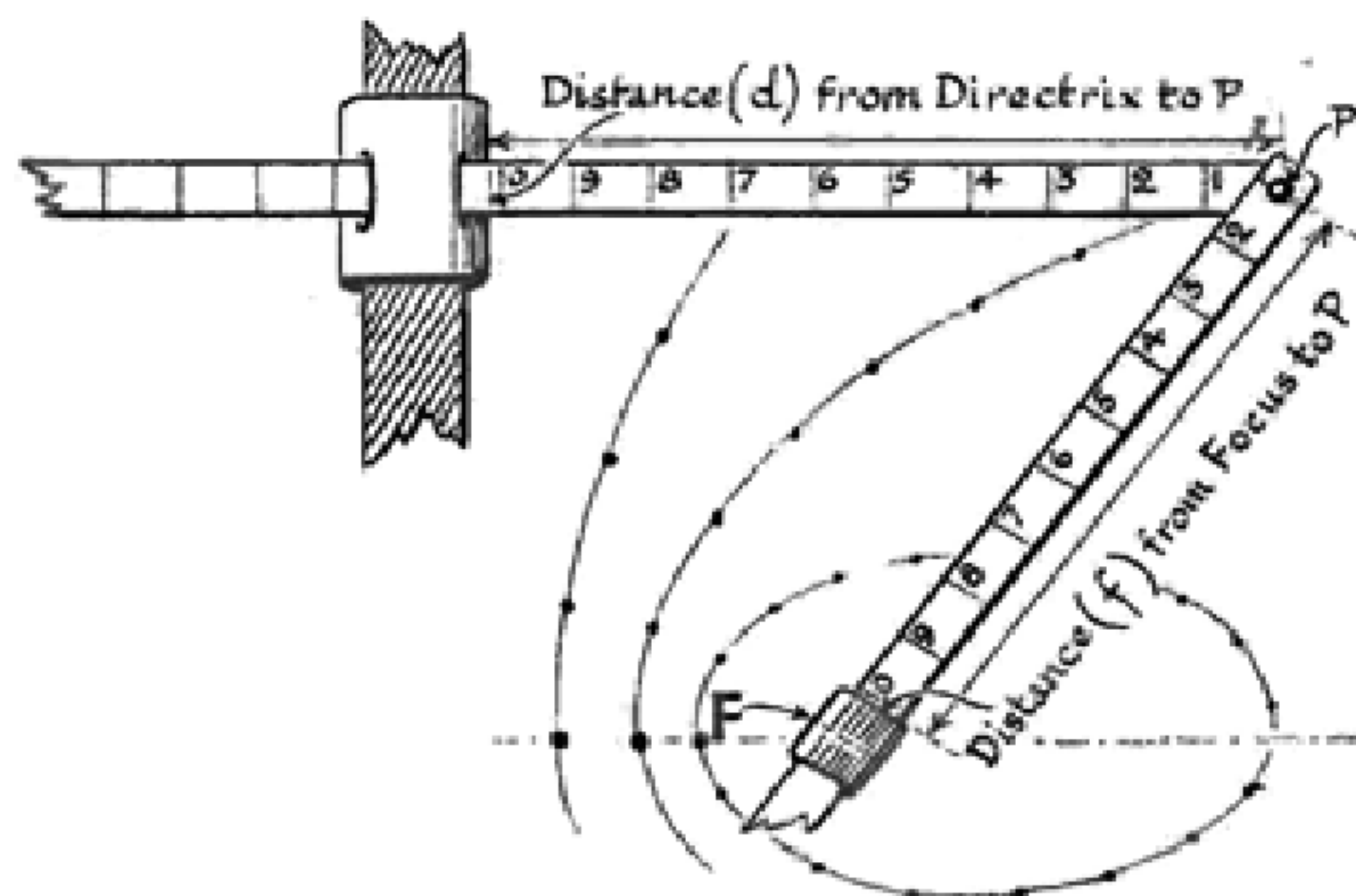
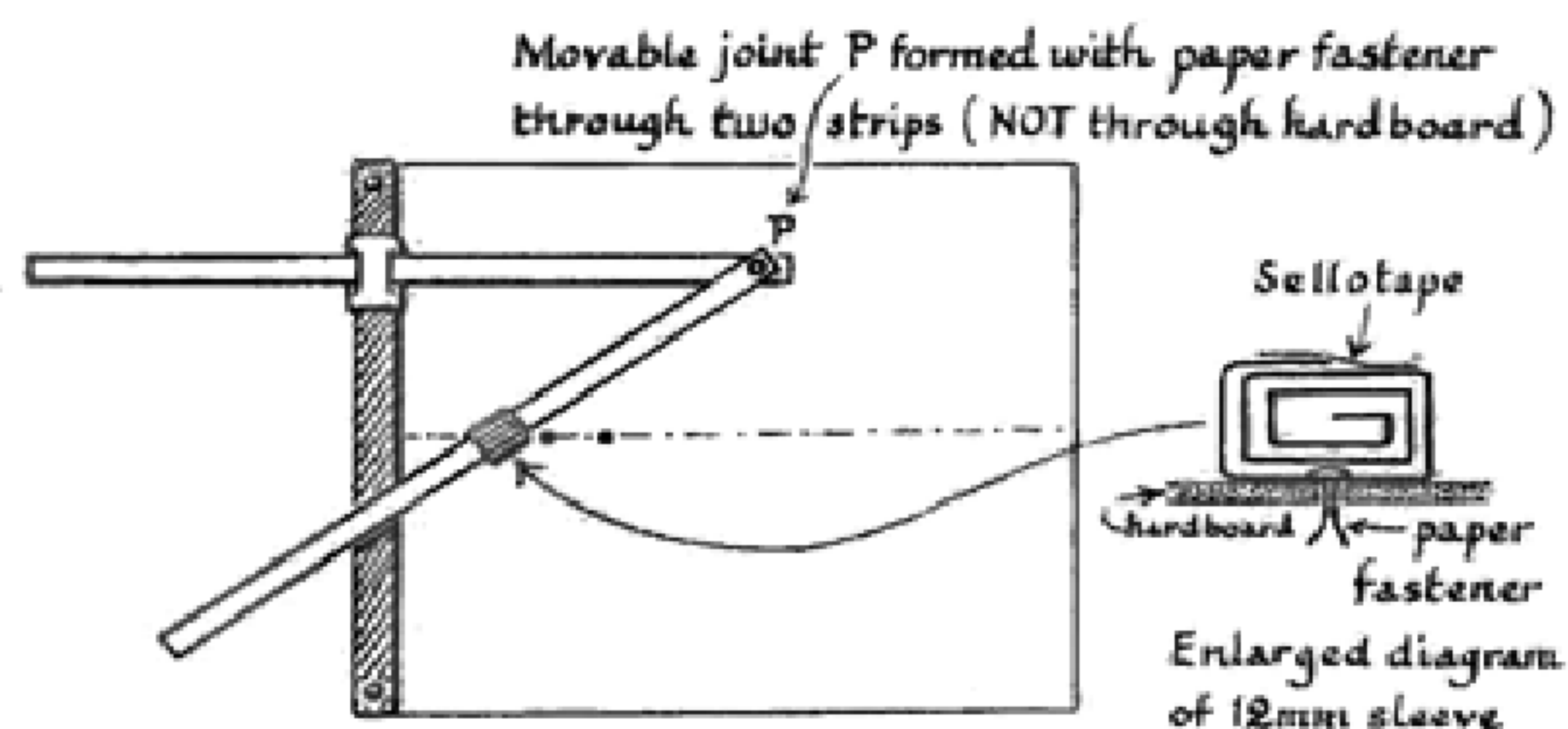
