SPROUTS

Have you tried the game of Sprouts? Noughts and Crosses has nothing on it! Take a sheet of paper and mark two points on it. Two players A and B take part and move alternately. A move consists of drawing a line starting on one of the points and ending on the same point or the other point and marking a new point on the line just drawn. The two possible moves for A are shown in figs. 1 and 2—the starting points are the two black ones and the point put in by A is white.

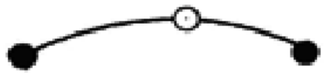


Fig. 1.

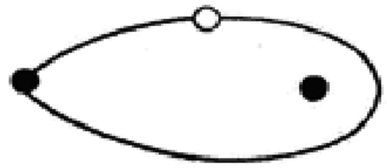


Fig. 2.

B then does the same (of course, he can use the new point) and thus the game goes on subject to the following rules.

1. A point has only three " lives ", i.e., no more than three lines can start or finish at it. The left-hand point in fig. 2 has used two of its lives and has only one left.
2. Lines may not cross.
3. The object is to leave your opponent without a move to make.

Figure 2 game might develop as shown (B's moves are in black points).

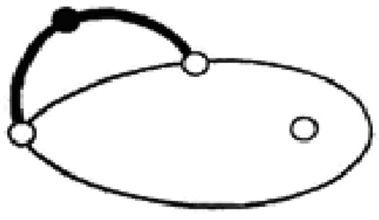


Fig. 3.

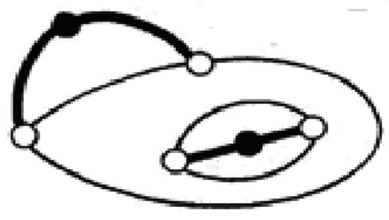


Fig. 4.

B wins because A cannot move. Although the two black points each have a life left, they cannot be joined because that would involve crossing an existing line.

We started the game with *two* points for simplicity but any number of points may be used as starters. If we start with n points, there is a maximum number of moves which the game can run and a minimum. Can you work out, in terms of n, what these maximum and minimum numbers are? Sixth formers may like to work out a winning strategy.

R.M.S.

join the mathematical pie
No. 68 *Editorial Address:* Alpha House, The Avenue, Rowington, Warwickshire, England SPRING, 1973

VECTOR GRAPHS

Most of you will be used to the idea of a pair of co-ordinates describing a point, for instance the ordered pair (2,1) represents a point with reference to a pair of axes x and y 2 units x-wards and 1 unit y-wards of the origin, as in fig. 1. Sometimes it is useful to think of (2,1) representing the movement 2 units East and 1 unit North as shown in fig. 2. Every movement shown can be described by (2,1) and so could be given the same label $\underline{a}$. Mathematicians call such movements *vectors*.

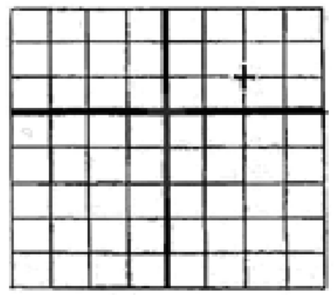


Fig. 1

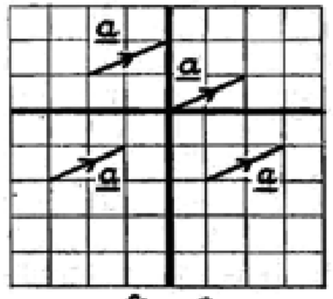


Fig. 2

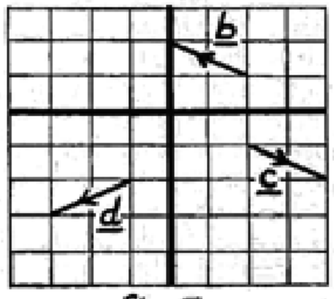
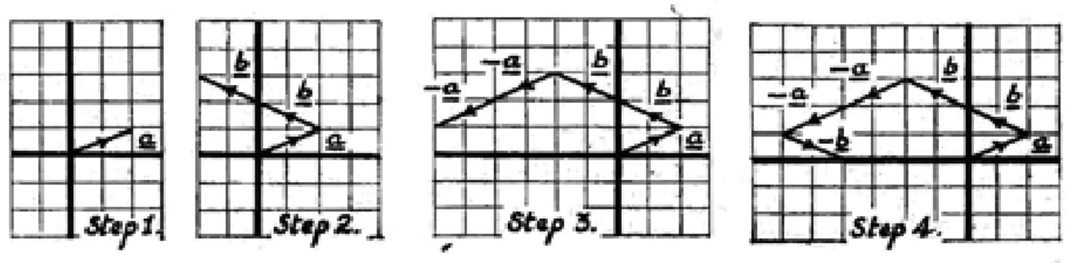


