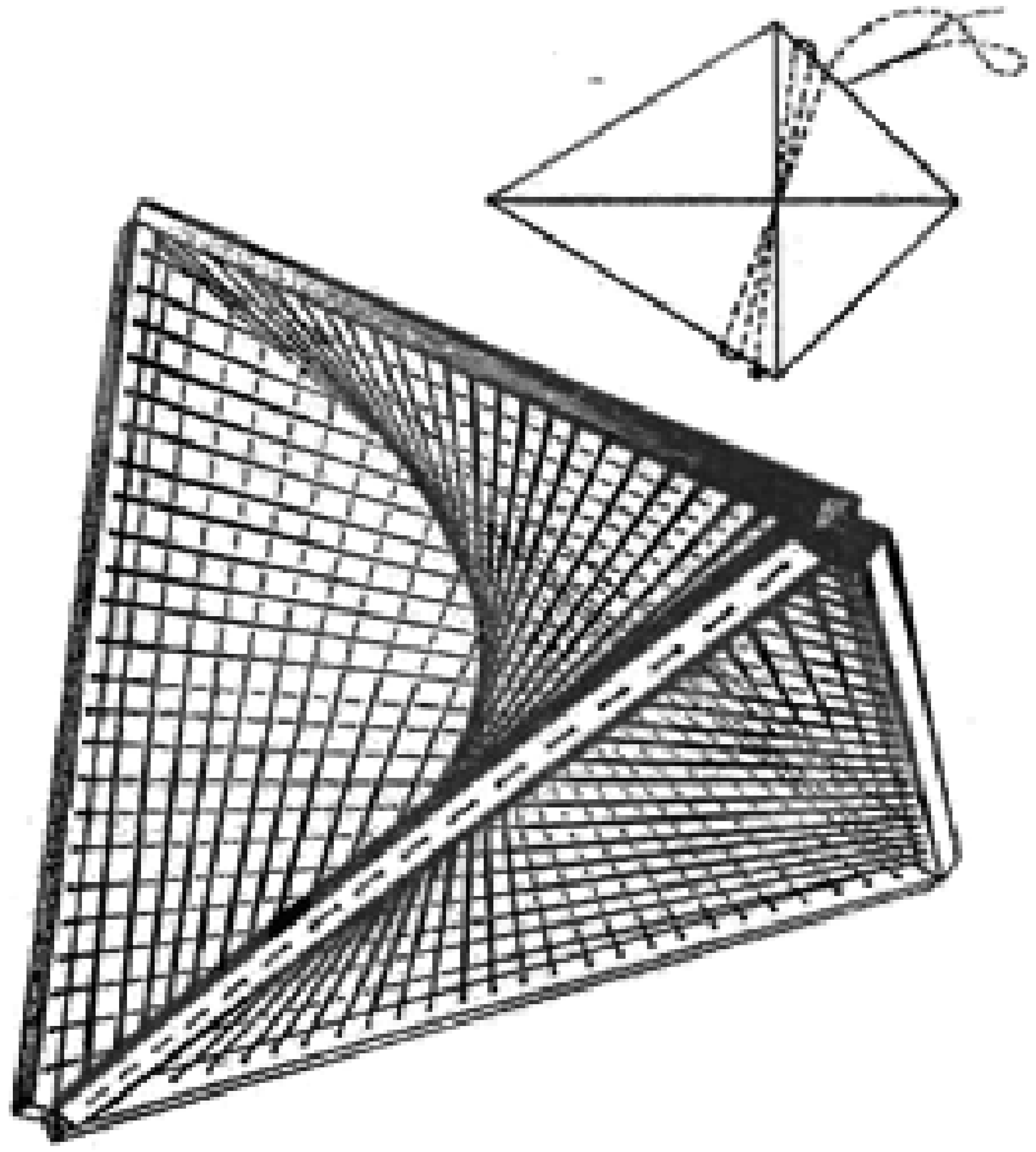
THREE-DIMENSIONAL CURVE STITCHING



This tetrahedron with ruled surfaces can be constructed simply from ¼-in. square-section balsa wood strip and "Coton à Broder" thread. Six 15cm lengths of the balsa wood are cemented together to form the edges in which holes are pierced with a needle at 5mm intervals. The diagram shows how the cotton is threaded between opposite edges. In this way, three intersecting surfaces can be developed, each one being a hyperbolic paraboloid.

D.I.B.

SORT YOURSELVES OUT

On fifteen postcards, write the numbers 1 to 15. In the table, you see the binary equivalent of these numbers. Along the side of each postcard cut holes and slots to represent the number in the binary scale; a slot represents 1 and a hole represents 0. Ensure that the holes and slots in each position are in the same place on each card.

