

Fig. 1.

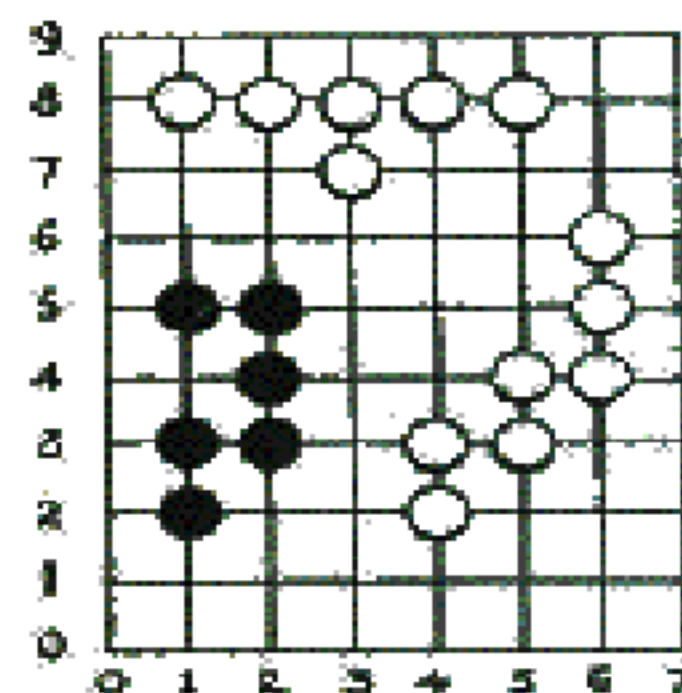


Fig. 2.

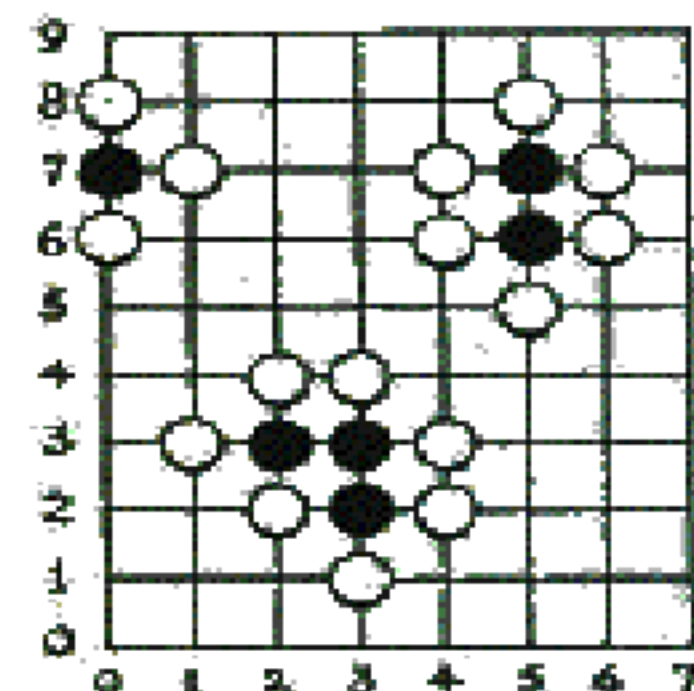


Fig. 3.

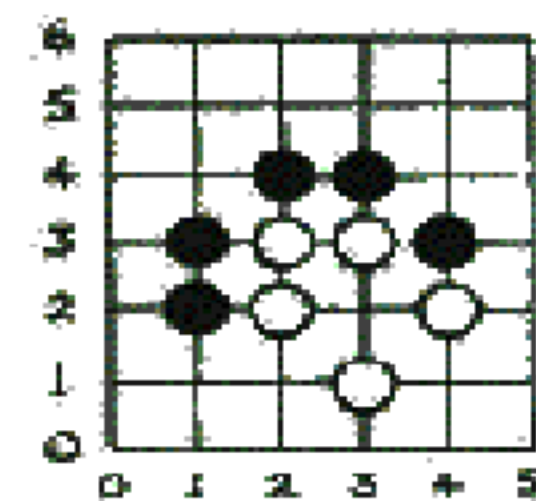


Fig. 4.

