POWERFUL

We have received a letter from R. L. Kenyon pointing out that he has found a fourth solution to the three digit number which is the sum of the cubes of its digits, issue No. 66. The solutions that we had found were 153, 370 and 371 to which our correspondent adds 407. Well done!

The problem of the two digit number which is the sum of the squares of its digits could be tackled as follows.

Let the digits be a and b and the number is $10a+b$, so that $a^2+b^2=10a+b$, giving $a^2-10a+(b^2-b)=0$.

If we run through the values of b from 0 to 9, we get a succession of quadratics in a which need to have solutions from 1 to 9 inclusive which are whole numbers.

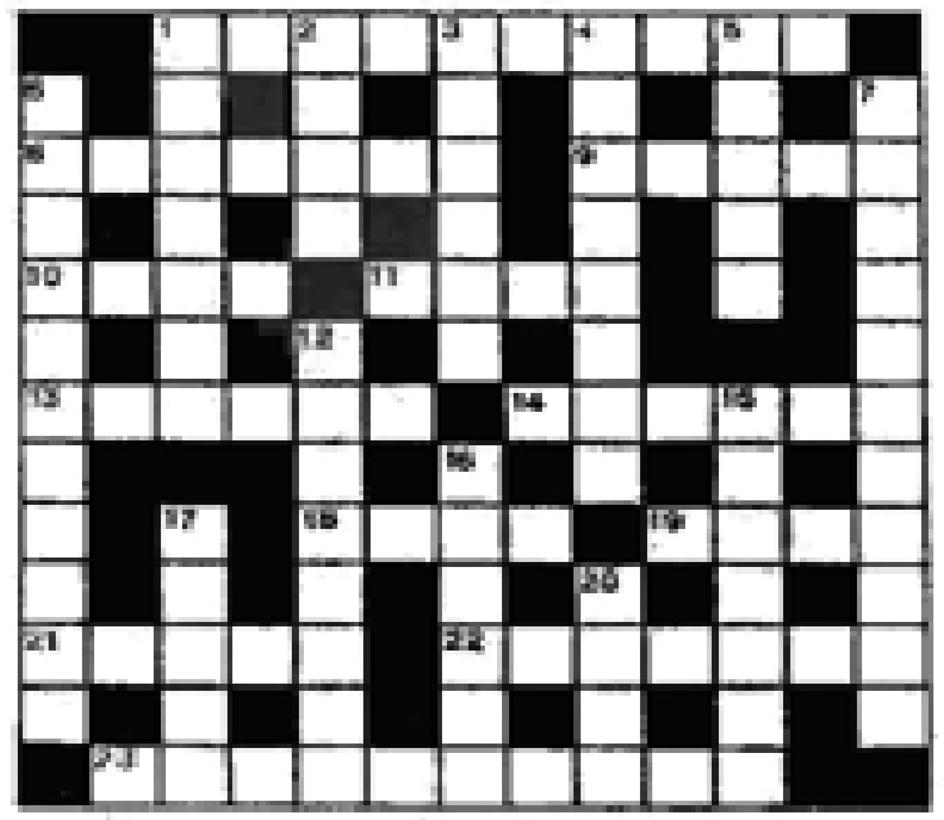
$b=0$ and $b=1$ both give $a^2-10a=0$ for which $a=0$ or 10.
$b=2$, 3, 4 and 5 lead to quadratics with non-integral roots and $b=6$, 7, 8 and 9 lead to quadratics with complex roots.

Thus there are no two digit numbers which are the sum of the squares of their digits.

R.M.S.

A MATHEMATICAL CROSSWORD

submitted by Lower V, Bournemouth School for Girls



Clues Across

1. $(a+b)+c$ is . . . with $a+(b+c)$.
8. You might be asked to . . . the process of addition on several numbers.
9. 4th letter of Greek alphabet represented by 3-sided figure.
10. Obtained by adding up a list of figures and dividing by the number of elements in list.
11. In code, these letters can be represented by : 19, 11, 9 and 20.
13. Graphs can be used to show these.
14. Decode : 5, 17, 20, 20, 3, 14.
18. ∞ means for . . .
19. First four letters of word used to describe a collection of numerical findings.
21. Arcs con . . . at a node.
22. HO + word describing a comparison of quantities = 1st name of a famous admiral.
23. These help you to calculate (two words).

Clues Down

1. Mode and mean are types of this.
2. Distance between thumb and little finger.
3. A good mathematician always does this to calculations.
4. The process of obtaining 246 from 123 and 123.
5. Mathematician who discovered $F+V=E+2$ or $R+N=A+2$.
6. The operation of subtraction is not . . .
7. The process of working out by arithmetic.
12. A vertex is a point where 3 or more . . . .
15. The earth does this on its axis.
16. You may be at the end of this when faced with maths homework.
17. A "O", minus the letter with four right angles.
20. Circumference × radius × height × length, symbolically.

join the mathematical pie

No. 69 *Editorial Address :* Alpha House, The Avenue, Rowington, Warwickshire, England SUMMER, 1973

SQUARES PUZZLE

developed from an idea by Spiller of 2W, Harold Malley School, Solihull

This puzzle is in two parts—the first is quite easy. Imagine a square with its diagonals drawn in, which is to be coloured in the following way : in any one of the four triangles, we may choose to leave the area blank, or to colour it with either of two colours, say red and blue—represented here by solid shading and stippling respectively. We may use blank, red or blue as many times as we wish, so that, for instance, the square may be coloured completely red. Two colourings are regarded as the same if they are rotations of each other, fig. 1, but as different if one is a "reflection" of the other, fig. 2.

How many different colourings are possible?

