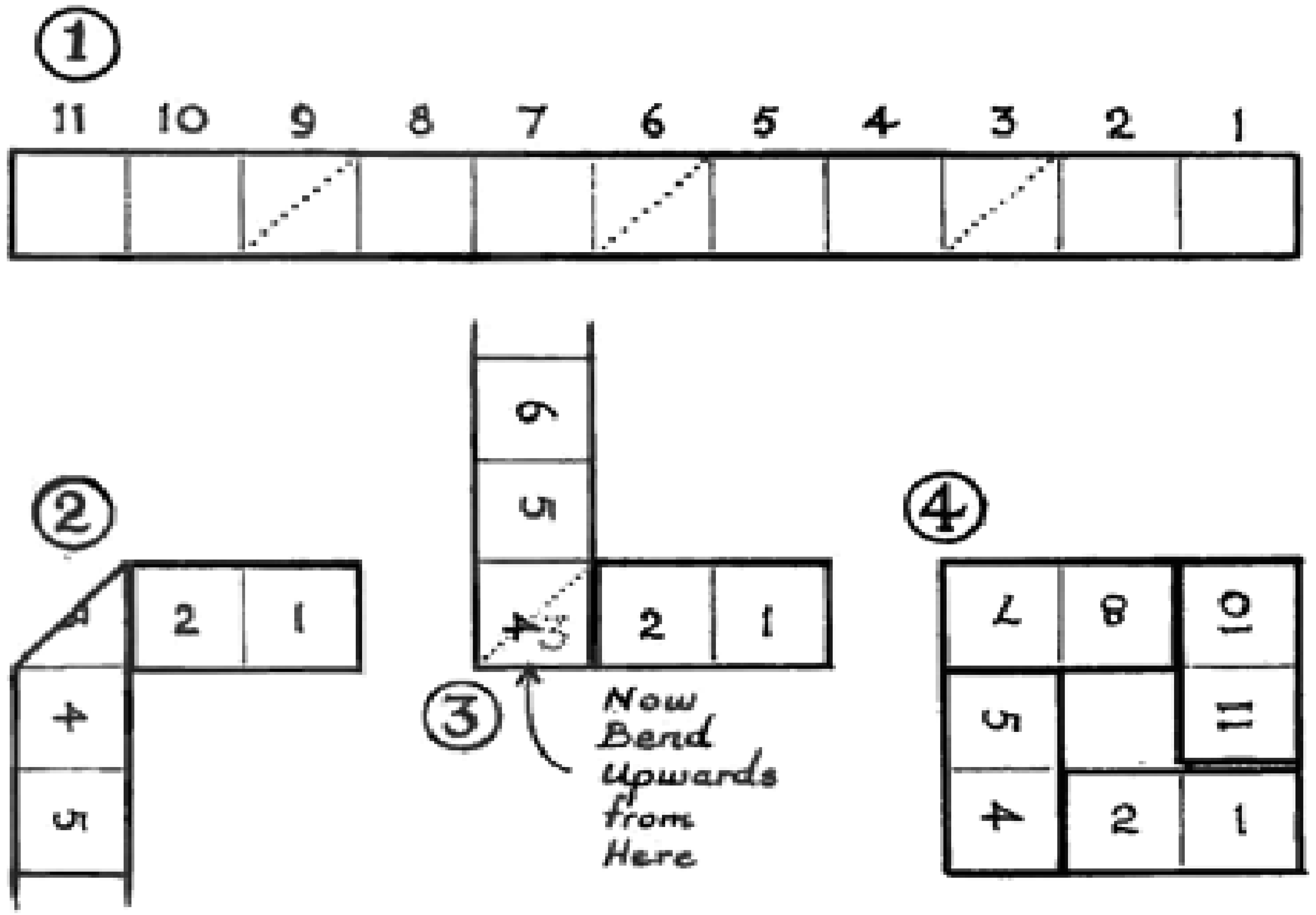
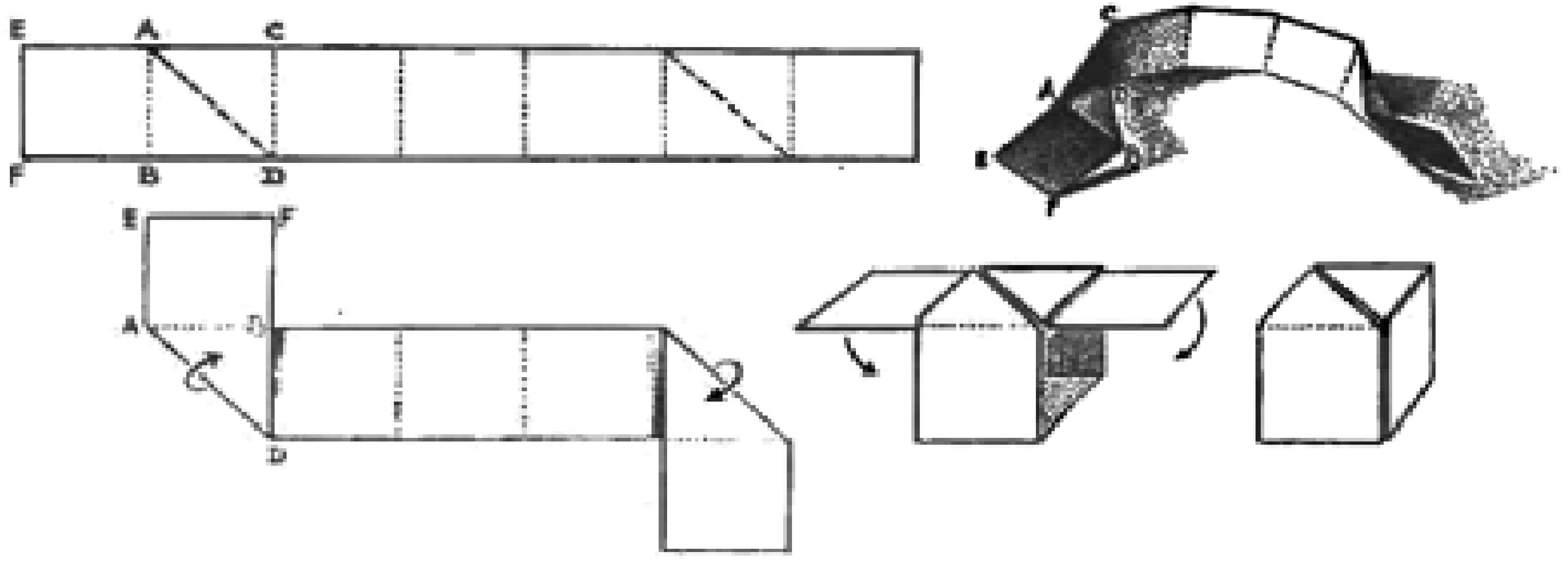
PAPER FOLDING



To enclose a square by eight squares, start with a strip eleven squares long, fold square 3 along the diagonal and then fold square 4 over square 3 as shown in figs. 2 and 3. Repeat the process with squares 6 and 9 to produce fig. 4.



In a similar way it is possible to produce a cube by folding a strip consisting of seven squares as shown in the figure above.