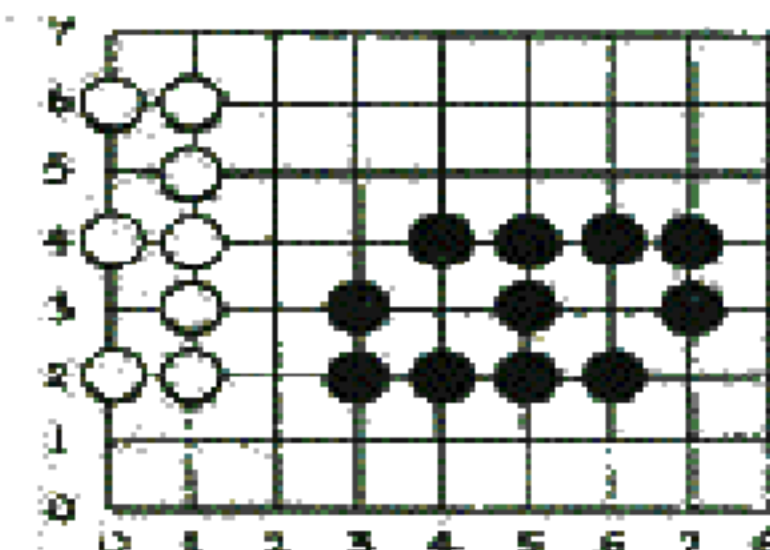


Fig. 5.

Did you know that chess is not played very much in oriental countries? The Japanese in particular consider it a much inferior game to the board game which originated in China some 4,000 years ago—the game which is known as "Wei'chi" or "Go". This, too, is a military type of game representing a battle, but its rules are simpler, since there are no pieces with different types of move. In fact the "men" are not lined up before the battle at all, and once they have been placed on the board they are not moved again! That seems a strange idea at first, but you will see the reason once the objectives of the opposing players are described.

The "Go" board has nineteen "horizontal" and nineteen "vertical" lines painted upon it, forming, well let me see, $20 \times 18 + 1^2 = 361$ intersections. If we label each set of lines with the numbers from zero up to eighteen (Yes, I know I said nineteen lines; zero is *one*, and one is *two*, . . .)—then every point can be described, as usual, by an ordered pair of numbers. (3, 4) will be the intersection marked in Fig. 1 by a star.

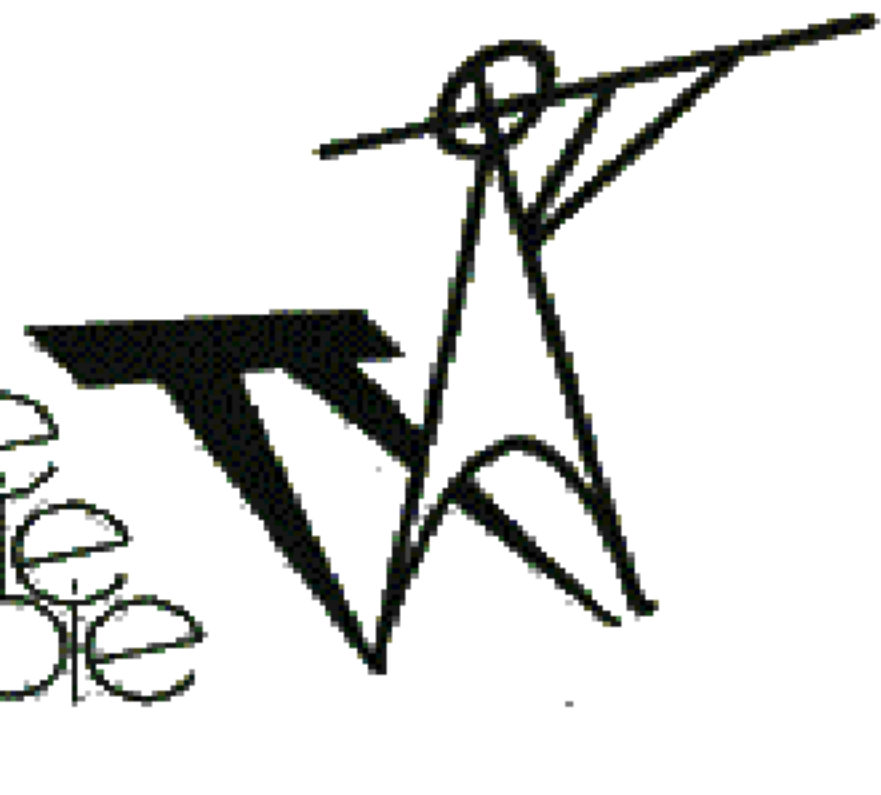
The two players each have men of a particular colour, white or black as in chess, but the men are all the same—a simple small disc about 15mm in diameter. The players take turns to place their discs on the intersections, anywhere on the board, with the object of surrounding as many empty intersections as possible. Meanwhile, they may upset each others plans by capturing enemy men as described in the next paragraph.