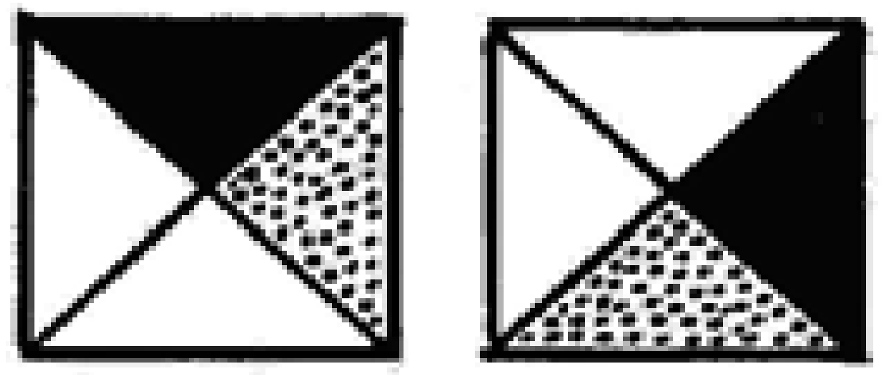


fig. 1

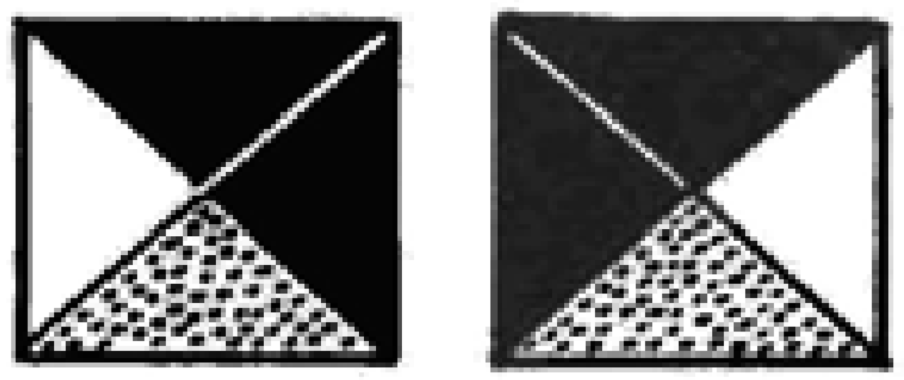
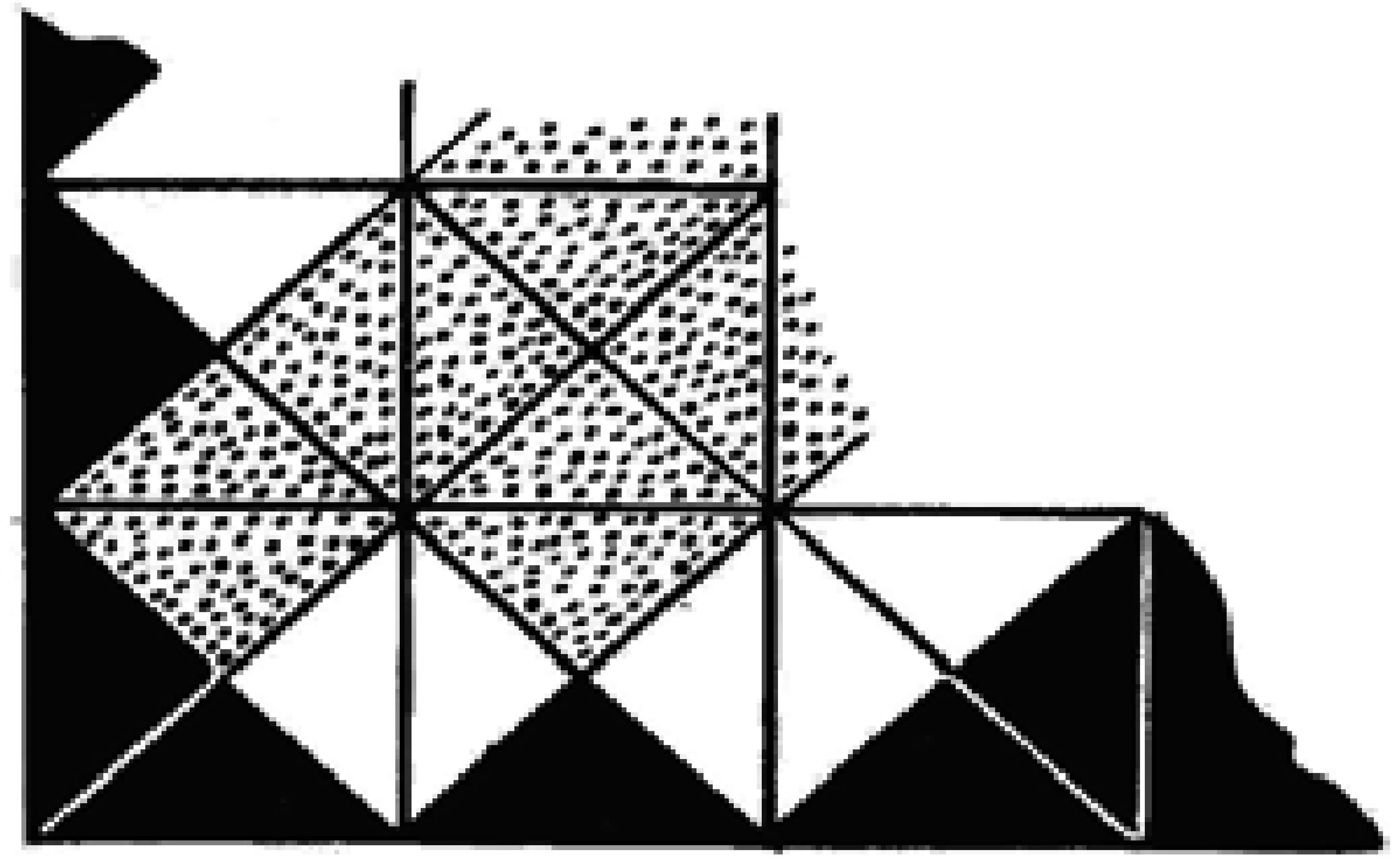


fig. 2



Now for the second part, which is a teaser. Draw a set of the "n" different squares, colour them and cut them out. Arrange them in a rectangle so that (i) all the outside edges are of the same colour, say red, *and* (ii) all the inside edges of the squares match edge to edge. The illustration shows part of one of the solutions, there are at least 30 but they are hard to find without careful thought. A complete solution will be shown in the next issue.

E.G.