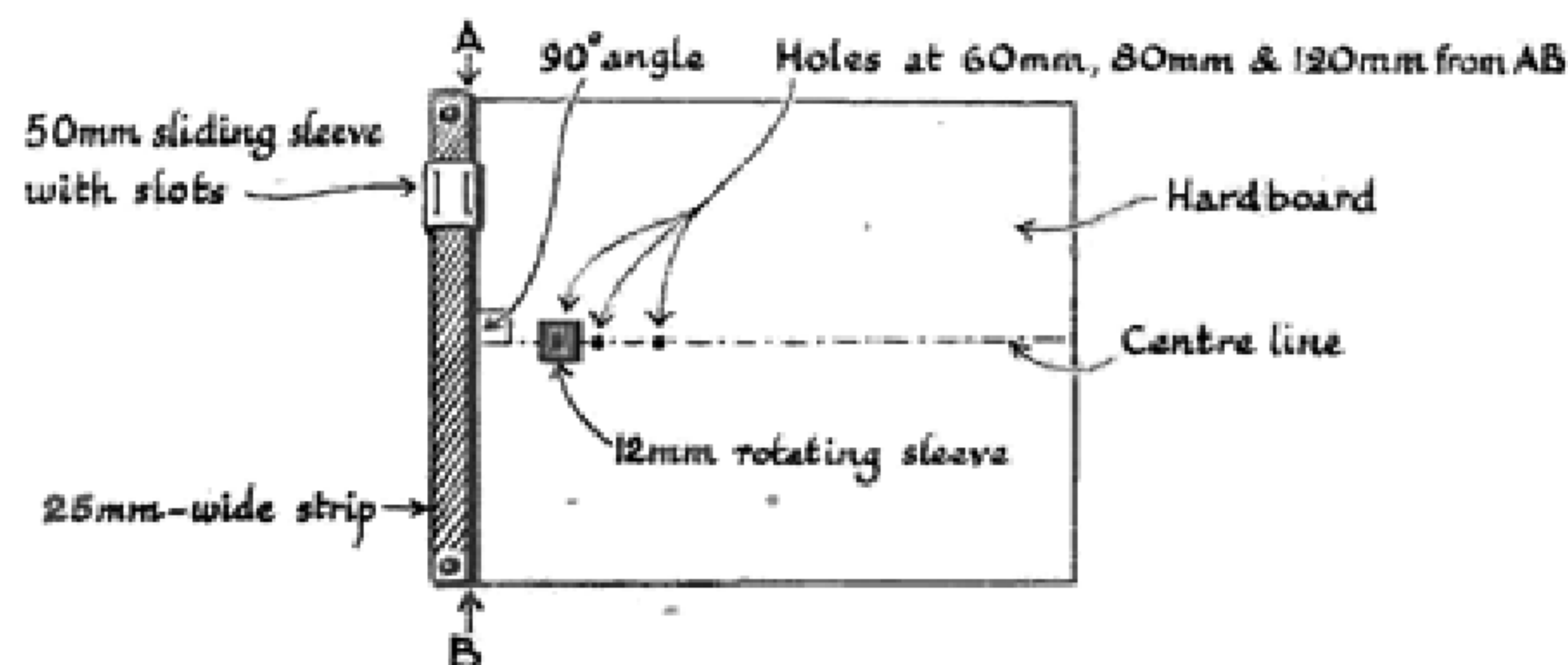