In order to describe the way of capturing opposing men, we need to be clear as to the idea of "connected". Any intersection is directly connected, by horizontal and vertical lines, to four other intersections, e.g. (2,7) is connected to (1,7), (3,7), (2,6), and (2,8). Of course points on the edge such as (0,4) are directly connected to only three points—in this case (0,3), (1,4), and (0,5), and corner points to only two (see Fig. 1).

Men of the same colour lying on connected intersections support each other and cannot be captured singly. A single "stone" or a set of connected "stones" is captured when all the possible connections to that stone or set are filled by stones of the opposing colour. Fig. 2 shows some connected sets, and Fig. 3 shows some black stones which have been captured by white. Captured men are removed from the board.

continued on page 522

Join the mathematical pie
in the mathematical pie
the mathematical pie

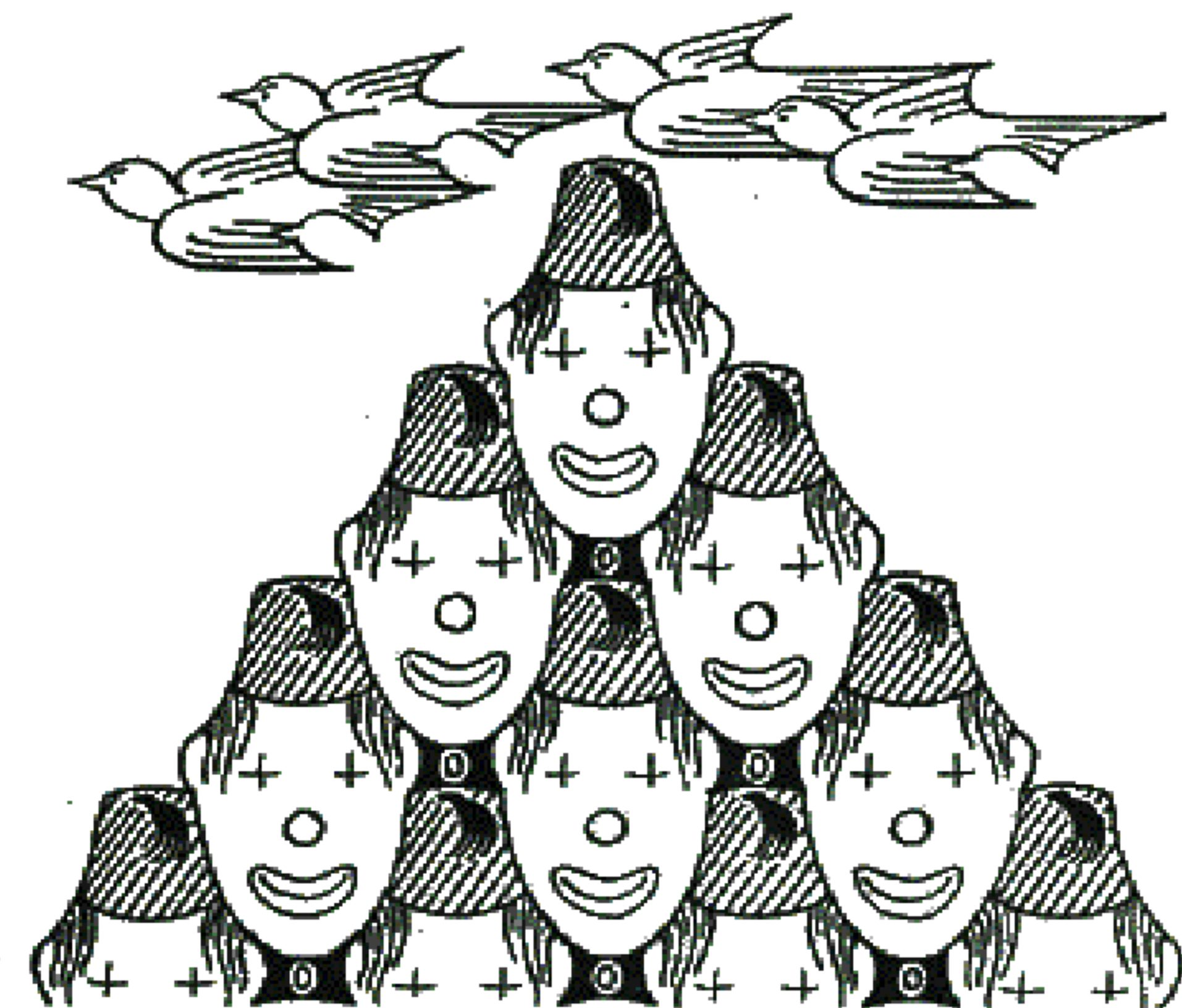


No. 66

Editorial Address: Alpha House, The Avenue,
Rowington, Warwickshire, England

SUMMER, 1972

FLY SOUTH THIS YEAR



The tessellation is based on designs submitted by Susan Raes of Coombe Dingle and Judith Turner of Stoke Bishop, both near Bristol. They each receive a book token. Ed.

CAREFUL PACKING

John left his umbrella, 1.7 ft. long, behind in the train and had to ask the LOST PROPERTY OFFICE to forward it to him through the post. Unfortunately, he caused the L P O quite a problem because the Post Office has a rule that no parcel must have a dimension greater than one foot. How did the L P O manager deal with the problem?

R.H.C.

THOUGHTS ON NOUGHTS

As you are aware, in the normal game of noughts and crosses, one person uses O's and the other uses X's. Consider, however, the game if it is played by two players, each of whom may use either a nought or a cross as he wishes.