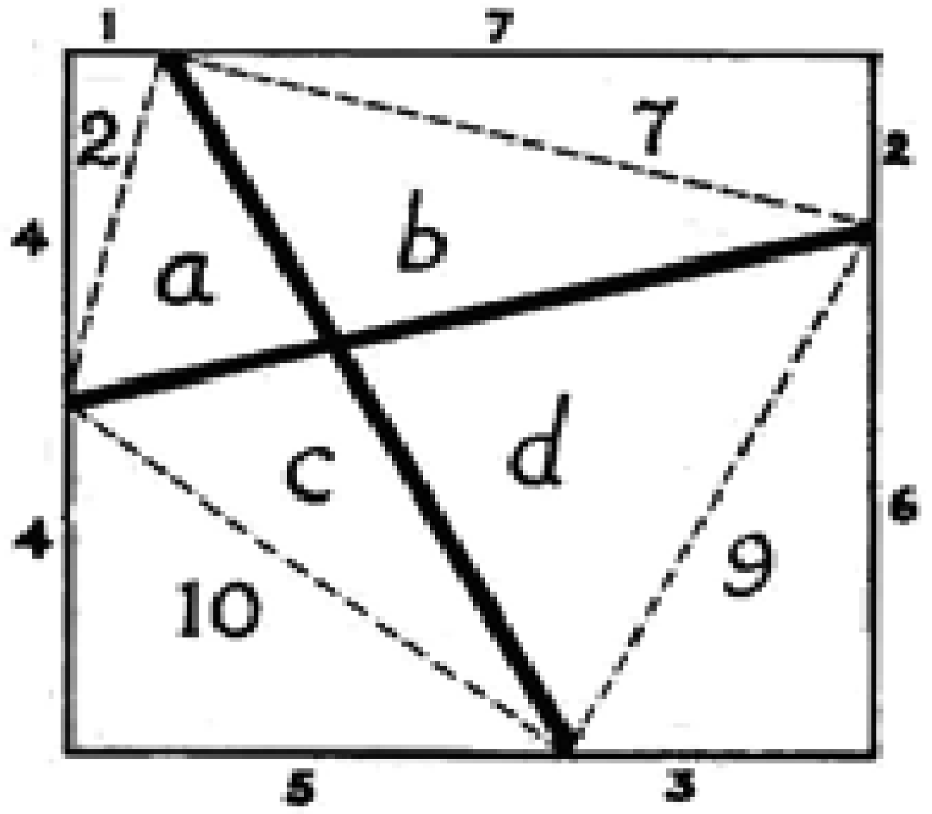
AN EXTRACT

from How to pass candidates, by W.G. adviser to S.P.E. (Society for the popularising of examinations).

THE TERM EXAM (MATHS.)

Here, the opening question should ease tension, be entertaining and helpful, and create pleasureful anticipation of Qs 2, 3, . . . We give an example.

Q.1. Read the following will and then either (a) calculate the ages of the sons (ten marks) or at least, (b) check the suggested answers ($3\frac{1}{2}$ marks).



" I, Squire of Squareisle, AITBY, 8, here in miles and square miles, indicate my plan, for four sons, a land share. To match age, at this date; which seems fair. To the eldest, I leave $(9+d)$, to the twins $(b+7)$, $(10+c)$; to the child $(a+2)$.

This, no doubt, some poor souls will need help to work out! a to b is as c is to d; b to d is as a is to c. Don't be driven round the bendo, for you just use addendo. Things like $(a+c)$ come so much easier, when you take 'this+that' from trapezia. Carry on, best of luck, Tom or Kate, you'll be tickled to notice the date! "

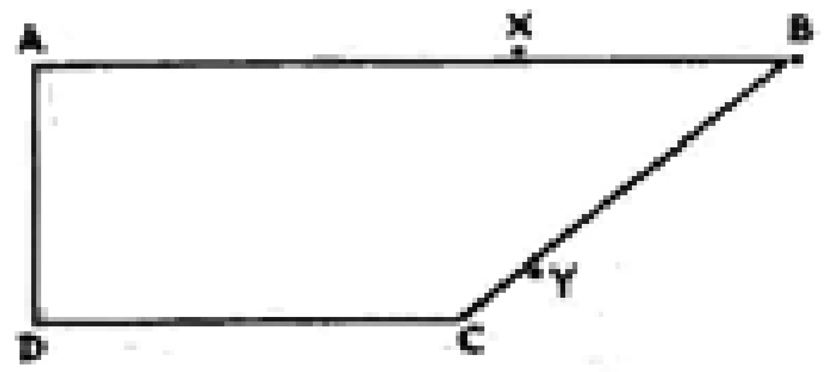
(*Signed*) SQUAREISLE, 23/7/1717.

(*submitted by W. F. Grieve, Harrogate*)

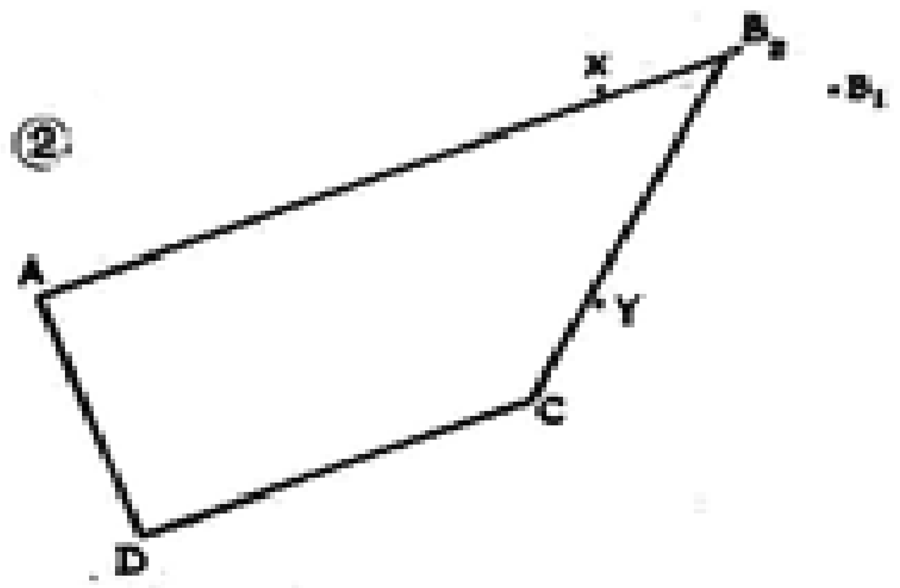
NOSEY

I wonder whether you will guess the outcome of this experiment before you complete it?

①



②



Take a small trapezium of card ABCD and place it on a clean sheet of paper. Mark two points X, next to the edge AB and Y next to the edge BC, on the paper.

Mark the point under B. Move the trapezium to some other position so that X and Y are still next to the sides AB and BC; rather as if the " nose " B were pushing through a hole XY: as in figure 2.

Plot the point under B again.