MAKE AN ECCENTROGRAPH



continued on page 554

join the mathematical pie
in the mathematical pie
the mathematical pie
mathematical pie



No. 70

Editorial Address: Alpha House, The Avenue,
Rowington, Warwickshire, England

AUTUMN, 1973

ON THE TILES



Cutie Pie was laying some tiles when she noticed that the number of square centimetres representing the area was equal to the number of centimetres representing the perimeter of the square tile. What was the length of the side of the tile?

What length of side of a tile would give the same relationship between the area and the perimeter of a rectangle formed by joining two identical square tiles?

Consider similar tilings of 3 and then 4 square tiles. Can you find a formula for such a tiling of "n" tiles?

D.I.B.

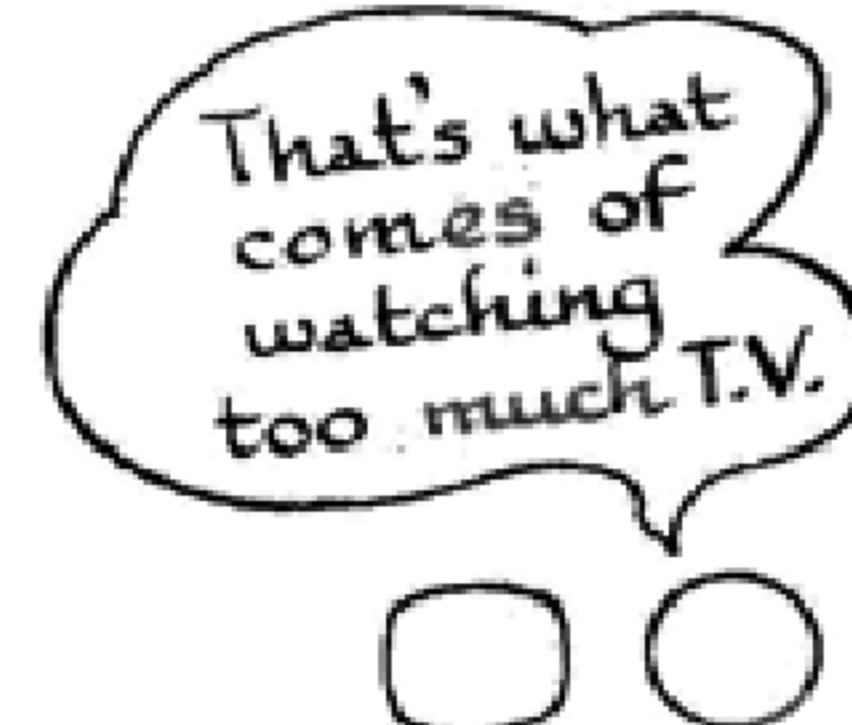
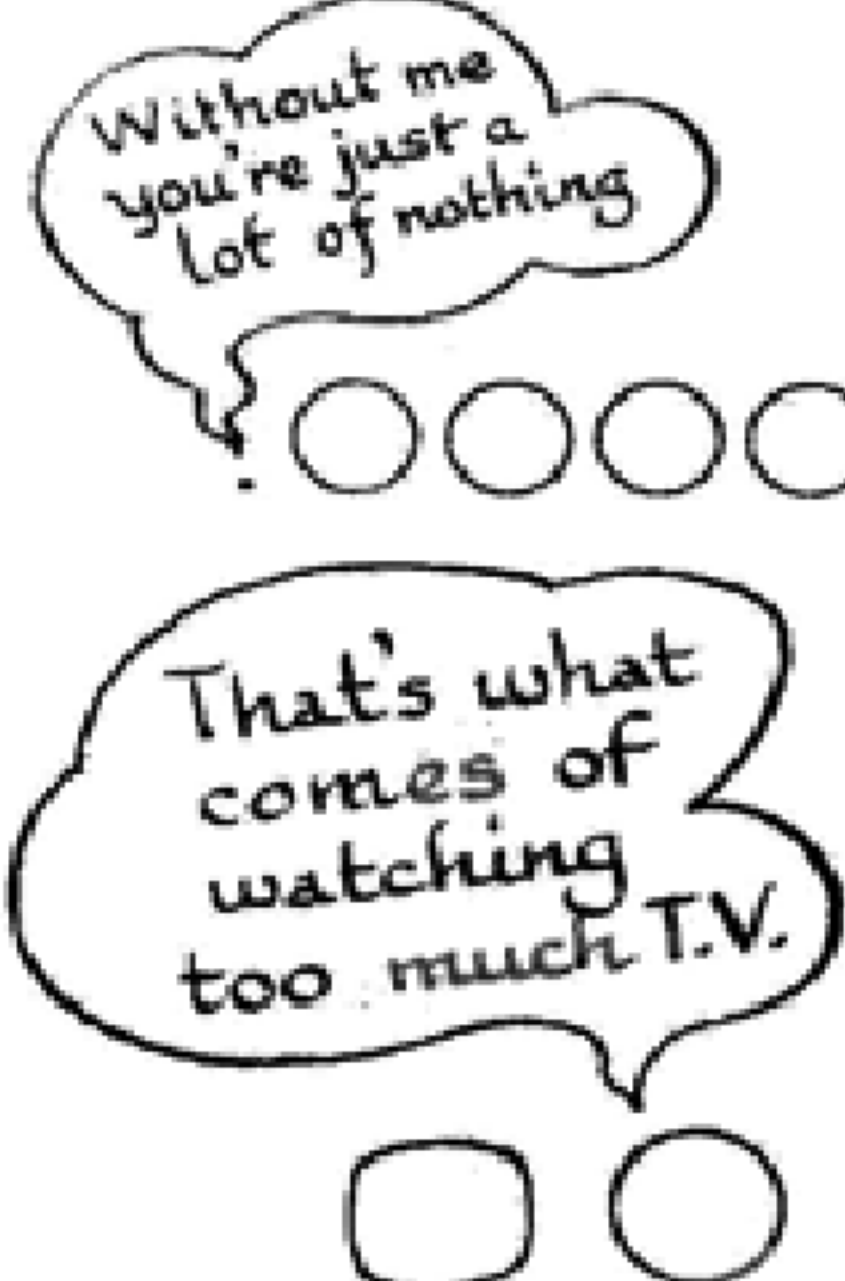


FIGURE IT OUT



111121111

THE CITY OF KONIGSBURG

submitted by E. N. Woodward of Middlesbrough

Times have changed in the City of Konigsburg. The citizens now take their families for a Sunday afternoon drive round the bridges. Unfortunately, like many other places, they often end in a traffic jam. Also to facilitate the traffic flow, the authorities have instituted a one-way system marked with appropriate signs. When the citizens are held up they use the one-way sign as a topological map of the bridges and plan their next turn.