The strategy of the game is not quite the same as ordinary O's and X's, in which stalemate is quite possible if the game is played properly. In this version, however, the first player has a distinct advantage if he starts with a cross in the middle square of a 3 by 3 set of squares. If the second player puts a nought in one corner—he cannot put an X because he will automatically lose—he must lose the game. Can you see why? If he puts his nought above, or to the side, of the cross he can produce stalemate, or lose if he is not careful. Can you produce a strategy by means of which he can produce a stalemate?

P.J.G.

POWERFUL

The number 371 has the property that it is the cube of its digits, i.e. $3^3 + 7^3 + 1^3 = 27 + 343 + 1 = 371$. There are just two other 3 digit numbers with this property. Can you find them?

Is there a two digit number which is the sum of the SQUARES of its digits? You may like to extend the problem. What n-digit numbers exist which are the sums of the nth powers of their digits?

This problem has not yet been solved for the general case. R.M.S.

NO SEVEN UP

Write down a three-digit number using only the digits 1, 2, 3, 4, 5, or 6 such that no two of the digits add up to seven. In how many ways can this be done (i) if repetition of a digit is allowed, (ii) if repetition is prohibited? E.G.

CHARLIE COOK

Charlie Cook has been at it again. He was asked to simplify $\frac{\log 4}{\log 8}$ so what he did was $\frac{\log 4}{\log 8} = \frac{4}{8} = \frac{1}{2}$. Is he correct?

With what calculation $\frac{\log X}{\log Y}$ should he have started in order that despite his howlers $\frac{1}{2}$ happens to be the correct answer? See issue No. 31. R.H.C.

GREATEST PRODUCT

Two positive integers X and Y add up to 16. If the product of the two numbers XY is to be as large as possible, what should X and Y be?

R.H.C.

IT'S EEEZY

If AB times CD is EEE where each digit is represented by a letter and AB taken from E times CD leaves CC, what is AB times D?

R.M.S.



GEMS AT THE ZOO

Each letter stands for a different digit. If we square ZOO we get TOPAZ, so what is TOP+PAT?

R.M.S.

JUNIOR CROSS FIGURE No. 58

1	2		3		4
5					
			6	7	
8		9			
		10			11
12				13	

Ignore decimal points.
Work to the required accuracy.

CLUES ACROSS

- $(a+b)^2$ when a is —1 and b is 4.
- Days in a leap year.