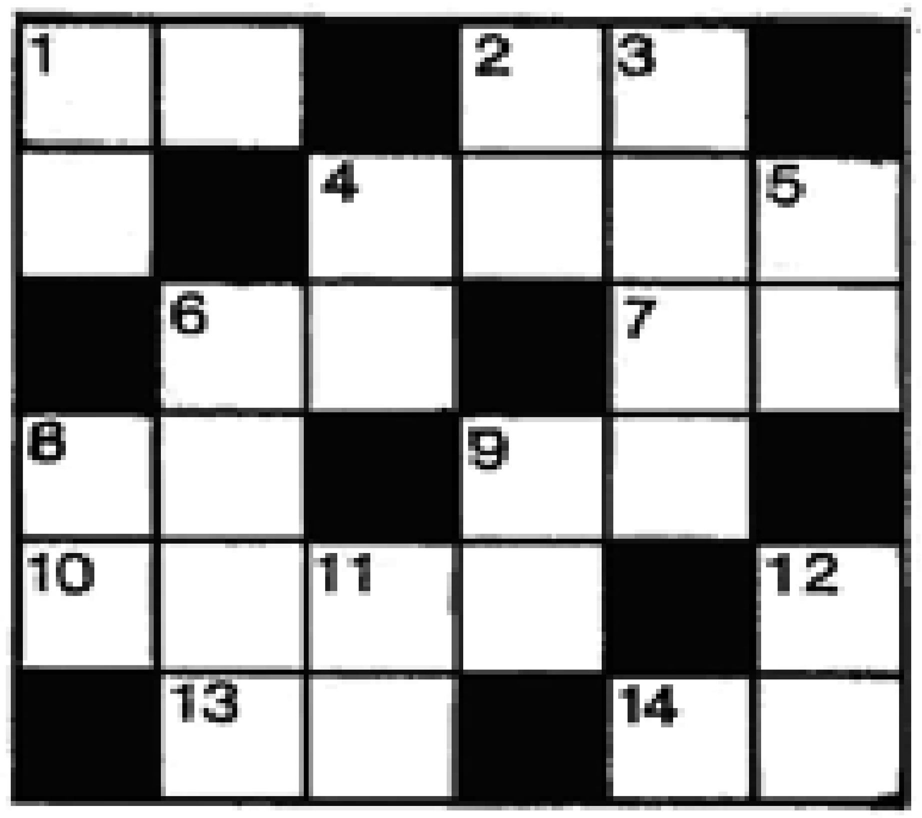
Repeat the procedure as often as you like. What figure results?

Now try pushing the " nose " C through the hole XY, but from the right-hand side this time. Can you name the two well-known properties of the completed figure which we have demonstrated?

More adventurous π addicts may like to investigate the effect of (a) varying the size of the " hole " using the same trapezium and (b) using the same hole but trying trapezia with different angled " noses ". Try drawing a graph to illustrate your results.

E.G.

JUNIOR CROSS FIGURE No. 61



Ignore decimal points, work to the required degree of accuracy.

CLUES ACROSS

1. Angle in degrees between the hands of a clock at 4.30.
2. One-third of this issue.
4. Sine of 47° 12′.
6. $(a+2b-3c)$ if $a=1$, $b=4$, $c=-1$.
7. Palindromic product of primes.
8. $(2n)^2+n$ where n is prime.
9. The last two digit prime.
10. Consecutive cubes.
13. Minimum days in a month.
14. Number of vowels in the Editorial address.