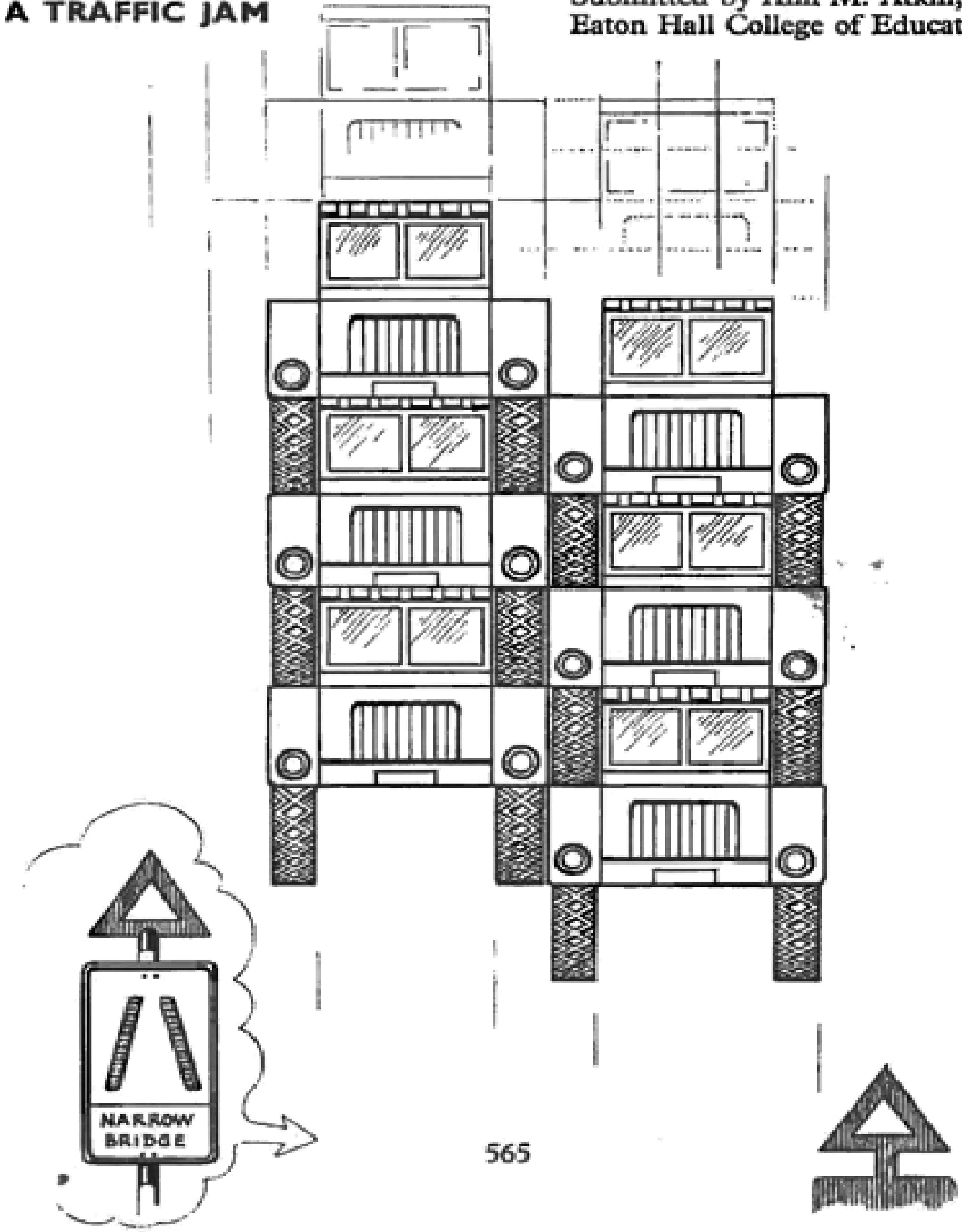
R.H.C.

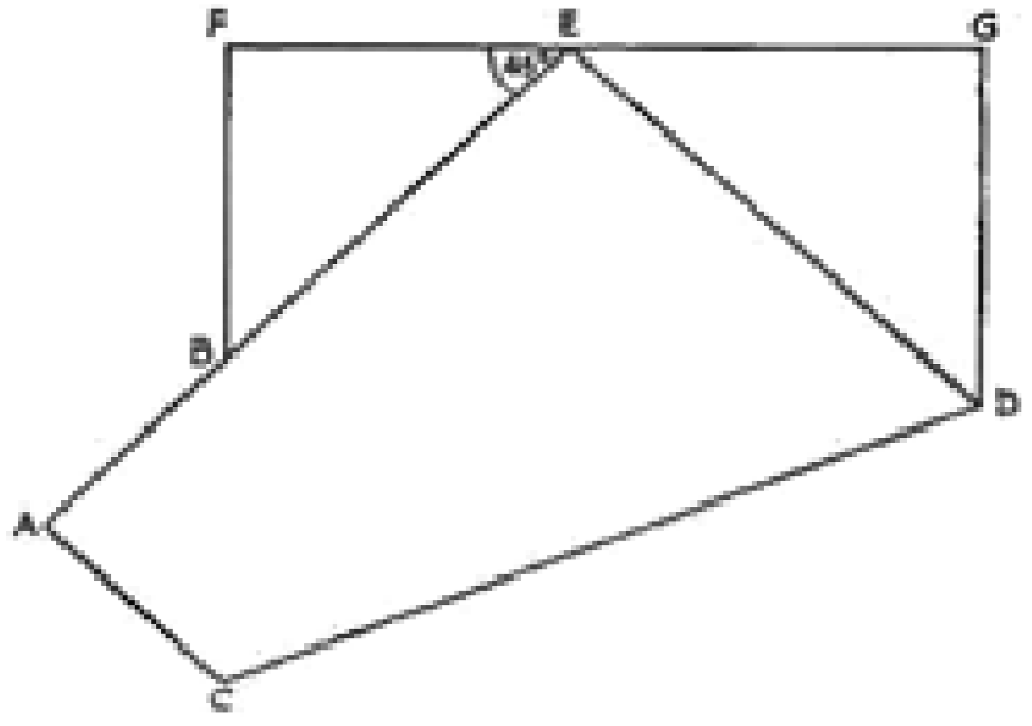
join the mathematical pie
in mathematical pie
mathematical pie
mathematical pie

No. 72 *Editorial Address:* 31 Oldway Drive, Solihull, Warwickshire B91 3HP SUMMER, 1974