- Pounds in 1 ton.
- Inches in one metre.
- (Unlucky Friday)² as written by an octopus.
- Last year reversed, last year.
- $(9 \text{ down})^2 \div \sqrt{(9 \text{ down}) \times (2^2 - 1)} \div \sqrt{16}$.
- $(11 \text{ down}) \times$ first digit of 6 across.

CLUES DOWN

- Hockey team plus football team.
- $(85)^2$.
- Thrice the first 3-digit prime.
- Reverse the digits of 3 across and add 1.
- $10^4 - (2^3 - 1)$.
- Wave length of Radio 1.
- Boiling point of water in degrees C.
- Transfer 2 old shillings into new pence.

P.J.G.



SOLUTIONS TO PROBLEMS IN ISSUE No. 65

WHO'S ZOO

The zoo had 14 animals and 22 birds.

THE COMMON MARKET

Pierre operated the lift in the Eiffel Tower. His friends call him Brighton Pierre.

A SQUARE PAD

Jimmy lived at number 961.

SUBSTITUTION

The solution is not unique but one is 0 is R, 1 T, 2 I, 3 A, 4 H, 5 S, 6 Y, 7 V, 9 E.

CONDENSED

$\frac{23}{30}$ pounds of nectar make 1 pound of honey.

MODERN CROSS FIGURE

Clues Across: 1. 53; 3. 32; 4. 4; 6. 1112; 8. 26; 9. 45;

Clues Down: 1. 55; 2. 32; 5. 125; 6. 12; 7. 16.

POLLUTION PROBLEM

Neither tank is more polluted. Each has the same, and it would not matter if the liquids were thoroughly mixed or not.

SENIOR CROSS FIGURE No. 61

Clues Across: 1. 866; 3. 175; 5. 65; 6. 36; 7. 153; 10. 104; 11. 257; 13. 97; 14. 67; 16. 171; 17. 252.

Clues Down: 1. 83; 2. 66617; 4. 725; 7. 14972; 8. 37; 9. 22; 12. 567; 15. 12.

JUNIOR CROSS FIGURE No. 57

Clues Across: 2. 1331; 4. 1991; 6. 13; 7. 444; 10. 366; 12. 63; 13. 1315; 15. 2346.

Clues Down: 1. 1113; 2. 19; 3. 314; 5. 936; 8. 461; 9. 4356; 14. 36.

FLOWCHARTING

There may be a stationary vehicle nearby.

In issue No. 63, Junior cross figure 8 down, the answer should read 525. Miss Whitehouse of Stourbridge brought this to our notice and receives a book token. Although great care is taken with the issue, misprints are sometimes missed. The editor will send a book token to the first reader who recognises a misprint and writes in to suggest what the original should have been, or how the misprint can be corrected. B.A.

FOR CRACK MATHEMATICIANS ONLY — 1

Most people, whether they belong to the Corgi Club, U.N.C.L.E., or the Archaeological Society enjoy code-breaking; so try your hand at this one. H GNHZF PAYUHQF EP IXHI EZ BXEQX HZD IBT GTEZIP VFEZM IHOFZ IXF PIYHEMXI NEZF VFIBFFZ IXFC NEFP BXTNND EZ IXF PAYUHQF.

It is a straight forward substitution code, i.e. each letter of the original message is always replaced by the same letter. (Mathematically this is called a one-to-one correspondence.)

You may start by guessing that H stands for A or I, as these are the only single letter words, but you will probably have difficulty in going much further. Many years ago, however, code-breakers observed that certain letters occur more frequently than others in the English language, and by analysing secret messages they began to crack codes systematically.

Here then is an interesting statistical investigation for you and your friends. Select a page at random in any book, and choose a passage with approximately 2,000 letters. Count the number of times each letter occurs in the passage, and in order to achieve a more accurate picture, combine your results. (This is desirable, as your figures will be biased if you happen, say, to choose a physics textbook with a page describing a 'bubble' experiment when you will get an unusually high proportion of b's). You can now make an analysis of the relative frequencies of the letters, and use this to decode the message. In the next issue we will give the results of an investigation involving millions of letters, and I think you will be surprised how closely they agree with the results of your small-scale sample. You will probably then begin to appreciate why polls, basing their results on the opinions of only a few thousand people, so often reflect national opinion very accurately. Get cracking! S.T.P.

MULTIPLICATION PROBLEMS

The product of two numbers which differ by ten is 1475. What are the numbers?

What is the smallest number whose digits are reversed when it is multiplied by 4? R.H.C.