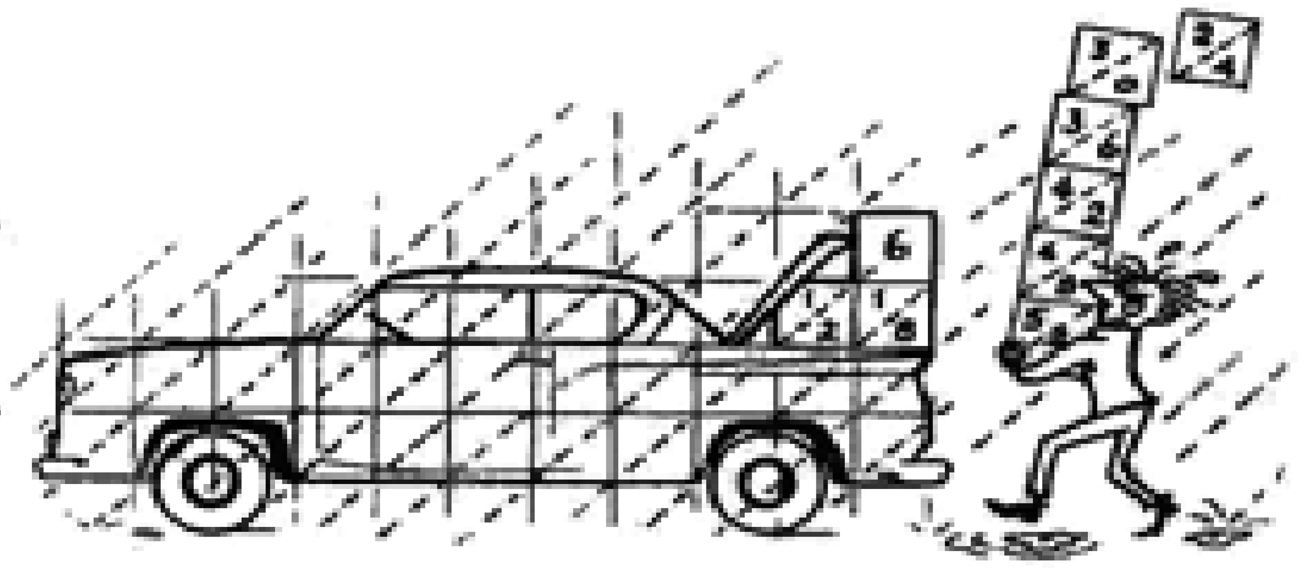
CLUES DOWN

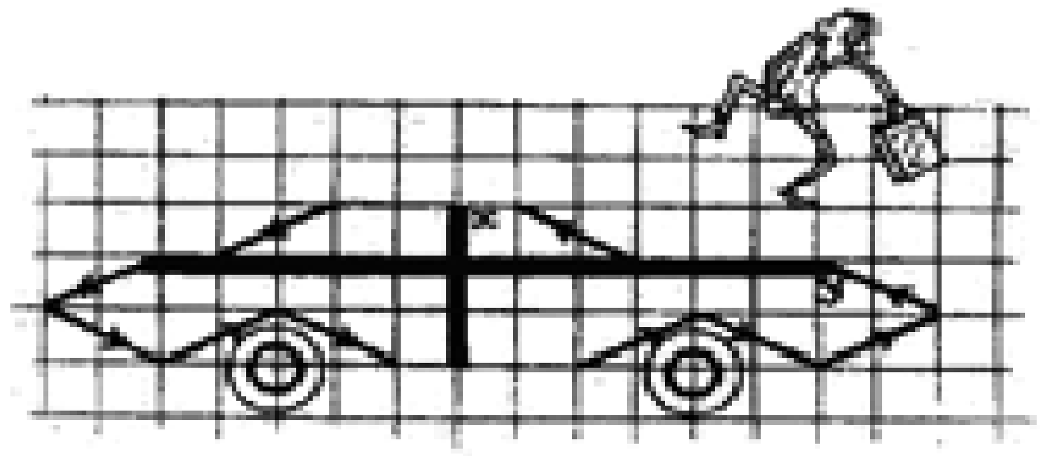
1. The sum of the other sides of a Pythagorean triangle with the smallest side 7.
2. The first prime whose digits are consecutive.
3. " Anagram " of 4 across.
4. Half a gross.
5. See 7 across.
6. The year of the last Olympics.
8. $2ab^2$ (see 6 across).
9. (Twice the square of a prime)—(first prime)^2.
11. The next term in the series 5, 8, 13, 20, 29, 40, 53,
12. The difference of two squares.

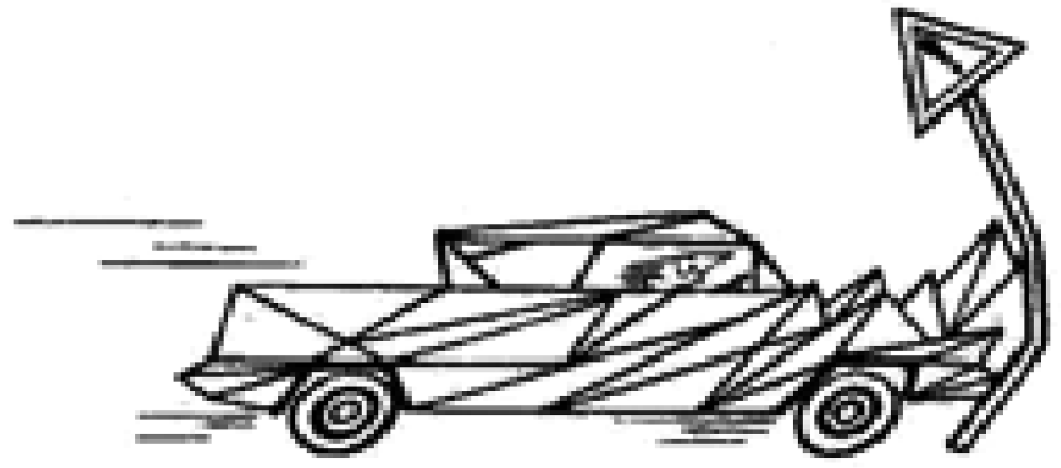
P.J.G.

MATHEMATICAL CARS

After mathematical Inn signs, C. A. Webster of Exeter suggests mathematical cars. A book token will be awarded to the sender of any further examples that are used in future issues.—ED.







SOLUTIONS TO PROBLEMS IN ISSUE No. 68

VECTOR GRAPHS

The shape that is drawn is π.

MAP READING

The area is 3 km².

POLLUTION

The tide was running at 5 ms⁻¹.

SENIOR CROSS FIGURE No. 64

Clues Across: 1. 22; 3. 525; 8. 111; 8. 24; 9. 1921; 11. 1968; 13. 10; 14. 512; 16. 131; 17. 30.

Clues Down: 1. 216; 2. 21; 4. 221; 5 54; 7. 1169; 10. 2685; 11. 103; 12. 120; 13. 11; 15. 13.

JUNIOR CROSS FIGURE No. 60

Clues Across: 2. 425; 4. 49; 6. 1251; 7. 2001; 8. 31; 9. 167.

Clues Down: 1. 64125; 3. 2021; 5. 51213; 6. 1066.

B.A.

Fig. 3 uses slightly different edges, but the rule is very much like that for the others. Is this pattern a glide reflection? Try some other patterns yourself, comparing the effects of the two different rules. Does the second rule always give integers? Do other variations of the rule still produce patterns?

E.G.

LETTERS TO THE EDITOR

Since we wrote in issue No. 67, we have had letters from a number of readers. Firstly, we spelt " justify " wrongly in issue No. 67 (jsutify); thank you M. Chapman, Southampton; Paul Grilli, Loughborough; A. Norton, Maidstone; and Miss C. Redhead, Norbury, S.W.16.

" Sort it out " produced $3.6.1+11=69$ and $3.6.1-1166=X$ from O. N. Hemming, Harold Malley Grammar School, Solihull; and $\frac{.9}{.1}=3+6=9$ and $9\div 1=3+6=9$ from P. M. Knight of the same school.

" A Cutting Problem " in issue No. 64. P. Handley of Prior Park College suggests that the third part of the solution should be 8 and not 7. We think that he is on to something and would like a few more readers to look at this problem again. Could P. Handley and Mathematical Pie both be correct?

D. Moore, Isle of Wight, sent a Flow Chart for the theorem of Pythagoras which will be commented on in a future issue. D. Mackinder, Edinburgh, sent two Flow Charts; the first on Noughts and Crosses seems to have been condensed to such a degree that it may be understood only by a programmer, and the second one on the solution of a Quadratic is mathematically invalid for $b^2-4ac=0$, although apart from this his additional steps are justified.

Miss T. Petty, Brighton, also spotted the mistake 1473 instead of 1575 in " Multiplication Problem " in Issue No. 66.

Richard Hook, Stroud, spotted the missing square root sign in the Flowchart in issue No. 67 but this was not the deliberate mistake but due to artistic licence. J. Taylor, Solihull, sent some suggestions for flowcharts and tessellations.

D. Eagle, Pudsey, noted that " Cutting the Mint ", issue No. 66, had already been " A Cutting Problem " in issue No. 64.

R. L. Kenyon, Aigburth, referred to the problem " Powerful ", issue No. 66, and then proposed that every multiple of 6 has a prime number next to it: what about $6\times 20\pm 1$?