A TRAFFIC JAM

Submitted by Ann M. Atkin,
Eaton Hall College of Education

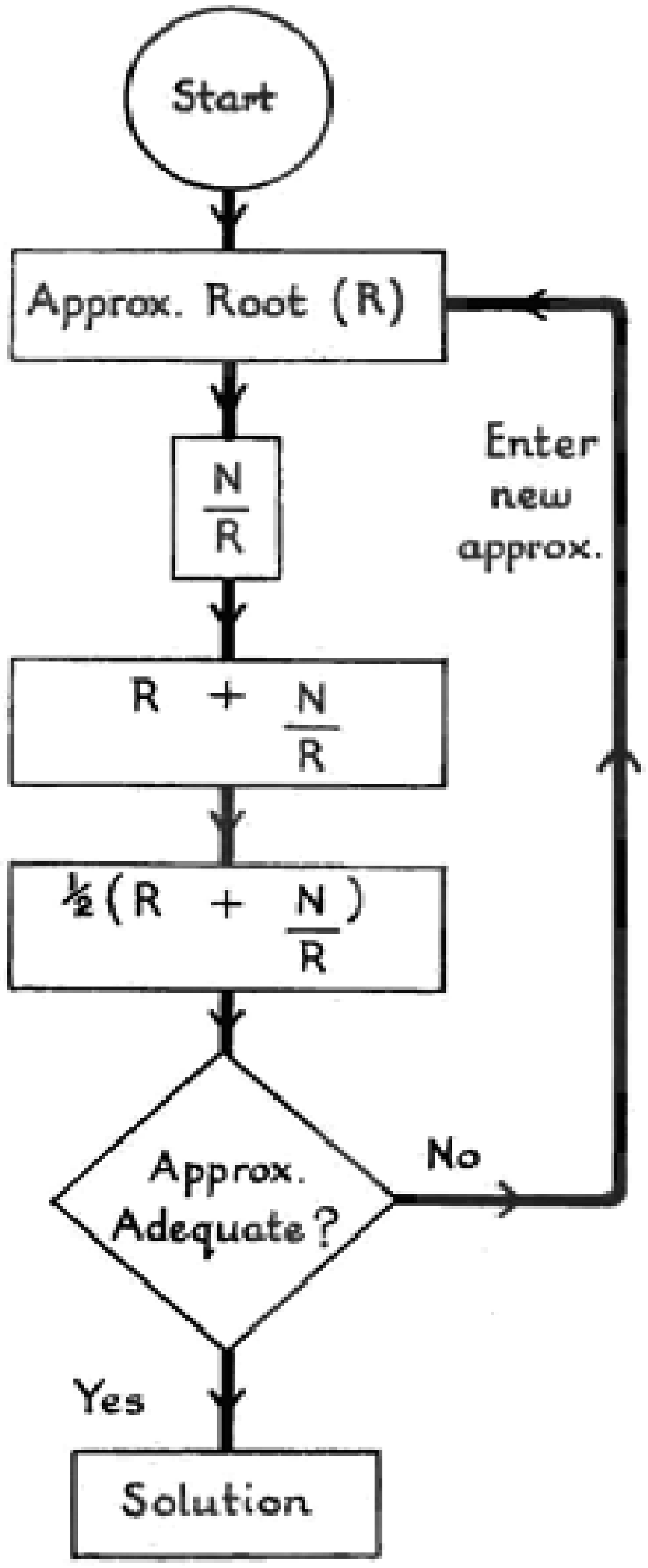




A RATIONAL PROBLEM

A rectangle is folded as shown so that one vertex falls on a side and the sides about the vertex make angles of 45° with the side on which the vertex rests. What can you say about the original rectangle?

R.H.C.



GETTING TO THE ROOTS

For thousands of years, mathematicians have attempted to improve the accuracy of square roots which are irrational numbers. In modern times, the advent of the computer has enabled the calculation of these roots correct to hundreds of decimal places.

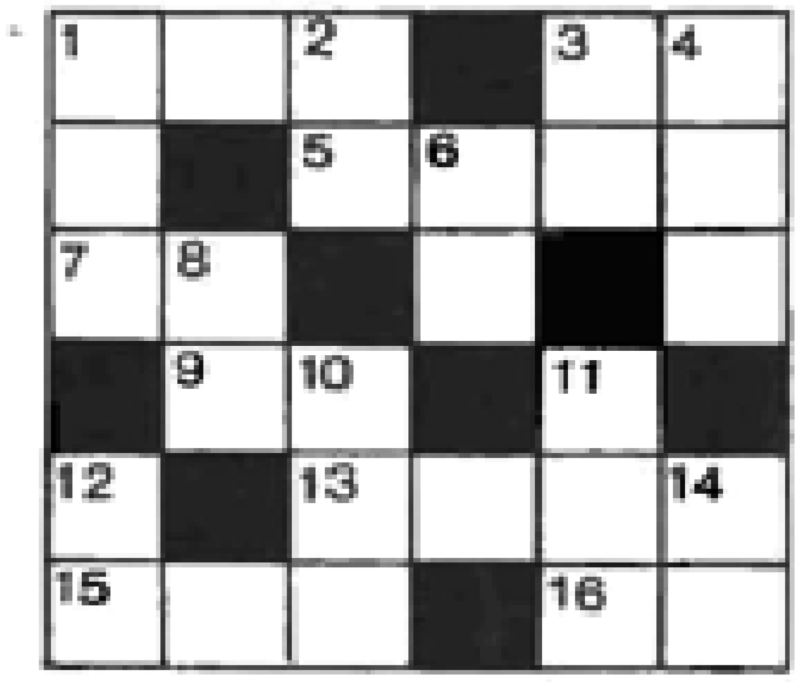
Starting with a rough approximation and followed by nearly two full days of computer time, Jacques Dutka of Columbia university has recently calculated $\sqrt{2}$ correct to over one million decimal places. Such a square root has a useful by-product; the digits of this number he explains are completely random and can be applied to statistical surveys.

By successive operations, try this iterative program to find the square root of a number N. e.g. find $\sqrt{78}$. 1st approximation let $R=8{\cdot}5$. 2nd approximation$=\frac{1}{2}(8{\cdot}5+\frac{78}{8{\cdot}5}) \approx 8{\cdot}84$.

3rd approximation$=\frac{1}{2}(8{\cdot}84+\frac{78}{8{\cdot}84}) \approx 8{\cdot}8318$, etc.

D.I.B.

JUNIOR CROSS FIGURE No. 64



Ignore decimal points.
Take π to be 3·14.

CLUES ACROSS

1. cm. in 1 inch.
3. An unlucky multiple of the square of a prime.
5. Next two terms in the series 1, 4, 9, 16,
7. 3^3+3^2+3.
9. Remaining angle of an isosceles triangle if the equal angles are 42°.
13. [illegible] as a decimal to 4 decimal places.
15. Interior angle of a regular 30-sided polygon.
16. xy if $x+y=13$ and $x-y=9$.