Fig. 3

A vector (−2,1) would be illustrated by $\underline{b}$ in fig. 3, and (2,−1) by $\underline{c}$. $\underline{b}$ is a movement 2 units West and 1 unit North, and $\underline{c}$ is 2 units East and 1 unit South. What ordered pair describes the vector $\underline{d}$? $\underline{d}$ is the opposite of $\underline{a}$ and we may write $\underline{d}=(-2,-1)=-(2,1)=-\underline{a}$. In the same way $\underline{c}=-\underline{b}$.

If we wished to draw a diagram using the above movements, we proceed as follows, starting at the origin. $\underline{a}+2\underline{b}-2\underline{a}-\underline{b}$ would be drawn step by step as shown, taking each movement in turn ($2\underline{b}$ means $\underline{b}$ and $\underline{b}$ again).



Try following the path $4\underline{s}+\underline{r}+\underline{s}-5\underline{r}-\underline{s}+\underline{r}-4\underline{s}+\underline{r}+4\underline{s}+\underline{r}-4\underline{s}+\underline{r}$ if $\underline{r}$ is (−1,−1) and $\underline{s}$ is (−1,1).

E.G.

MAP READING

On a map-reading exercise, Cutie Pie had to travel on a triangular course given by the following Ordnance Survey references: from 120040 to 138070 to 146050 and back to the start. The navigation presented her with no serious difficulties but, on completing the course, she was asked to calculate the area enclosed by her route. What solution should she have obtained?

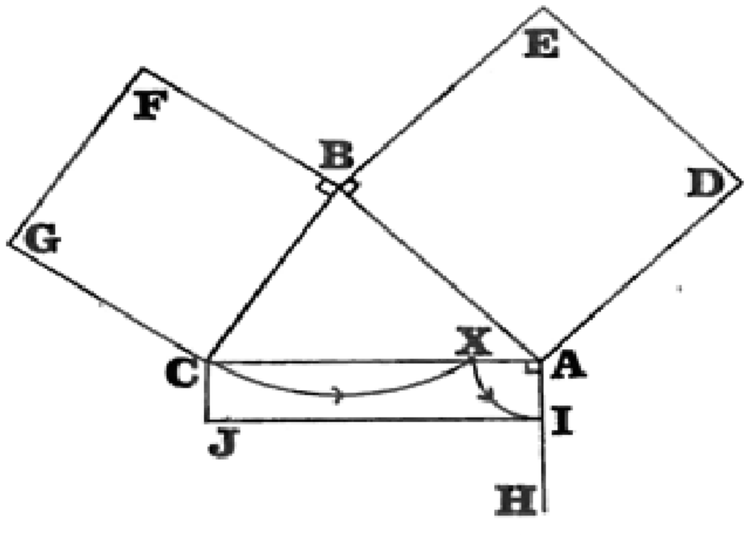
D.I.B.

POLLUTION?

Cutie's boy friend was taking his motor launch full out up river against a falling tide. Cutie, who was sunbathing on the cabin roof was swept overboard as the launch went under a bridge and was carried down stream. Luckily she was using her life jacket as a pillow and had the presence of mind to grab it as she fell. It was five minutes before she was missed, her boy friend put about immediately and went full out down stream in search of her and caught up with her one mile below the bridge. If we ignore the turn round time how fast was the tide running?

R.M.S.

CUTIE IS CUTE

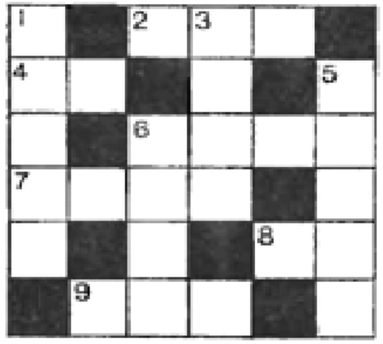


Teacher had asked for an illustration of the theorem of Pythagoras, but Cutie Pie is not very good with her protractor, and when she had finished, she realised that ABC was a *wrong* angled triangle! Doodling despondently with her compasses she drew an arc centre B, radius BC, until it met AC, at X in the figure. Then with centre A, radius AX,

Using this method, it might take two weighings or it might take eleven weighings to find the odd ball. Using a different method one can guarantee to succeed with only three weighings. Start by putting four balls in one scale pan and another four in the other. Whatever the result, the number of possibilities is very much reduced.