E. Woodward, Hustler School, Middlesbrough, has " signposted " one way(!) of looking at the Konigsburg bridges.

B. K. Booty, Wolton-under-Edge, sent us some suggestions for " Sort it out ", issue No. 67, including a musical one, " Powerful Numbers ", issue No. 66, and a portrait of a Mathematician.

Finally, C. Webster, Exeter, made the suggestion about Mathematical cars which you will find in this issue.

RANDOM THOUGHTS ON π

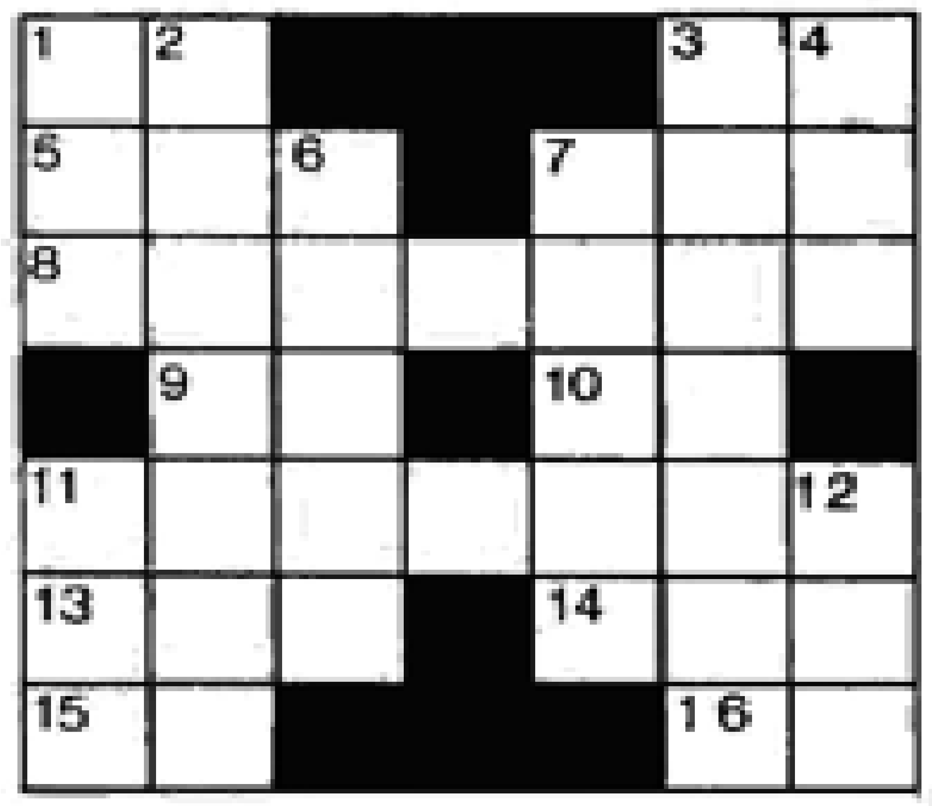
based on a thesis by Mrs. H. B. Turner, Denton, Manchester

De Morgan was explaining to an actuary the chance that a certain proportion of a group of people would be alive at the end of a given time, and he quoted the actuarial formula, which involves π. He explained that π was the ratio of the circumference to the diameter of a circle. His acquaintance interrupted, " My dear friend, that must be a delusion; what can a circle have to do with the number of people alive at the end of a given period of time? " The definition of the ratio of circumference to diameter for π is not the only one possible and does not give the best analytical definition.

If sticks are dropped on to a plane marked with parallel lines, spaced the same distance apart as the length of the stick, the chance that a stick will fall on a line is given by the ratio $2:\pi$. This does not give a very good approximation for π unless millions of observations are taken.

If two numbers are written down at random, the probability that they will be prime to each other, i.e., have no common factor other than 1, is $6:\pi^2$. This is an interesting exercise to carry out with your friends. Ask 25 of them to write down 5 pairs of numbers each, without saying why you want them. From your results you can find an approximate value of π.

SENIOR CROSS FIGURE No. 65



Ignore decimal points, work to the required degree of accuracy.

CLUES ACROSS

1. Coefficient of x^3 in $(1+x)^8$.
3. Exterior angle of a 15-sided polygon.
5. $(9 \text{ across})^2+1$.
7. Reverse gravity.
8. Roots of $x^2+20x-58=0$, larger first.
9. The smallest palindromic prime.
10. A prime which is the sum of four non-equal primes.
11. Angles of a triangle in the ratio 32:9:4.
13. 20° below absolute zero.
14. A prime number whose last two digits are consecutive and add up to the first digit.
15. The 13th term of an Arithmetic Progression with the first term 2 and common difference 7.
16. Nine hands?