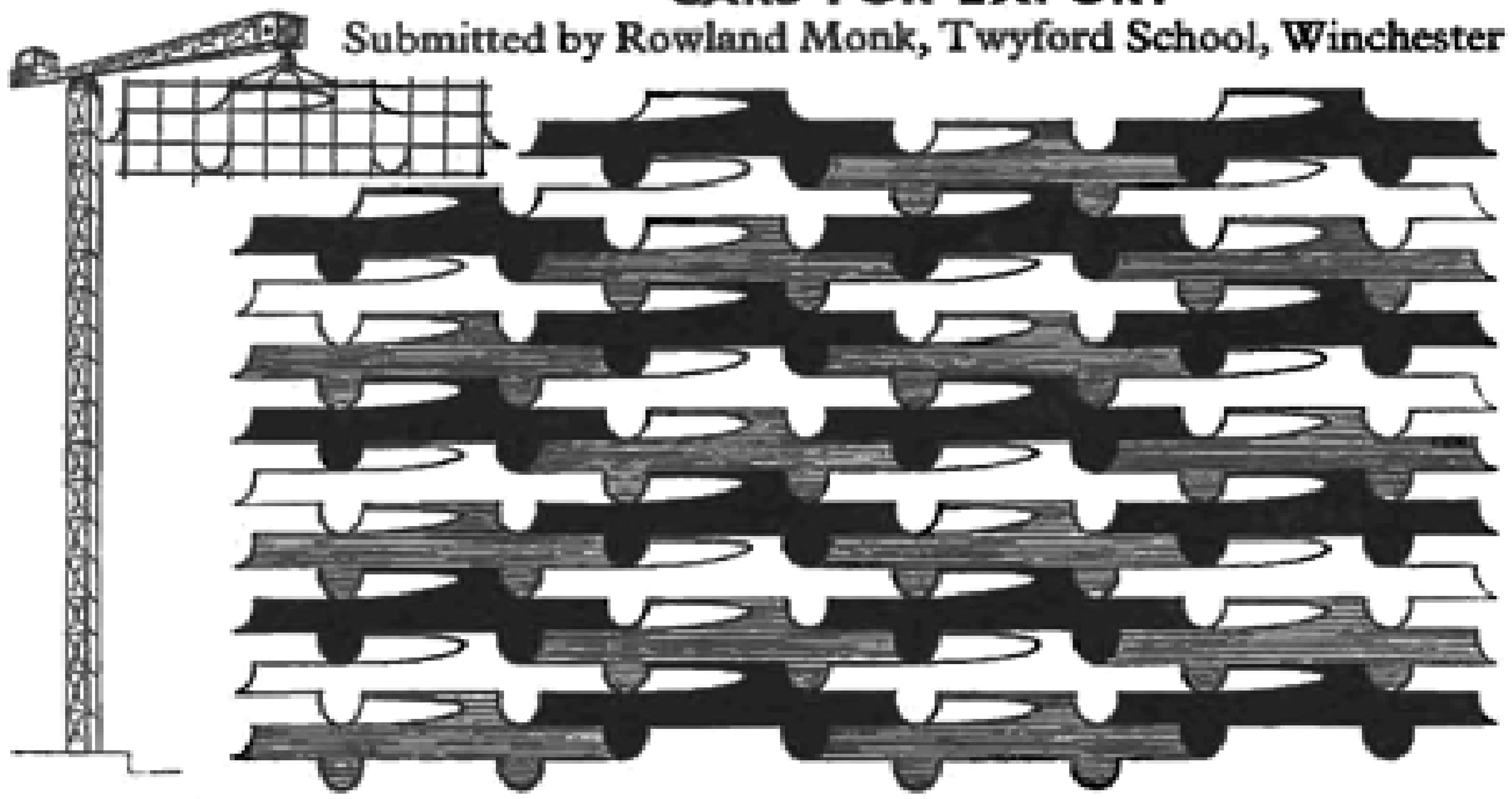
CLUES DOWN

1. First 10's prime multiplied by first 20's prime.
2. Number of cards left in a full pack (i.e., with jokers) when the picture cards have been removed.
3. CCXII ÷ IV.
4. Anagram of 10 down.
6. Square number + 1.
8. $(x-1)(x+1)$ when $x=10$.
10. 2π.
11. Number of days in the first 5 months of a leap year.
12. $(\sqrt{3})^8$.
14. Half a dozen dozen.

D.I.B.

CARS FOR EXPORT

Submitted by Rowland Monk, Twyford School, Winchester



SOLUTIONS TO PROBLEMS IN ISSUE No. 71

SENIOR CROSS FIGURE No. 67

Clues Across: 1. 77; 3. 1013; 6. 253; 8. 47; 9. 56; 10. 135; 12. 516; 13. 122; 15. 14; 17. 79; 18. 835; 20. 5100; 21. 20.

Clues Down: 1. 72; 2. 755; 4. 143; 5. 3750; 7. 3652; 10. 1618; 11. 1175; 14. 291; 16. 432; 19. 50.

JUNIOR CROSS FIGURE No. 63.—Clue 4 Down should have read $b=-4$

Clues Across: 1. 24; 3. 193; 5. 3142; 6. 510; 8. 533; 10. 2048; 12. 575; 13. 25.
Clues Down: 1. 23; 2. 4113; 3. 125; 4. 380; 7. 1742; 8. 525; 9. 325; 11. 85.

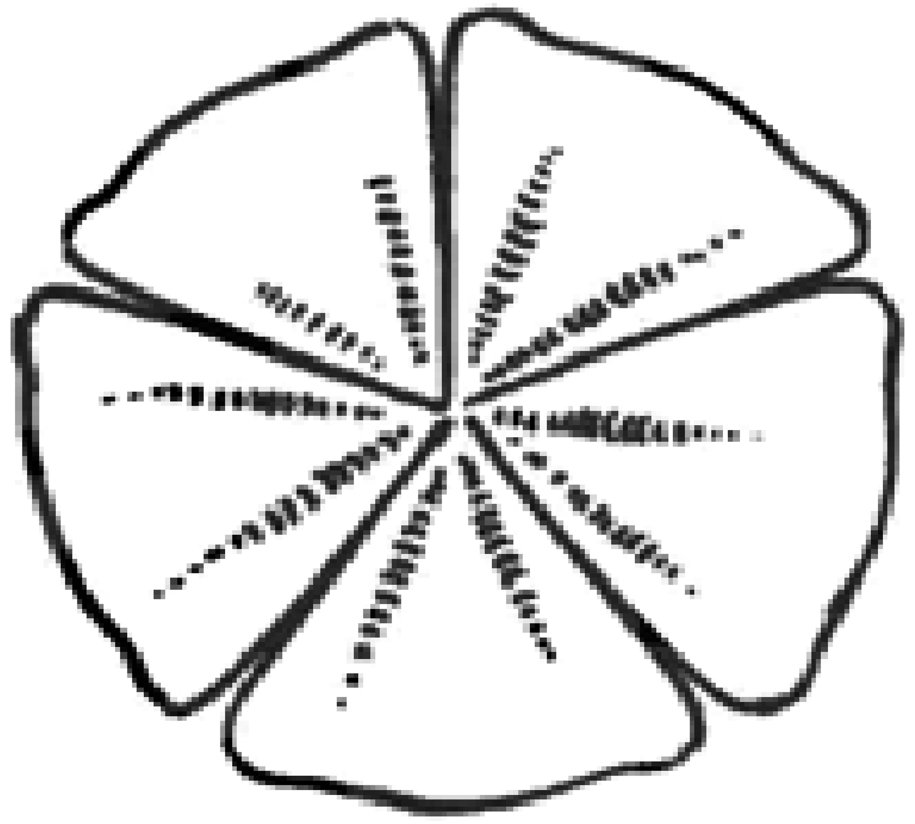
A CROSS PUZZLE

Clues Across: 3. THEOREM; 6. 10000; 8. 11001; 10. SQ; 11. COSINE; 16. PROBABILITY; 17. 20; 18. 048372; 22. 16; 23. RATIO; 27. 15312; 28. INVERSE.
Clues Down: 1. CIBUC; 2. ROOTS; 3. TON; 4. 001; 5. ENO; 7. EIGHTY; 9. 13; 10. SECANT; 12. 1600; 13. 301206; 14. 2168; 16. POWERS; 19. 38534; 20. 23222; 21. 11; 24. TEN; 25. ONE; 26. PIE.

B.A.