Did someone say "Le plus ça change, le plus ce le même"?



REPLAYS

The number of matches played in the F.A. Cup competition in 1912 was 1st round 50, 2nd round 22, 3rd round 10, 4th round 6, semi-final 3. How many of these were replayed ties?

P.J.G.

ODD OR EVEN

What is the difference between the sum of the odd and the sum of the even numbers up to 100?

R.H.C.

WRAP IT UP

A man has a number of books to pack into parcels. If he puts 3 or 4 or 5 or 6 books into each parcel, there is one book left over each time. However, if he puts 7 books into each parcel, there is no book left over. What is the least number of books for this to be true?

P.J.G.

BEHIND THE IRON CURTAIN

We all become used to a certain way of setting out our solutions to problems and tend to think that ours is the only way. In Eastern Germany, they have a way of setting out long multiplication which may be new to you. When we multiply 342 by 247 we set it out as in fig. 1, but when Kleine Hans von Leibzig does it, he would set it out as in fig. 2.

$$\begin{array}{r} 342 \\ \times 247 \\ \hline 68400 \\ 13680 \\ 2394 \\ \hline 84474 \end{array}$$

Fig. 1

$$\begin{array}{r} 342 \times 247 \\ \hline 684 \\ 1368 \\ 2394 \\ \hline 84474 \end{array}$$

Fig. 2

$$\begin{array}{r} 4^{**} \times 3^{**} \\ \hline ***5 \\ 3*** \\ 8^{**} \\ \hline ***3^{**} \end{array}$$

Fig. 3

Unfortunately, he has a tendency to smudge his figures and one Solution appeared as Fig. 3 where each * represents a smudged figure from 0 to 9. Can you reconstruct his sum and solution?

R.M.S.

JUNIOR CROSS FIGURE No. 62

1		2	3		4
		5			
6	7		8	9	
10		11		12	
		13	14		
15					

Ignore decimal points and work to the accuracy stated.

CLUES ACROSS

1. Perfect squares between 140 & 170.
5. Maximum number of days in two consecutive months.
6. Date, in June, of the longest day of the year.

CLUES DOWN

1. Next three terms in the sequence 1, 4, 9, ...
2. Product of two primes which have one digit in common.
3. $x^3 + 1$ if x is an integer such that $4 < x < 6$.
4. IX \times III "translated" into normal numbers (including the sign!) followed by the answer.
7. Sum of two squares.
9. Half a "ton" plus 1.
11. $x^4 - 2x^2 + x - 4$, when x has the same value as in 3 Down.
14. Number of pounds in half of 1 cwt.

P.J.G.

IN THE SWIM

Two men can each swim at a speed of 3 m.p.h. in still water and agreed to race in a river which flowed at $1\frac{1}{2}$ m.p.h. The first man swam once across the river and back to the bank, heading straight across the river and allowing himself to be carried down by it. The river was $\frac{1}{4}$ mile wide and the second man chose to swim for $\frac{1}{4}$ mile upstream and then back. If they started together, who would cover " $\frac{1}{4}$ mile" first and by how many minutes would he win?

P.J.G.



SOLUTIONS TO PROBLEMS IN ISSUE No. 69

SQUARE PUZZLE

One complete solution appears at the top of the opposite page.

THE TERM EXAM

The sons received 23, 17, 17 and 7 square miles of land.

SENIOR CROSS FIGURE No. 65

Across: 1. 56; 3. 24; 5. 122; 7. 189; 8. 2572257; 9. 11; 10. 17; 11. 1283616; 13. 293; 14. 743; 15. 86; 16. 36.
Down: 1. 512; 2. 6251296; 3. 2857143; 4. 497; 6. 27183; 7. 12167; 11. 128; 12. 636.

JUNIOR CROSS FIGURE No. 61

Across: 1. 45; 2. 23; 4. 7337; 6. 12; 7. 77; 8. 39; 9. 97; 10. 2764; 13. 28; 14. 19.
Down: 1. 49; 2. 23; 3. 3377; 4. 72; 5. 77; 6. 1972; 8. 32; 9. 94; 11. 68; 12. 39.