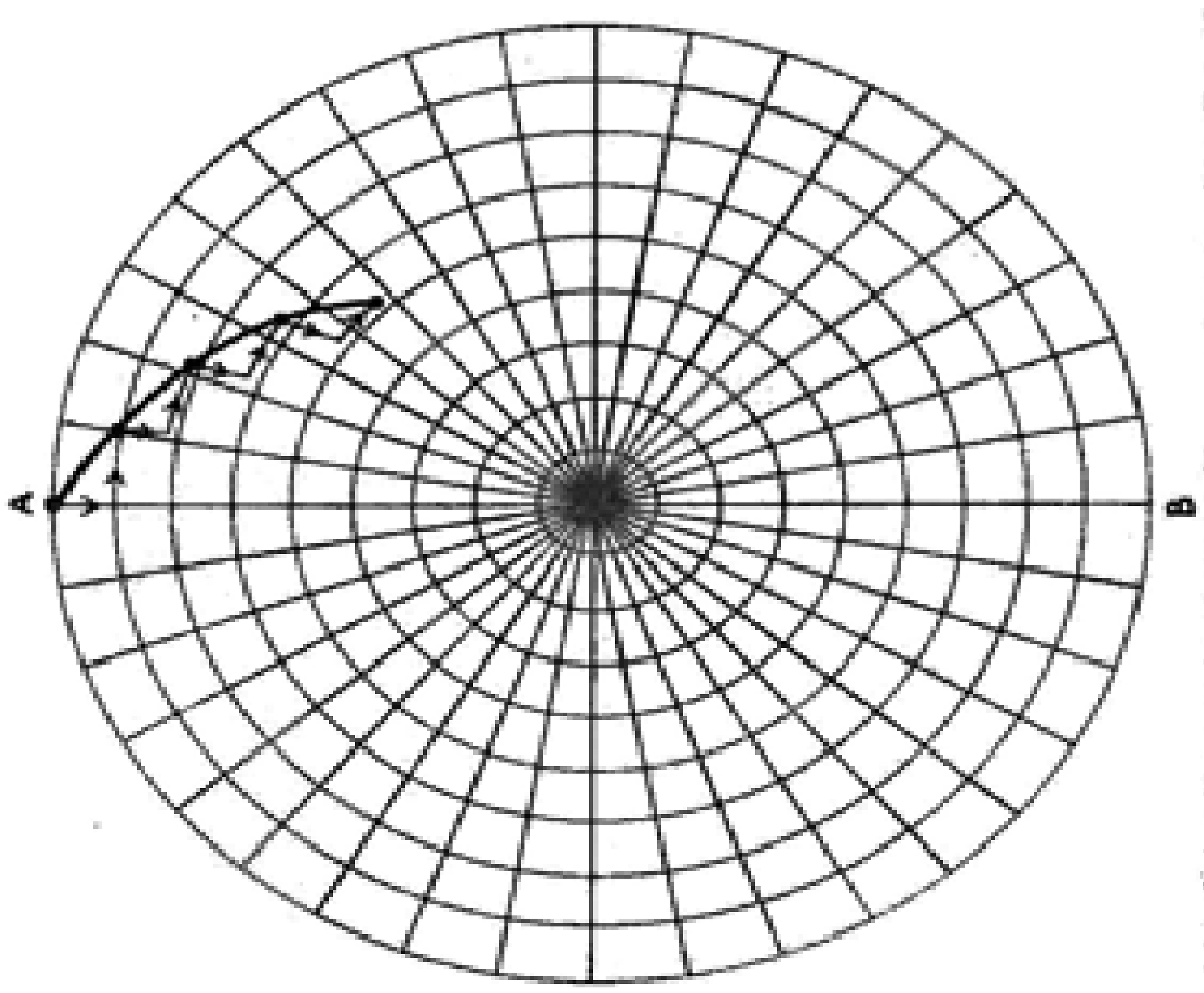
CLUES DOWN

1. 2^n where n is a square number.
2. The next two terms in the sequence 1, 16, 81, 256,
3. Twice the probability of obtaining the same score when a 7-sided die is thrown twice.
4. The first two digits form the square of the third.
6. " e " to four decimal places.
7. $(4!-1)^3$.
11. 2^n when n is prime.
12. The sum of the first 24 terms of an A.P., whose 5th term is 4 and whose 10th term is 19.

P.J.G.

E.G. who is a member of the Editorial Board has moved to Australia to become our first overseas member. This is the first of many, we hope, letters from down under.

Flying to Melbourne via Singapore, there was little of Mathematical interest except noticing that it was 02.30 G.M.T. when the sun rose as we approached Bahrain. However, I was delighted to see, when strolling about the old quarter of Singapore, that shopkeepers have an abacus on their counter to calculate the shoppers' bills.



Since arriving, our most interesting discovery was a revolving restaurant in Katoomba, in the Blue Mountains near Sidney. A circular area of the floor in a square room revolved so that the diners all had a chance of appreciating the view from the windows. Unfortunately visibility was poor due to low cloud, and so we were more interested in watching people walk across the circle. Although the angular velocity was not great, it was still necessary to walk in a curve and I began to wonder what would happen if the rotation were speeded up.

If your Mathematics teacher can spare some polar graph paper it would be useful, otherwise draw ten concentric circles, equally spaced and radii every 10 degrees as shown in the diagram.

Suppose that someone wishes to walk across the circle from A to B. If he takes a step equal to one-twentieth of the diameter AB each time the circle revolves by ten degrees, what will be his likely path? We will suppose that he looks up after each step and takes his next step in the direction of B. The beginning of his walk is shown. If you continue this you may get a surprise! Try to discover what will happen if the circle revolves only five degrees at each step. Suppose he just closed his eyes and walked in a " straight line "? It's quicker by Boomerang!

Best wishes

E.G.

An interesting type of number pattern was described recently by Martin Gardner in " Scientific American ". Take a row of zeros and ones :

0 0 0 0 0 0 0 . . . . . .