LETTER FROM AUSTRALIA No. 3

Searching around the beaches, I have noticed that all the shells seem to curl the same way, and wondered whether (like the bath water) things run the opposite way to those in the Northern Hemisphere. Looking from the "small" end, the curves run clockwise.

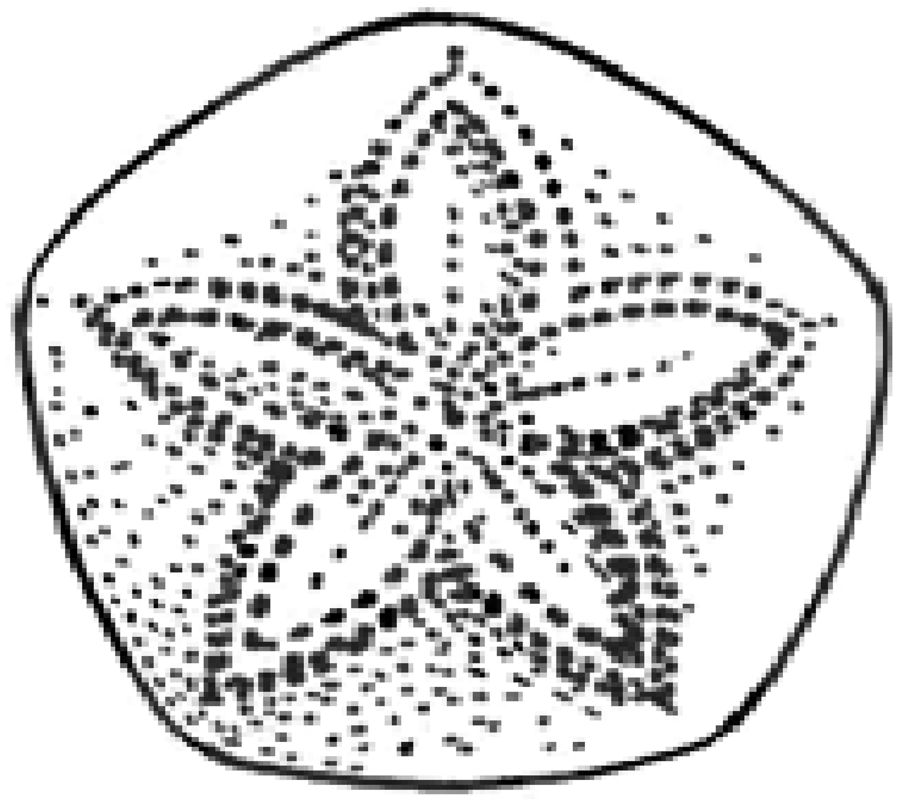


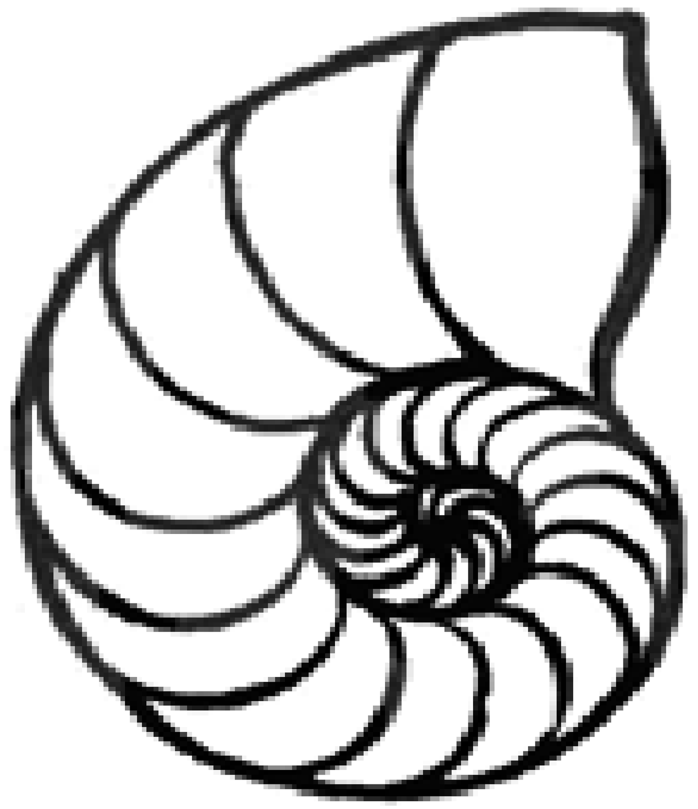
Am I right in my recollection that the good old British shells run counter-clockwise?

Some interesting creatures grow rather flat shells, of which these are two typical examples.

They rather show that base five and base ten arithmetics do have some place in the ocean, and that the starfish is not the only "fiver".

How *did* the octopus ever come about?





E.G.

BASIC ARITHMETIC

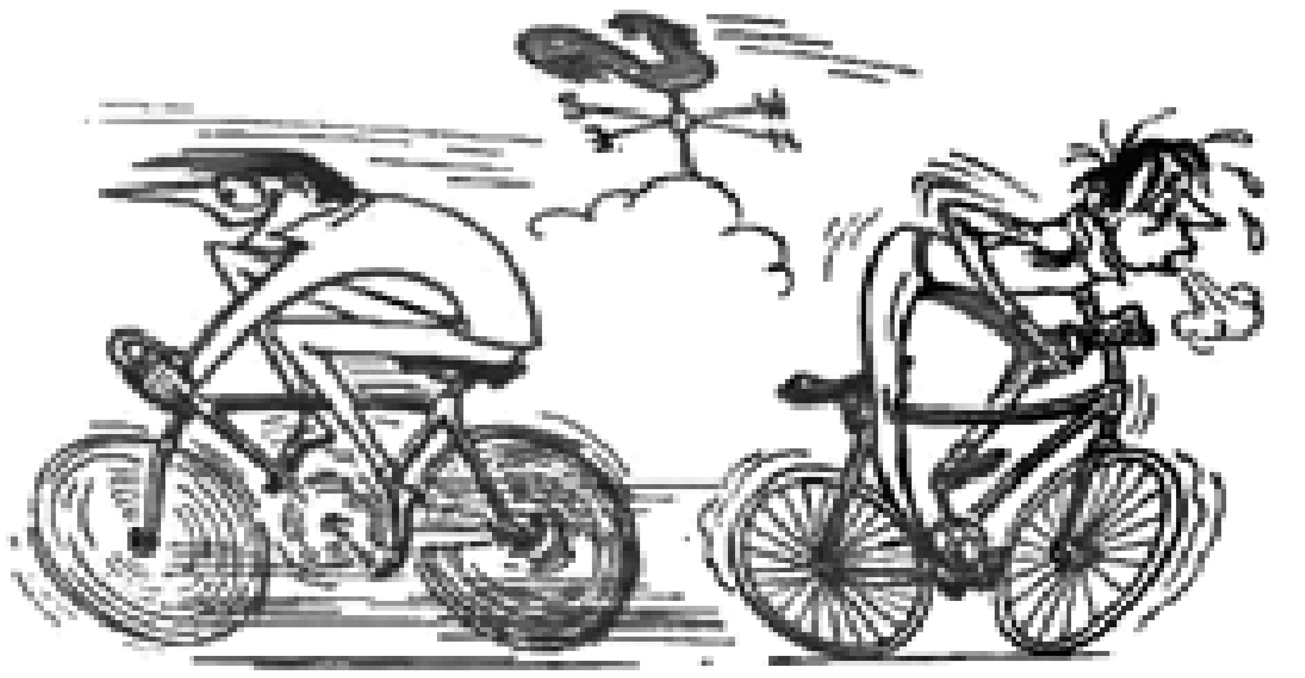
Add the following lists, using the given base in each case.