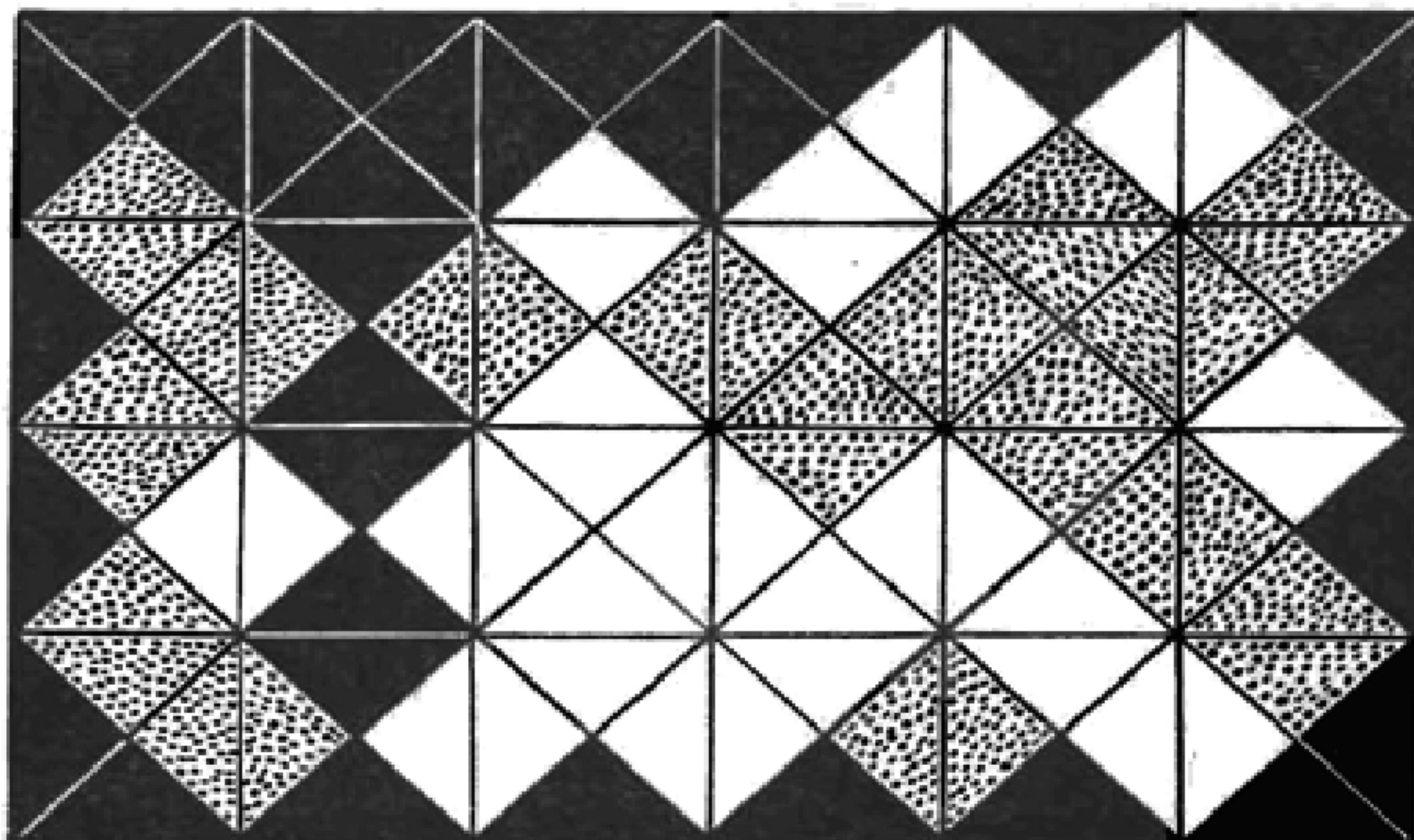
MATHEMATICAL CARS

The cars were Sunbeam Napier, Vauxhall Vector and Ford Angular.

A MATHEMATICAL CROSSWORD

Across: 1. ASSOCIATED; 8. OPERATE; 9. DELTA; 10. MEAN; 11. SKIT; 13. TRENDS; 14. CORRAL; 18. EVER; 19. STAT; 21. VERGE; 22. HORATIO; 23. SLIDERULES.
Down: 1. AVERAGE; 2. SPAN; 3. CHECKS; 4. ADDITION; 5. EULER; 6. COMMUTATIVE; 7. CALCULATION; 12. EDGESEND; 15. ROTATES; 16. TETHER; 17. CIRCL; 20. CRHL.

B.A.



MAKE AN ECCENTROGRAPH continued from page 556

Simply constructed from a piece of hardboard (400mm×350mm), three strips of thick card (one 350mm×25mm and two 350mm×12mm), some thin card and four split paper fasteners, this model can be used to construct ellipses, parabolas and hyperbolas.

From the thin card, cut a 50mm long sleeve which will slide along the 25mm wide strip: after cutting two slots which allow one of the 12mm wide strips to slide through it at right angles. Join the strip at the back with Sellotape.

Attach the 25mm-wide strip to the left-hand side of the hardboard with a paper fastener through a hole pierced close to each end. The edge AB of the strip is known as the *directrix*.

Again from thin card, make a sleeve which will allow a 12mm-wide strip to slide through it. Made from a treble thickness wrapping of card, this sleeve is attached with one fastener to one of the three holes on the centre line of the board: this fastener is at the *focus*. The 12mm-wide strips are marked out in centimetres as shown in the diagram.

To draw a parabola, first place a sheet of white paper on the board and hold firmly under the 25mm-wide strip and 12mm sleeve set in the first hole. Set both *f* and *d* at 3cm and mark the position of *P* on the paper. Reset both *f* and *d* at 4cm and mark the position of *P*. Repeat at cm intervals marking the positions of *P*. Join the points by a smooth curve, the locus is a *parabola*.

The ratio *f* : *d* is called the *eccentricity* of the curve. For a parabola, *e* is 1; for an ellipse *e* is less than 1; for a hyperbola *e* is greater than 1.

Taking *e* to be $\frac{1}{2}$, set *f* at 2 and *d* at 4, then at 3 and 6 respectively, and so on. This produces an ellipse.

Taking *e* to be 2, set *f* at 4 and *d* at 2, then at 6 and 3 respectively, and so on. This produces a hyperbola.

Try other eccentricities of your own.

D.I.B.

EQUALITIES

Are the following statements true?

(a) $1+2+3=4+5+6-3^2$ (b) $1+2+3+4=5+6+7+8-4^2$.

Now increase each of the above terms by 1 so that (a) becomes $2+3+4$ and $5+6+7-4^2$. Are the two equal?

Try the same thing with (b).

Add 2 to each term and see if the two sides remain equal. Experienced mathematicians are invited to generalise the result and prove it.

R.H.C.