1 1 1 1 1 1 1

Fig. 1

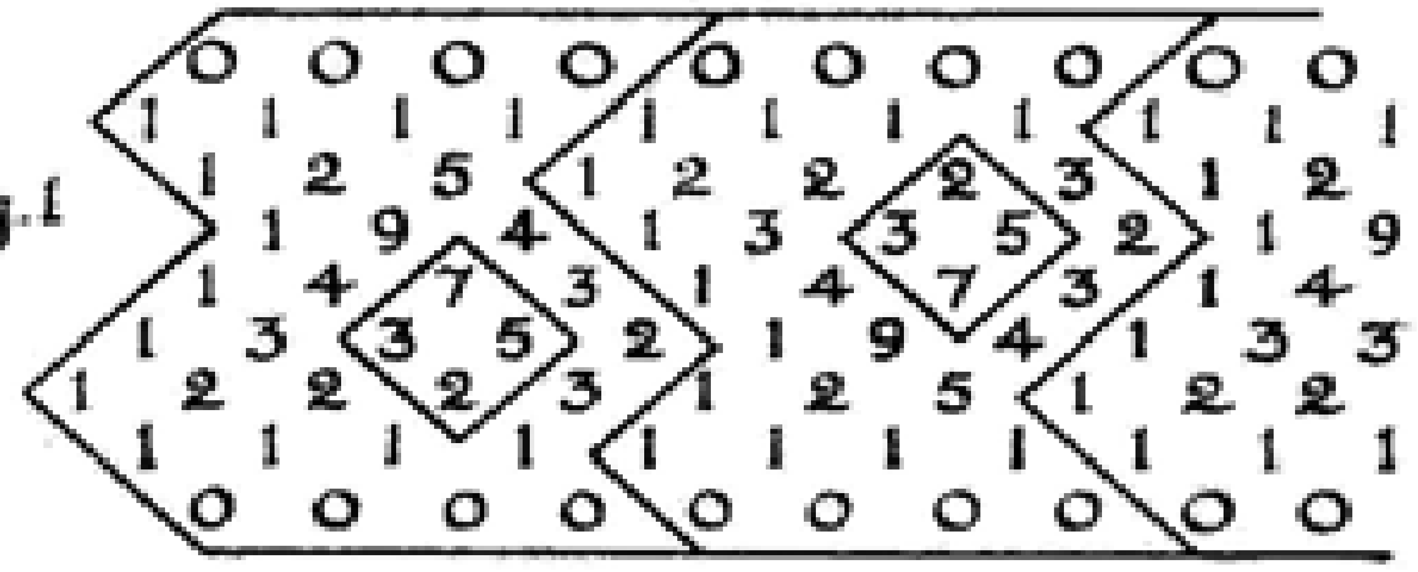


Fig. 2

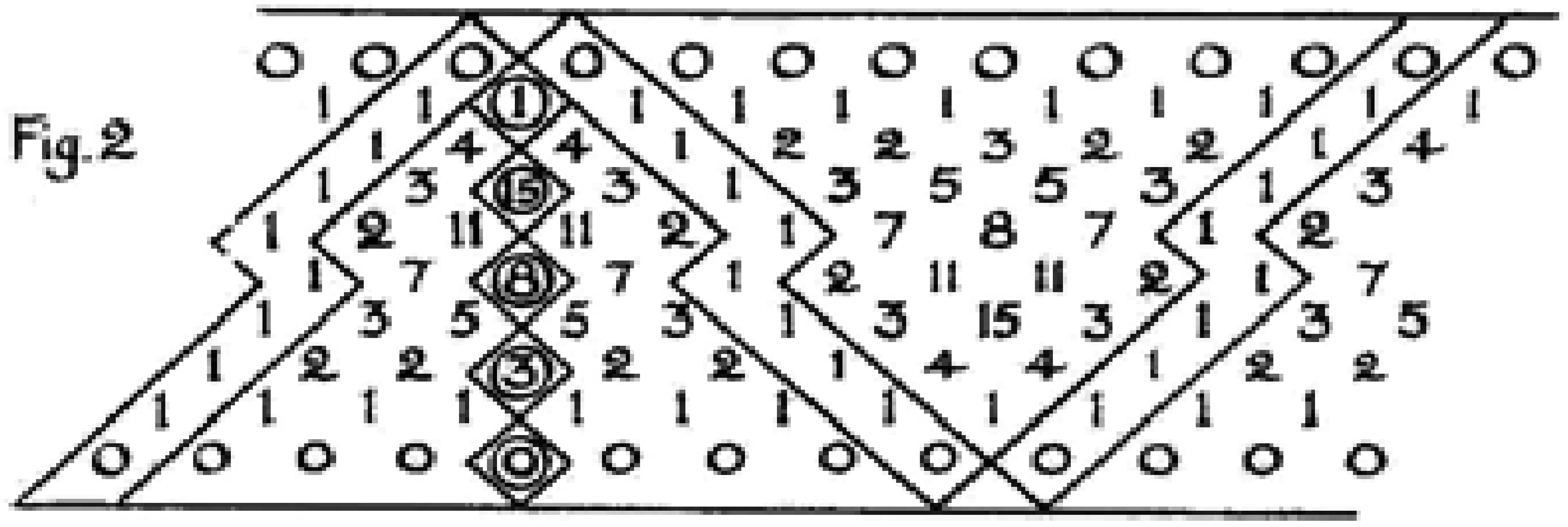
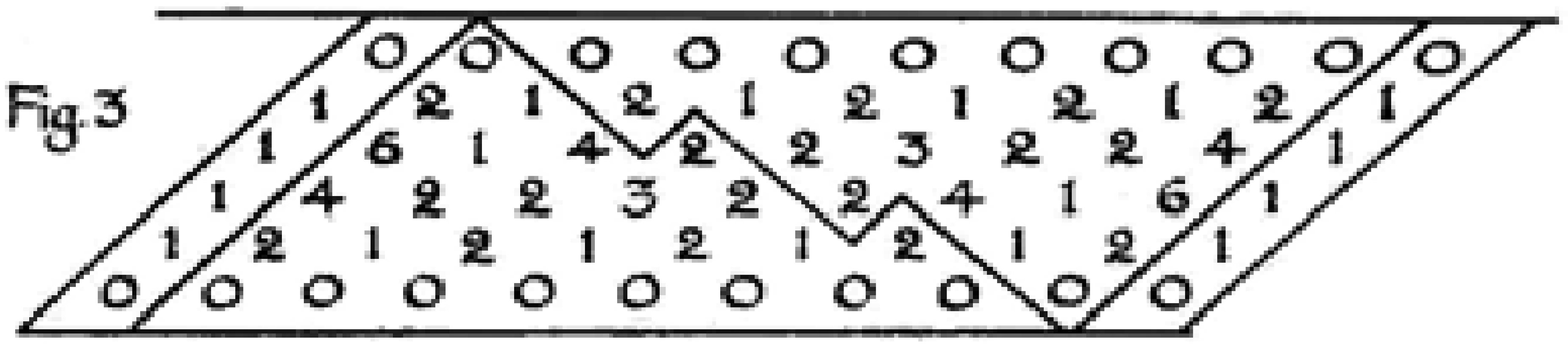
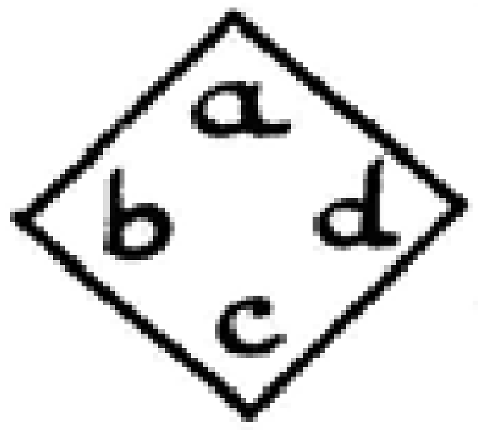


Fig. 3



At the left-hand end, write a line of ones downwards, diagonally or zig-zag as shown in fig. 1 and fig. 2, for several rows, and then complete a bottom array of zeros and ones. The numbers within the strip are then calculated from left to right using a simple rule. Fig. 1 and 2 are based on the same rule and many other patterns can be formed by using it. To find the rule, look at different square cells such as marked in fig. 1. Calculate ac and bd. Write down the relationship between ac and bd and rewrite it to express d in terms of a, b, and c.



It seems surprising that using this rule, any array built up in this way will contain only integers, but even more unexpected is the repetition of the pattern of numbers. Fig. 1 shows the type of " move along and turn upside down " pattern which is called a glide reflection. Fig. 2 is of the same type, but the " A " shape of the pattern is symmetrical and so the glide reflection is the same as a rotation.