Base ten	Base ten	Base five	Base five	Base seven	Base seven
123	1234567	123	1234	12	123
456	8901234	401	0123	34	456
789	5678901	234	4012	56	012
012	2345678	012	3401	01	345
345	9012345	340	2340	23	601
678	6789012			45	234
901	3456789			60	560
234	0123456				
567	7890123				
890	4567890				

Try other lists of this type in various number bases. Can you explain the patterns in your results?

E.G.

CHARLIE COOK AGAIN



Charlie Cook's father is a keen cyclist and decided to go for a ride one windy day. Because of the wind he was able to average 12 m.p.h. on his outward journey, but only 4 m.p.h. on the way back. His journey took five hours and when he asked Charlie how far from home he had been, Charlie said "15 miles of course" immediately. When asked how he arrived at this answer so quickly, Charlie said that he had divided the faster speed by the slower speed and multiplied by the time. The answer is numerically correct, but obviously, if worked out like this, is not in the correct units. Derive a formula connecting F (faster) and S (slower) units of speed if he is to get a correct value for the answer by his method.

R.M.S.

SENIOR CROSS FIGURE No. 68

1			■	2	3	4
	■	■	■	■	5	
6		7	■	8		■
	■	9			■	10
■	11		■	12		
13		■	■	■	■	
14			■	15		

Ignore decimal points.

CLUES ACROSS

1. y is inversely proportional to the cube root of x. $y=39$ when $x=1\cdot25 \times 10^2$, find y when x is 8×10^{-3}.
2. $p+q+r$ if $(a^3b^{-2}c^{-4}) \div (a^{-4}b^{-2\frac{1}{2}}) = a^pb^qc^{-r}$.
5. Maximum value of $2x^3-3x^2-36x+12$.
6. $\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{4+\frac{1}{5}}}}$
8. Area between the graphs $y=x^2-2x+6$, $y=2x-3x^2-2$, $x=0$, $x=3$.
9. Sum of the longer sides of a right-angled triangle whose shortest side is 13.
11. Gradient of $y=x^3+x^2-x+4$ at (1,5).
12. $\dfrac{1}{\sqrt{\pi}}$
13. Value of a if $(3x-2)$ is a factor of $9x^4-6x^3-3x^2+ax-14$.
14. Difference of two cubes, the difference of the original numbers being one-third of the digit sum of the answer.
15. 43'rd term of the A.P. whose 3rd term is 8 and whose 7th term $=4$ (2nd term).

CLUES DOWN

1. Larger root of $x^2-4x-47=0$.
3. 6th term of the series (1×2), (4×3), (9×4), . . .
4. Coefficient of x^5 in $(1+x)^8$.
7. First palindromic number greater than 300.
8. Reciprocal of 2·02.
10. $\log_p \dfrac{(p^2)}{4}$ if $\log_p 2=0\cdot6786$.
11. Smaller of two consecutive odd numbers whose squares differ by 480.
13. Cube root of x^3-3x^2+3x-1 when x is 30.

D.I.B.

THREE DIMENSIONAL CURVE STITCHING

In issue No. 71, the basic ideas of two dimensional curve stitching were discussed and no doubt many of you have made up your own designs. In issue No. 65, one example of three dimensional curve stitching was given and now the idea will be extended. Pegboard is best for this purpose as it already has holes in it and elastic thread rather than ordinary thread to give a more lasting model.

The simplest three dimensional form can be made using two pegboard squares separated by four screwed rods, with locking nuts above and below the boards. Many effective designs can be achieved on this as a base, perhaps the easiest being to stitch from one hole in a circle in the top to the corresponding hole in the bottom one, thus forming a cylindrical shape, see **Fig** 1. If the top board is now removed and rotated through 90°, a "cooling tower" shape is produced—mathematically this is known as a hyperboloid. Many three dimensional models to illustrate properties of 3-D trigonometry can be made using single threads on this two board system. Fig. 2 shows a pyramid illustrating vertical and slant heights and angles between edge and base and face and base.