COVER UP

A carpet for a rectangular room is to be made of strips cut from a roll and sewn together. The cut ends are to be bound. Which involves the least labour, to make the carpet with the strips running parallel to the length or parallel to the width of the room?

P.J.G.

SENIOR CROSS FIGURE No. 66

submitted by S. P. King, Royal Liberty School, Romford

1		2	3		4	
		5				
	6			7		8
9					10	
11		12		13		
	14		15			16
17			18			

Ignore decimal points.

CLUES ACROSS

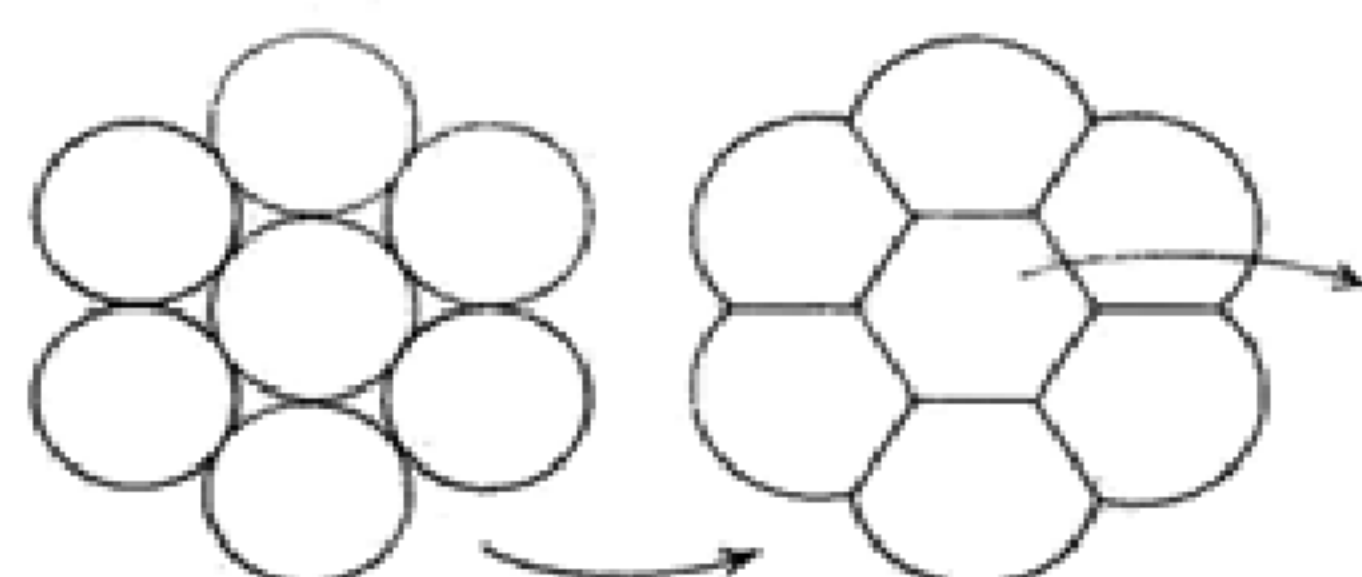
- A number whose digit sum is 21.
- The product of odd primes.
- The cosine of $64^\circ 6'$.
- The cube root of $x^3 + 6x^2 + 12x + 8$ if *x* is 16.
- $(a-b)(a^2+ab+b^2)$ if *a* is 10 and *b* is 1.
- The sum of the coefficients of x^4 in $(1+x)^6$ and $(1+x)^8$.
- If $(2^n)^3 = (2^{n+1})^2$, the value of either.
- The larger root of $2x^2 - 339x + 169 = 0$.
- x* if $\log x = \log\left(\frac{8}{9}\right) + \log\left(\frac{9}{20}\right) + \log(25)$.

- Palindromic multiple of 6 greater than 2,000.
- The smaller of two consecutive odd numbers whose squares differ by 384.
- $(x+y+z)^2 + 0.25$ if $a^2b^{-2}c^{-1} = \frac{b^xc^y}{a^z}$.

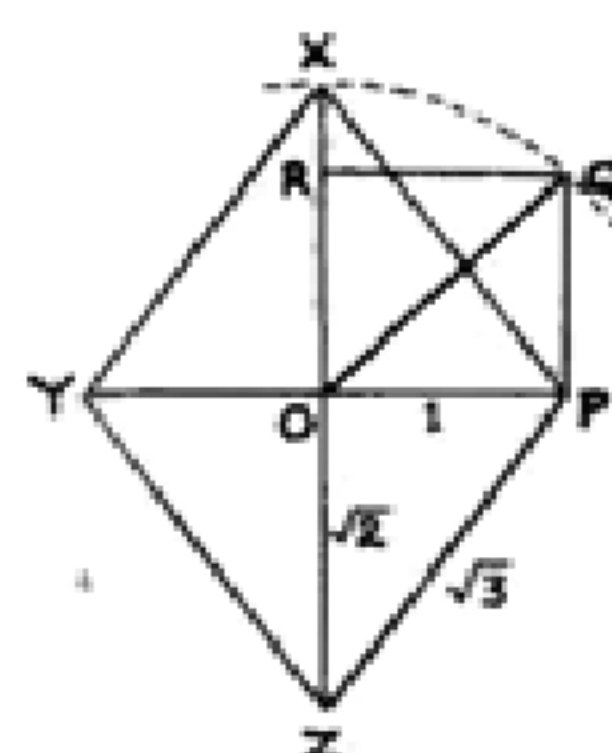
CLUES DOWN

- $5! - 4! - 3.3!$
- The 5th term in a geometric progression whose third term is 18 and the fourth term is 36 times the second.
- $(x+y)$ if $2x+y=49$ and $\frac{1}{2}x + \frac{1}{3}y + \frac{1}{6} = y$.
- The logarithm of $\sqrt{6194}$.
- $(x+y)^2$ if $x=91$ and $y=34$.
- A man buys *x* articles at 1½p each. If he had bought $\frac{2}{3}(x-1)$ articles at the same price the cost would have been 93p. Find *x*.
- The sum of the greater numbers in a Pythagorean triad whose smallest number is 9.
- x*, if $\frac{2(x-11)}{5} = \frac{x+5}{3}$.
- One more than the cube of a prime.
- The reciprocal of 0.9091.
- The 13th term in an Arithmetic Progression if the 3rd is 5 and the sum of the 5th and the 7th is 22.