"GO" continued from page 524

A stone cannot be played on an intersection whose connections are filled by opposing men, UNLESS by such a play some of the surrounding men are themselves captured. In Fig. 4 (3,2) has its connections filled by white men. Black cannot play there. However, if black plays at (2,1) he may on his NEXT move play at (3,2) since this would then capture the men at (2,2), (2,3), (3,3).

It follows that if a set of stones is to be safe from capture, it must contain two separate vacant intersections—for instance the sets of stones in Fig. 5 can never be captured.

The game ends when the players agree that neither is able to capture more vacant territory. The number of men lost by a player is subtracted from the number of intersections he has surrounded, and the player with the larger number of "points" left is the winner. A shorter game can be played on a "quarter" board with 100 intersections. Our school chess club has a "Go" section; we use pegs in peg-board as an alternative to the usual board.

E.G.

NUMBER SYSTEMS

In the binary system, you can have two positive integers which contain not each digit not more than once, e.g. 1, 10. In base three, you will have ten, 1, 2, 10, 20, 12, 21, 210, 120, 201, 102.

What happens in base five? and when you have done that try base ten.

R.H.C.

CUTTING THE MINT

What is the maximum number of pieces into which an annulus (i.e. the area between two concentric circles) can be cut with (a) 1 cut, (b) 2 cuts, (c) 3 cuts, . . . ?

Can you find a general formula?

E.G.

A FAMILY RELATIONSHIP

Each of the Smith brothers has as many sisters as he has brothers. Each of the Smith sisters has twice as many brothers as sisters. How many boys and girls are there in the Smith family?

P.J.G.

SENIOR CROSS FIGURE No. 62

	1	2	3	4		
5		6				7
8	9				10	
11				12		
13			14		15	
		16		17		
	18					

Take π to be 3.1416.
Ignore decimal points.

CLUES ACROSS

- Sum of the first 8 terms of the series 4, 12, 36, . . .
- Palindromic number which is prime when halved.
- The second prime number such that the product of its digits is prime and the sum of its digits is a perfect square.
- Square of a prime number, the answer having a digit sum equal to the next prime number.
- Evaluate $(\sqrt{5+2})^4 + (\sqrt{5-2})^4$ exactly.
- $\sqrt{(61\ 000)}$.
- Shortest side of a right-angled triangle whose other sides are 2113 and 2112.

- Coefficient of x^3 in the expansion of $(1+x)^5$.
- $\log_{10} \sqrt{(3020)}$.
- $(\pi)^2$ correct to 4 decimal places.