C.V.G.

JUNIOR CROSS FIGURE No. 60



CLUES ACROSS

2. $(\frac{1}{4}+\frac{1}{8})\div(\frac{2}{3}-\frac{1}{6})$.
4. A square number whose digit sum is prime.
6. $(2\times5)^3+2\times5^3+1^3$.
7. First year in the next century whose digit sum is an odd prime.
8. Maximum number of days in a month.
9. $1\frac{1}{9}$ to two decimal places.

CLUES DOWN

1. Next two terms in the series 1,8, 27, . . .
3. Product of two primes between 40 and 50 whose average is 45.
5. Sides, in order of size, of a right angled triangle.
6. One in the eye for Harold?

P.J.G.

SOLUTIONS TO PROBLEMS IN ISSUE No. 67

ROUND ABOUT 'RITHMETIC

$13\times2=31$ base 5 and $25\times2=52$ base 8.
The general expression is $5+3k$ for $k=0, 1, 2\ldots15\times3=51$ base 7.
The relation between the base and the multiplier is $b=2n+1$.

MIND YOUR HEADS

The 'bus is 4 metres high.

SQUARE THE QUAD

The length of the side is 12 units.

WHOOPS

In each case the increase in the radius is $5/\pi$ metres.

CUBICS

There are five cubes in the figure.

SENIOR CROSS FIGURE No. 63

Clues Across: 1. 975; 2. 425; 5. 56; 6. 483; 8. 42; 9. 169; 11. 13; 12. 564; 13. 21; 14. 999; 15. 128
Clues Down: 1. 9140; 3. 252; 4. 56 7. 313; 8. 495; 10. 6428; 11. 119; 13. 29.

FLOWCHARTING No. 2

In the flowchart for solving a quadratic equation, the fifth instruction should have had a square root sign over the b^2-4ac. No provision was made for this number to be negative.

JUNIOR CROSS FIGURE No. 59

Clues Across: 1. 254; 3. 52; 5. 2536; 7. 39; 9. 96; 13. 2857; 15. 168; 16. 22.
Clues Down: 1. 233; 2. 42; 3. 53; 4. 268; 6. 50; 8. 99; 10. 628; 11. 152; 12. 81; 14. 72.

THE DISAPPEARING WINE

The steward placed 4 bottles at each corner and one in the middle of each side.

AS FAR AS THE EYE CAN SEE

The distance is nearly 35·7km. The general expression is $\sqrt{h(12640000+h)}$ but as h is usually small compared with the diameter of the earth, a good approximation is $\sqrt{(12640000h)}$ metres.

WHAT AM I?

A fraction.

B.A.

continued from page 537

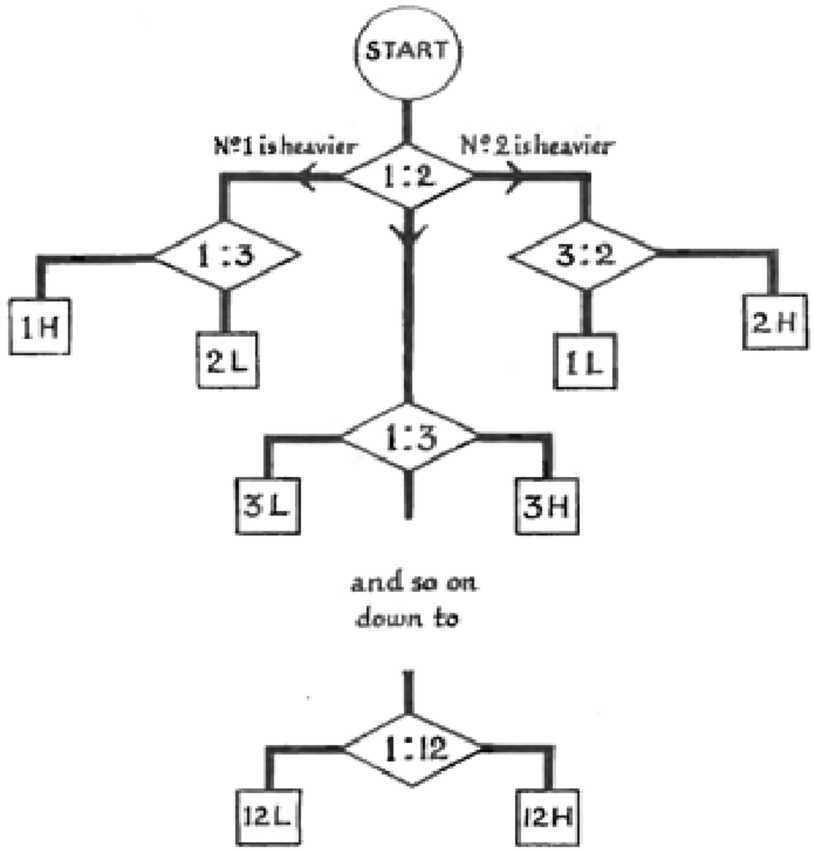
4a to 4g are tetrominoids. Here we have shown 7 of them—are there any more? How many *pentominoids* are there?