Fig. 1

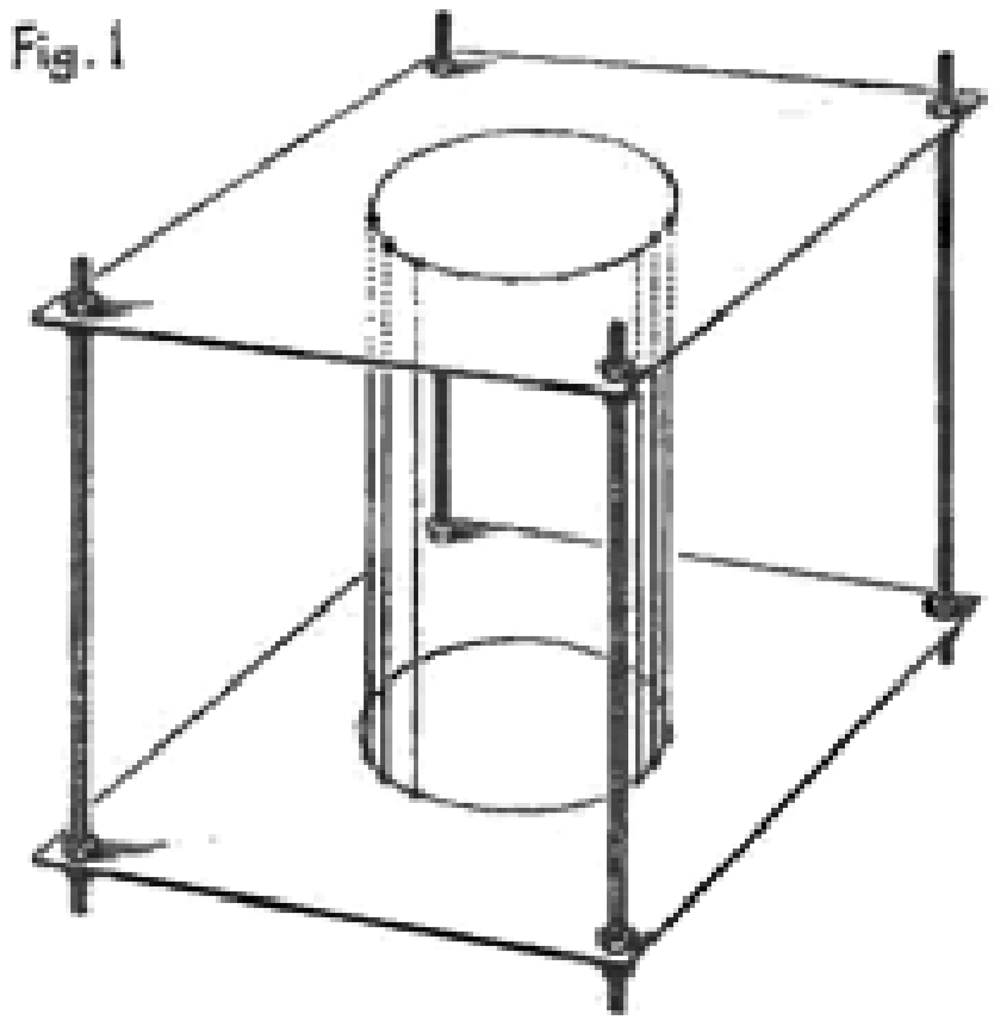
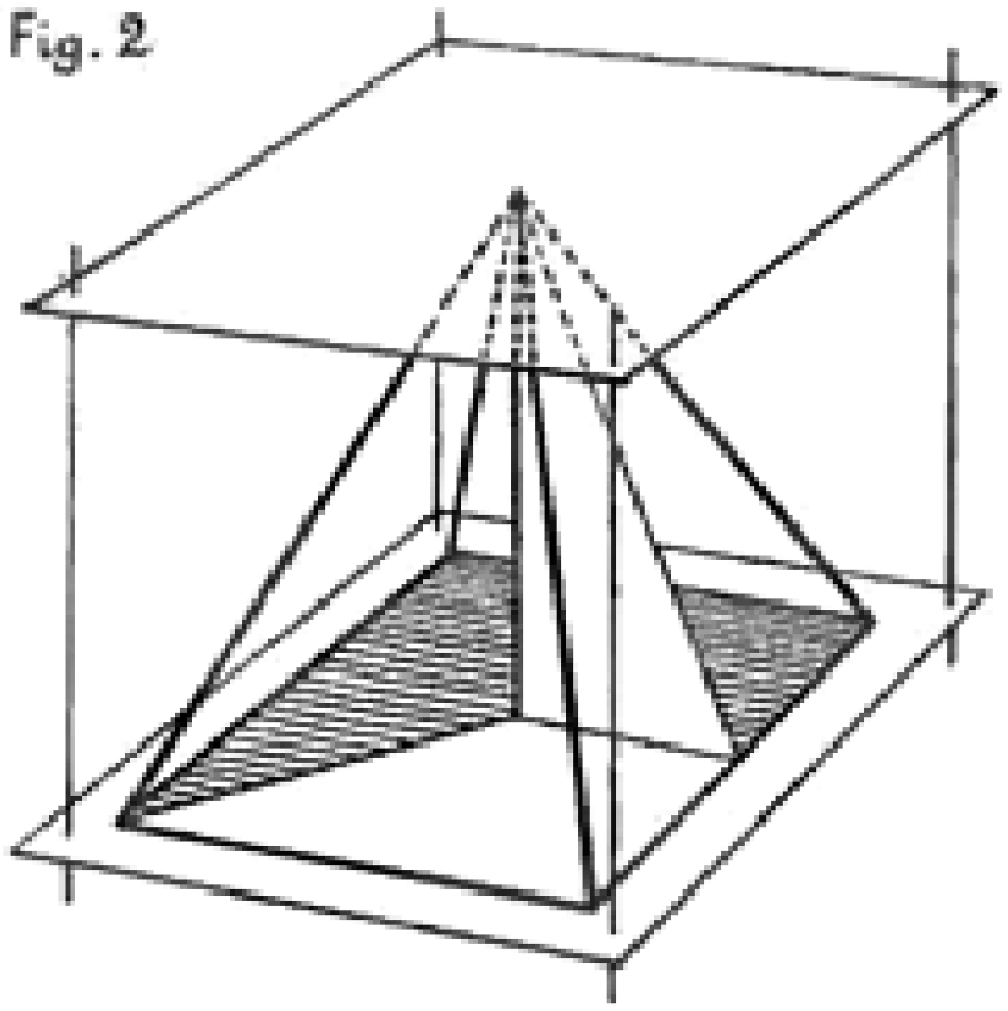


Fig. 2



If three boards are constructed as shown in Fig. 3 and fitted together, they form a solid angle. Many designs based on circles and straight lines can be produced—Fig. 4 shows a typical straight line design. Starting at the back of Board 1 (B1), tie a knot in the thread and stitch through Hole 1 (H1) of (B1), through H1 of B2, up through H2 of B2 to H2 of B3, back through H1 of B3 to H1 of B1. It is then taken up through H2 of B1 to H2 of B2, through H3 of B2 to H3 of B3, back through H2 of B3 to H2 of B1. This is continued connecting H3 of B1 to H3 of B2 which is already connected to H3 of B3, and so on. Use slight tension in the thread to keep it from sagging but do not pull it too tight and if joins are necessary, make them at the back of the boards. An alternative design can be made by using the diagonals of the faces instead of the horizontal and vertical lines, using the following sequence. H1 B1 to H1 B2, through H2 B2 to H2 B3, through H1 B3 to H1 B1, through H2 B1 to H2 B2, through H1 B2 to H1 B3 and then through H2 B3 to H2 B1, etc. The above sequence produces two triangles. The

design can be further enhanced by using two dimensional designs in the corners of the boards.

Circular designs tend to be more abstract, but are nevertheless most attractive if done properly. One such design would be to use two concentric circles on each board and follow the instructions :

a. stitch the large circles on B1 and B2 by joining each point on B1 to its diametrically opposite one on B2 and progressing in a clockwise direction on both boards.

b. stitch the small circles on B1 and B3 using the same principle.

c. stitch the large circle on B3 and the small circle on B2 using this idea of diametrically opposite points.

This means that the small circle on B2 is stitched twice and is one reason for recommending the use of pegboard with its "large " holes. Obviously the use of arcs against radii or diameters is another possibility and the number of designs is infinite. The editor would be pleased to receive photographs of your creations and will print suitable examples, and reward the sender with a book token.

P.J.G.