CLUES DOWN

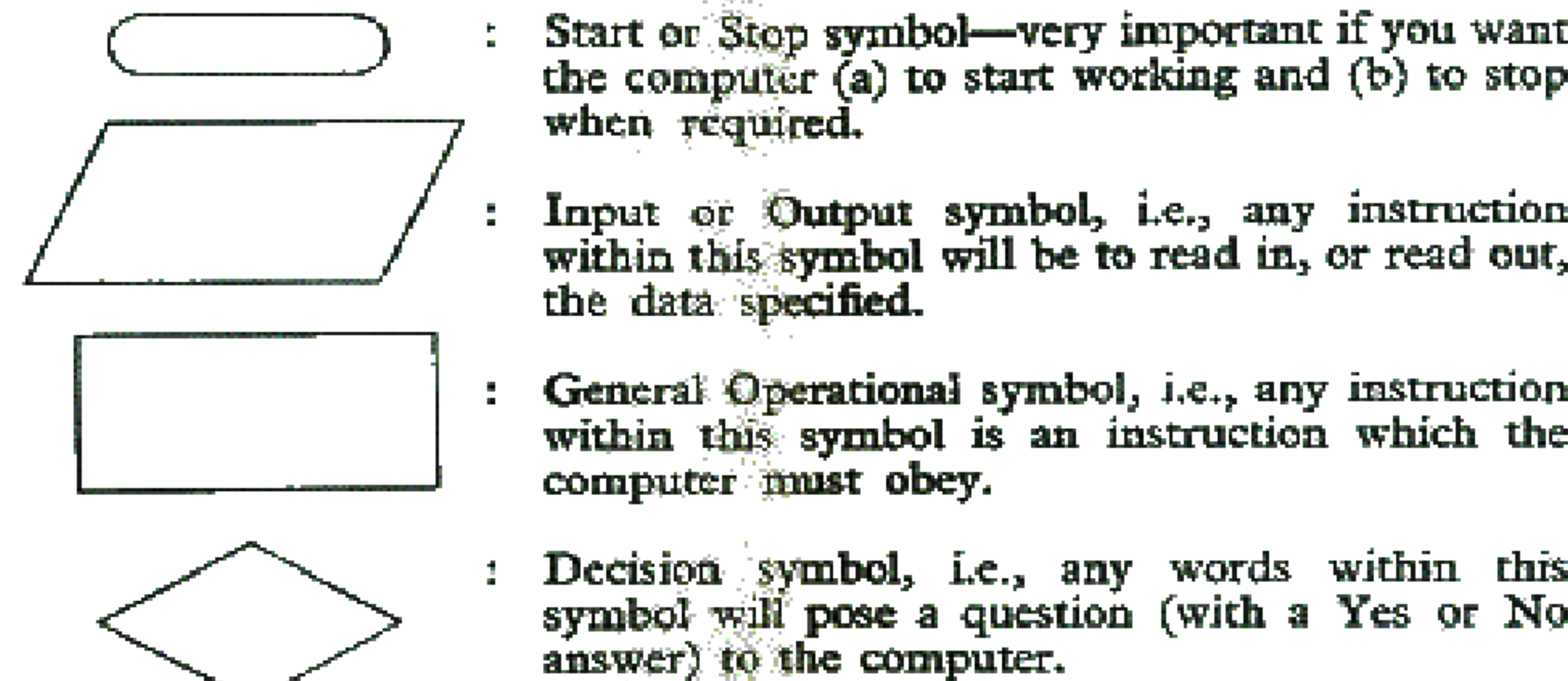
- Sum of the first 12 terms of the series $\frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \dots$
- Decimal value of $\frac{x}{y}$, correct to 2 decimal places, if $\frac{x+n}{y+n} = \frac{x}{y}$ for all n .
- Easter Sunday in 1973 falls on the . . . nd of April.
- Sum of the first 21 cubes.
- Volume of revolution formed when $y = \sqrt{x}$ is rotated through four right-angles about the x -axis in the range of x from 0 to 1.
- The percentage increase of the area of a square if its side is increased by 50%.
- $\frac{x^3 - 1}{x - 1}$ when $x = 15$.
- Interior angle, in degrees, of a regular 90-sided polygon.
- $n^{2n} + n$, for some integral value of n .
- The probability of a baby being a boy is 0.7, what is the probability that the first two children born to a family being boys?

P.J.G.

FLOWCHARTING

In issue Number 54, in the article entitled "Writing to a Computer", we discussed how a programmer can communicate with a computer by using one of the many languages now available to him. However, before he can write to the computer he must know exactly what things to write and in what order he must write them, since a computer obeys orders given to it in the same order as they are given. In order to make this job easier, the programmer draws a "FLOWCHART" of the problem he is trying to solve. This is merely a pictorial breakdown of the problem into its logical steps, each of which must be as simple as possible in order for the computer to understand the orders.

For such flowcharts, there are a number of internationally recognised symbols (27 in number according to the European Computer Manufacturers Association—ECMA—), but for our purposes we are going to use only four of these symbols:



These symbols are joined by straight lines with arrows on them to indicate the direction in which the computer must carry out the sequence of operations. The problem involved may be either a mathematical process or a logical problem, possibly involving a minimum of mathematics. The example on the left is a mathematical one to find the average age of the pupils in a class, the one on the right is one for crossing a normal two-lane one-way street (with traffic coming from the right).

In the arithmetical example, the words MONTHS and TOTAL merely indicate locations in the memory of the computer into which the required number is to be stored. We need two locations because a computer, like a tape recorder, "erases" any previous number when writing a new number into the same location. This flowchart assumes that the data is available in the form of a list of ages as pairs of numbers, the first of the pair being the age in years and the second the remaining months in the pupil's age.

The flowchart on the right is, as mentioned, for crossing a one-way street, but it is not complete. Can you see what step has been missed in the flowchart, or rather, what thing have we not considered in drawing it up? (If you wish to cheat, the answer is in this issue's solutions!) If you find this one too easy, see next issue's thrilling instalment.

P.J.G.