If you remember, the tetrominoes would fit together to make a 5×4 rectangle. Can all the tetrominoids be fitted into a rectangular block? How about pentominoids?

These are just a few of the games you can play with poliominoids. Perhaps you can invent some more. If you find any interesting ones, we would like to hear from you.

R.M.S.

ODDBALL



When the doctor asks "Where is the pain?", you might answer, "In the right shoulder" or you might answer "In my left ear". The doctor's next step will depend on your answer.

There are many problems in which the next step is decided by the answer to a question. Drawing a flow diagram often helps to sort out the method. You will remember the old chestnut about the twelve billiard balls one of which is the wrong weight. Using an ordinary pair of scales one has to find which is the odd ball.

An obvious way to start is to put No. 1 in one pan and No. 2 in the other. Either No. 1 is heavier than No. 2, or it is the same weight, or it is lighter. The next step depends on the result. In the flow diagram 1:2 inside the diamond means compare No. 1 with No. 2. According to the result one branches one of three ways. If they are equal, we know that both are standard weight and go to test the next ball. If No. 1 and No. 2 are not equal then we know that No. 3 is standard and so we . . . It is really much easier to follow on the flow diagram.

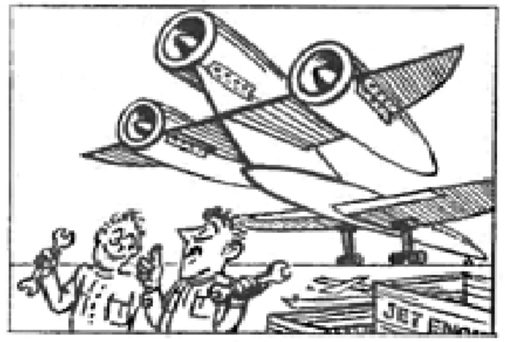
she drew another arc to meet the line AH she had prepared for the third square. Thinking about that wrecked angle was probably the inspiration for AIJC. To Cutie's surprise, she found that she was right after all. Neatly she wrote next to her diagram—

$$EDAB=FBCG+AIJC$$

Try it for yourself with any triangle.

Teacher said it wasn't the theorem of Pythagoras, but he gave her a mark and said she had extended herself that time!

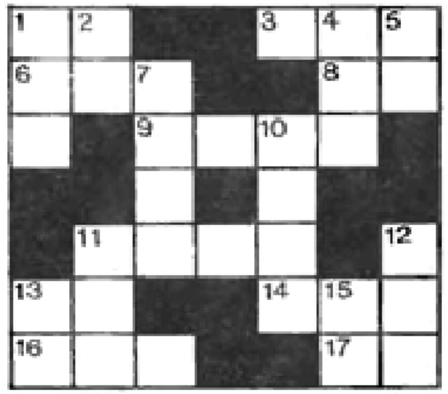
E.G.



EUREKA

The specifications of the Tristar aircraft included the statement that it should have more than two and less than four engines. This was met, as you know.

SENIOR CROSS FIGURE No. 64



Ignore decimal points and work to the required degree of accuracy.

Clues Across

1. (Difference of squares of 2 odd primes) divided by twice (sum of these primes).
3. Sum of the series $\log(6{\cdot}917)+[\log(6{\cdot}917)]^2+[\log(6{\cdot}917)]^3+\ldots$.
6. Percentage profit made by buying lira at 1,000 to the £1 and selling at 900 to the £1.
8. Algebraic sum of the coefficients of $(13x^2-2x-3)(2x^2+1)$.
9. Root of $x^2-40x+399=0$.
11. Percentage error in taking 1 inch to be 2·54 cm if the true value is 2·53995 cm.
13. Calculate b if 0·05 and 1·25 are the roots of $bx^2-ax+0{\cdot}625=0$.
14. $(1-x)^{-9}$ if x is 0·5.
16. Product of 130 terms of $(1+1)(1+\frac{1}{2})(1+\frac{1}{3})\ldots\ldots$
17. Integral part of the number of cm. in 1 foot (use the value given in 11 Across).