Fig. 3

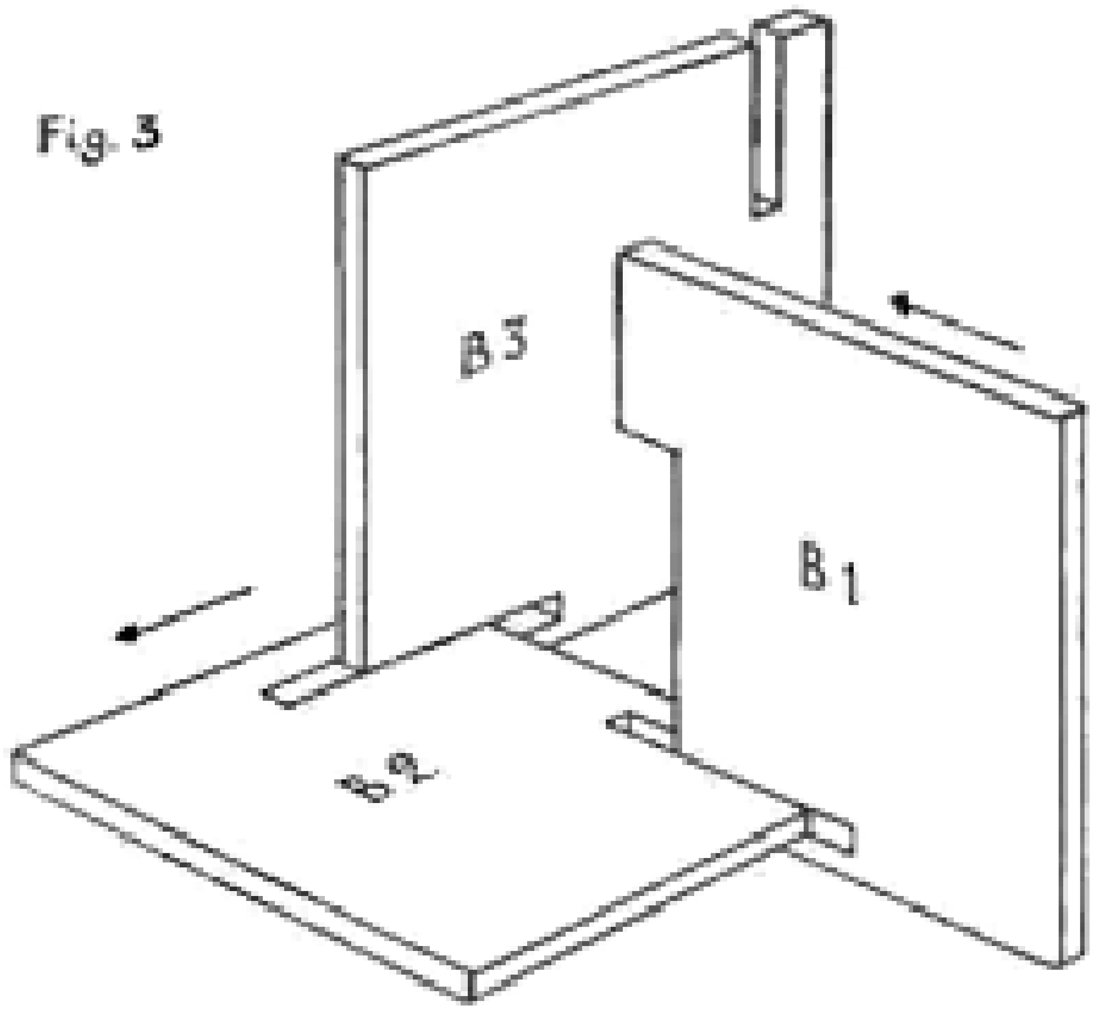
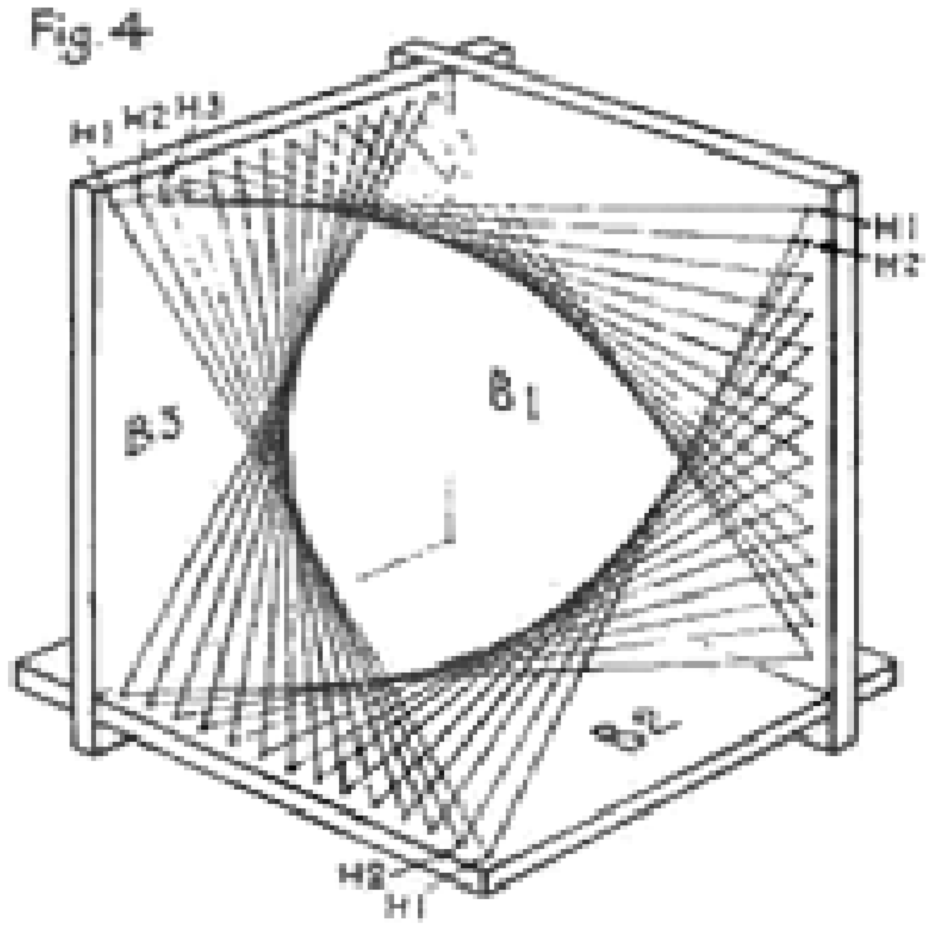
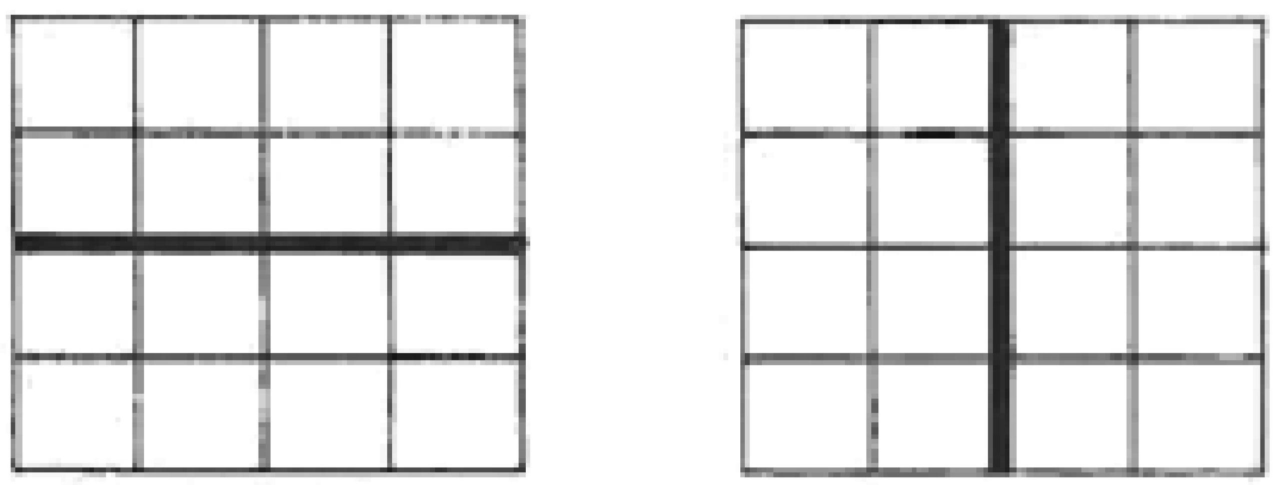


Fig. 4



HALVE IT



When asked to divide a four by four square into two equal parts, a boy produced the two diagrams shown on the left. It was pointed out to him that they were the same as the second is the first rotated through 90°.

He then considered other dissections along the lines of the squares and found four others which were different from the one shown and from each other. Can you ?

R.H.C.