The bee-cell



Rhombic Dodecahedron



$$\tan \hat{OYX} = \frac{\sqrt{2}}{1}$$

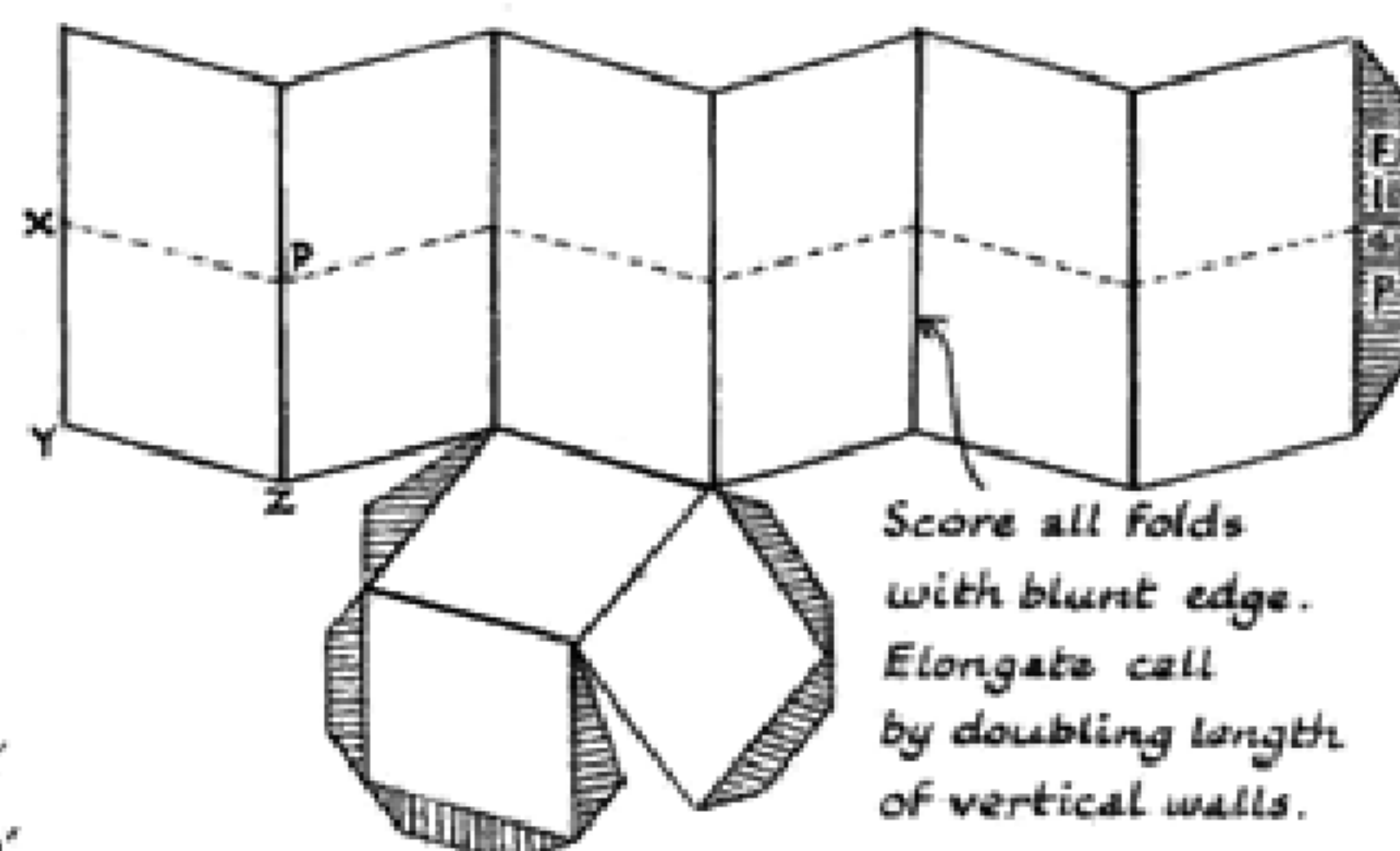
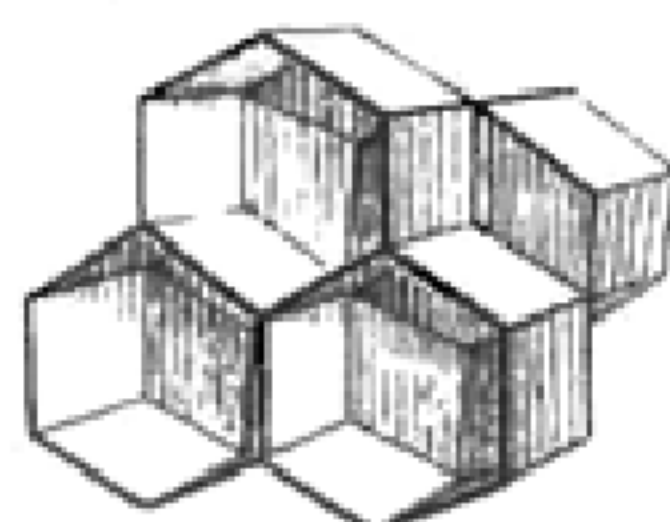
$$\therefore \hat{OYX} = 54^{\circ}44'$$