Clues Down

1. $(2n)^n$ given that n is an integer.
2. Digit sum of 10 Down.
4. Product of two numbers whose difference is 4 and the difference of whose squares is 120.
5. One equal angle in an isosceles triangle if the sum of the equal angles is an anagram of the number of degrees in a triangle.
7. Value of
$$\frac{4a^2-b^2}{a^2+ab+b^2}\times\frac{3a^2-4ab+b^2}{2a^2-ab-b^2}\times\frac{a^3-b^3}{6a^2-5ab+b^2}$$
if a is 23·69, b is 22·521.
10. $(\sqrt{6}-\sqrt{2})\div(\sqrt{6}+\sqrt{2})$. Take $\sqrt{3}$ to be 1·7315.
11. Smaller of two consecutive integers whose product is 10712.
12. $(125^{1/2}\times4^{3/2})\div(\sqrt{25}\times27^{-1/3})$.
13. Reciprocal of 0·909.
15. Value of a in 13 Across.

P.J.G.

FRACTIONS

Start with a square and divide each side into three equal parts. Join the pairs of points by lines drawn parallel to the original sides and block out the four corner squares. What fraction of the original area remains unshaded?

Repeat the process with the centre square, what fraction of the original square still remains unshaded?

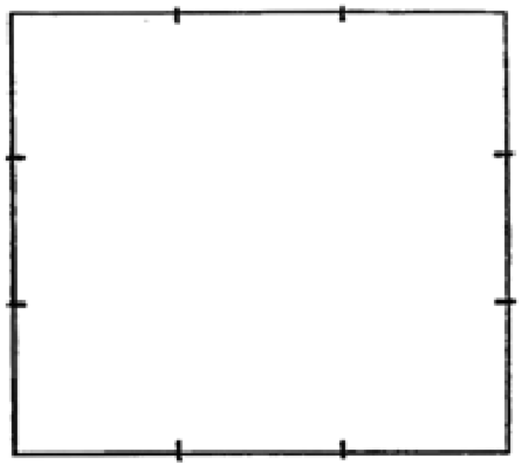
Continue the process with the new central square and find the area of the unshaded part as a fraction of the original area.

If the process were continued indefinitely, what fraction of the original square would remain unshaded?

Starting with a stellated hexagon, consider the process of blocking the stellations at each stage and find the fraction of the original figure that remains unshaded. It is not as difficult as it appears initially.

R.H.C.

1.



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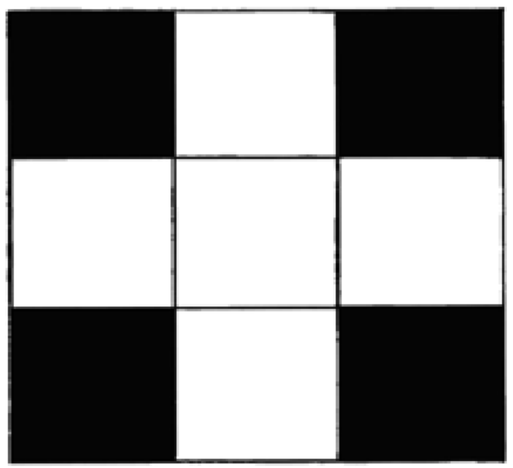

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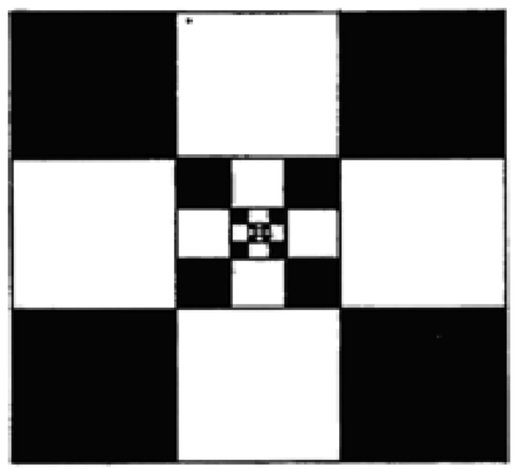
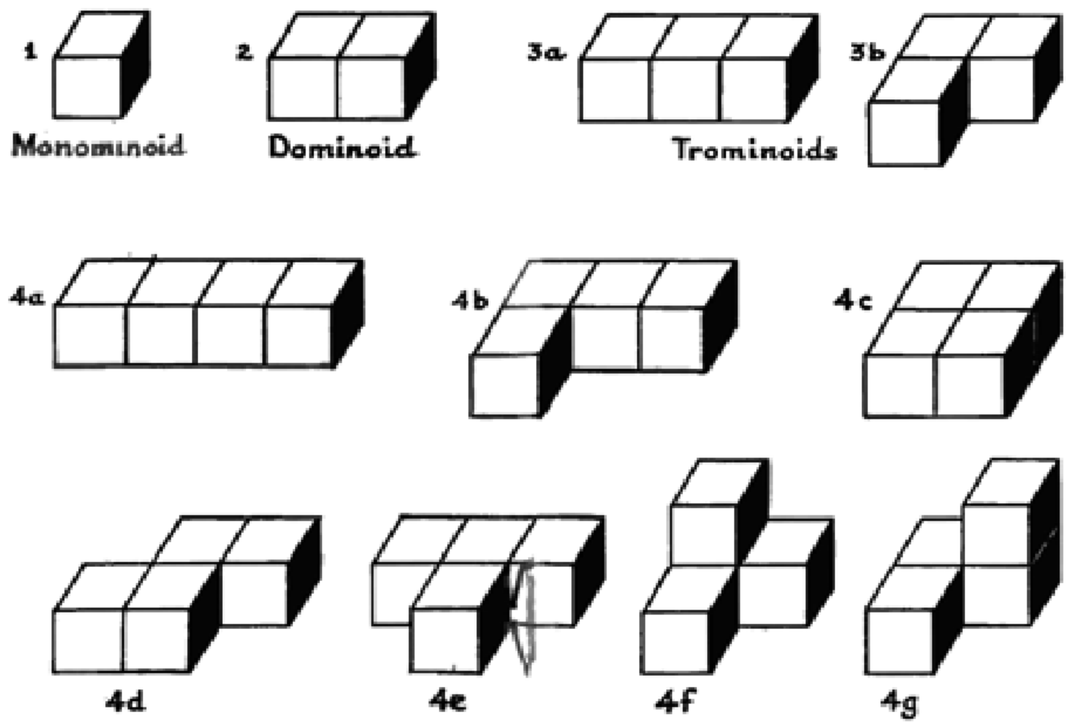


POLYOMINOIDS

In issue No. 62, we talked of polyominoes and polywhatsits. Here is a further extension you might like to try. Instead of using unit squares, try using unit cubes and go into the third dimension. The cubes must be joined together *face to face*. You could make this out of cardboard or (but don't let Mother know) sugar cubes which will stick together well if the two faces are wetted slightly, pressed together and left to dry. If you are fortunate enough to have access to a set of Centicubes (made by Osmoroid) you will find these ideal. Here are a few starters.



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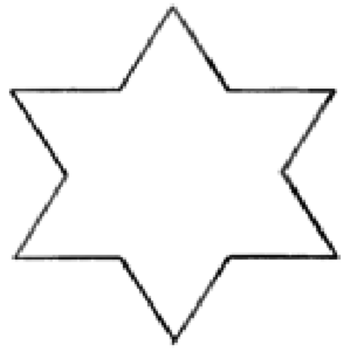

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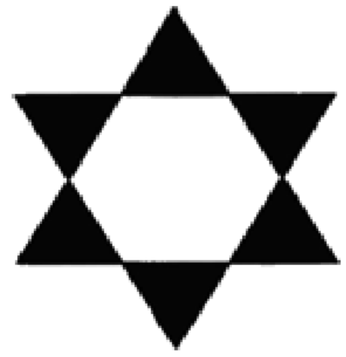

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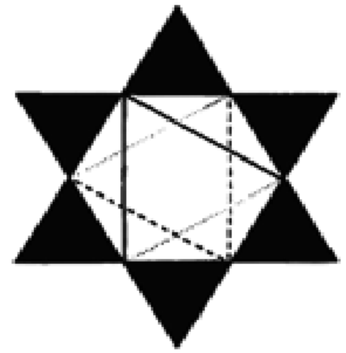

4.