$$\therefore \hat{XYZ} = 109^{\circ}28'$$

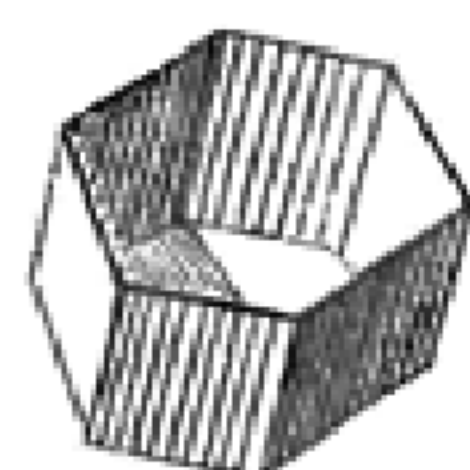
NET FOR CELL

Construct square OPQR of side 1 unit (say 15mm).
Produce OR to X such that OX = OQ. Produce PO to Y such that PO = OY.

PXYZ is the Rhombus from which the net is drawn.

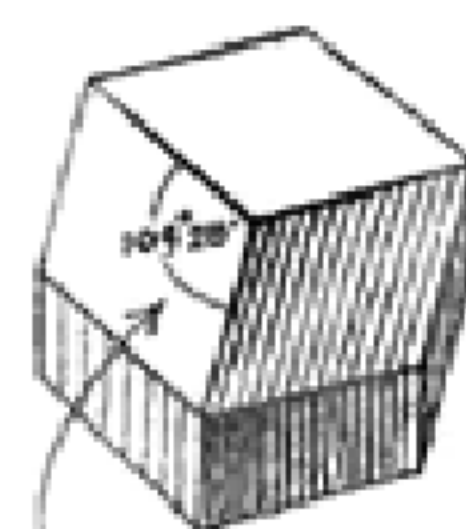


Score all folds with blunt edge.
Elongate cell by doubling length of vertical walls.



ONE CELL

All open-ended Rhombic Dodecahedron



BASE OF CELL

showing MARALDI ANGLE.

Examination of a pile of table-tennis balls or marbles will show that one sphere has contact with twelve equal spheres, six in its own plane with three above and three below. If the spherical surfaces are pliable, pressure acting inwards develops twelve plane surfaces at the points of contact, forming a rhombic dodecahedron. In this way, spherical globules of wax secreted by the bee produce the characteristic shape of the cell which provides maximum storage space (honey) with a minimum of material (wax). A further economy is achieved in the assembly of the cells which are laid almost horizontally in a double series back to back and offset: the three rhombuses which form the base of one cell form part of the base of three other cells. Such an arrangement strengthens the structure of the comb.

Maraldi was the first to calculate the large angles of each rhombic face and the apex angles of the end faces as $109^{\circ}28'$. This angle of natural fit has since been known as the Maraldi Angle.

Construct several identical cells from stiff paper and build them together to form a model of a piece of comb. Use a quick-drying adhesive cement.

D.I.B.

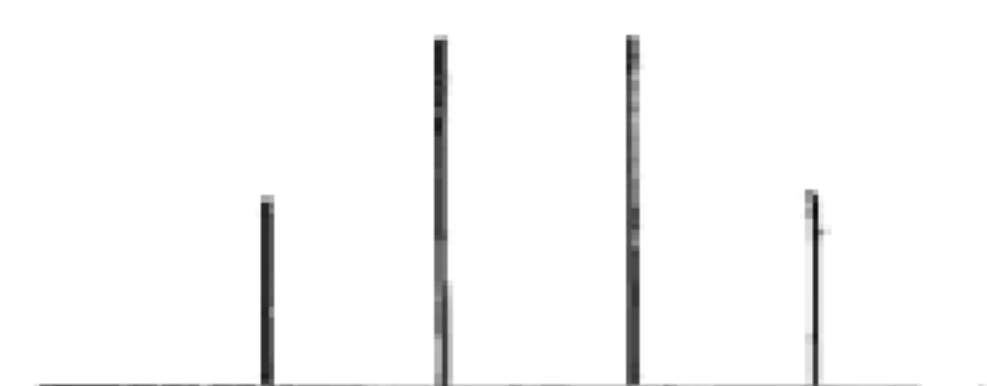
REVERSING TIME

How many times in twelve hours are the hands of a clock in a reversible position, i.e., if the hands are interchanged a true time is still shown?

P.J.G.

LETTER FROM AUSTRALIA No. 2

"Footy" mania is here again in Melbourne. To an Aussie, "footy" is not soccer, but a game rather like rugby—except that there are eighteen players on each side and the pitch is oval as well as the ball! The game is made very fast by kicking and "marking": when a player kicks the ball high and far towards a team-mate, that team-mate is allowed another unobstructed kick if he succeeds in catching the ball. Of course, opposing players are jumping for the ball too, and if they win possession may use hand-passes or kicks. The goalposts look like this.



Points are scored for a "goal" between the centre posts or a "behind" between the outer pairs. Here are some extracts from a newspaper report.

"There was absolutely no connection between the St. Kilda that meekly retired to a half-time deficit of 29 points and the team that blazed 18,5 in the second half for a 58 points victory. The "riot" began at half-time in the dressing rooms and the mass breakout soon after, at the start of the third quarter. The rejuvenated "Saints" kicked 11,3 in that 25 minutes and crushed footscray with their power."

The scores show	St. Kilda	1,2	4,2	15,5	22,7	139
	Footscray	3,0	9,1	10,3	12,9	81

How can you tell that the "Saints" "Blazed" 18,5 in the second half and kicked 11,3 in the third quarter? What do you think 18,5 stands for? Finally, how many points are scored for a goal and how many for a behind? Check your answers by calculating the Saints' half-time deficit.

G.E.