Denary	*Binary*
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

When you have completed the cards, shuffle them into any order. Push a knitting needle through the right hand set of holes in the cards; as you lift the needle, those cards with a hole in this position will stay on the needle, those with a slot will be left behind. Place those left behind at the back of those lifted out and repeat the operation using the next hole along, placing those cards that are left behind at the back of those which are lifted out. Continue with the remaining two holes. What do you notice when you have finished? The cards should be in numerical order. Can you see why this works?

If you have an old pack of playing cards that you do not want and you can extend the binary equivalent of the denary numbers up to 52, you can perform a very interesting trick. Number the cards from 1 to 52; spades 1 to 13 starting ace, deuce, etc., to ten, jack, queen, king; hearts 14 to 26; diamonds 27 to 39; and clubs 40 to 52. Drill six sets of holes along one side of the cards and cut slots for the 1's in the binary equivalent of the card and leave the holes intact for the 0's. In the same way as before, using a knitting needle and placing the cards that are left behind each time at the back of the pack, you can sort the cards into suits in numerical order.

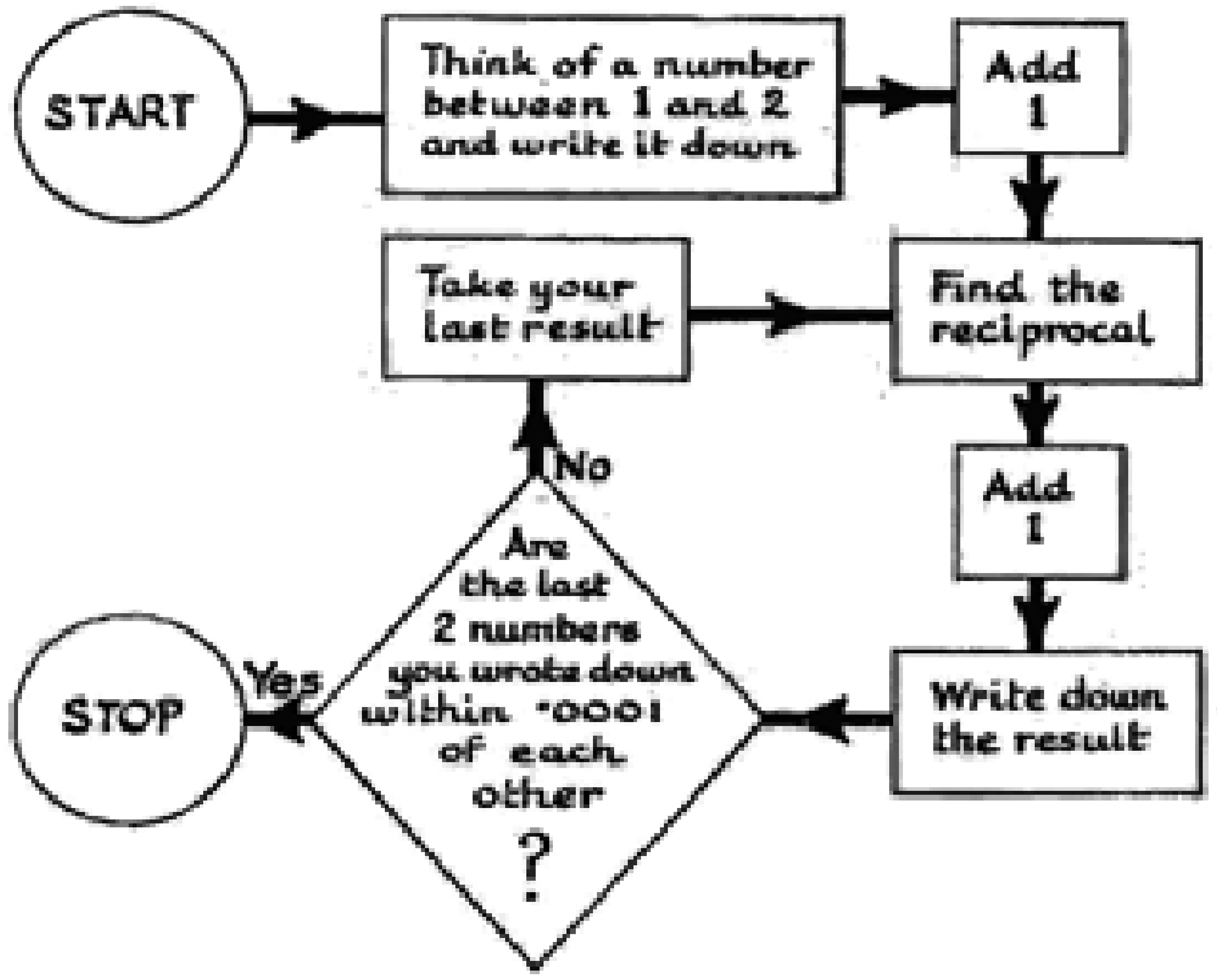
A.W.B.

join in the mathematical pie

No. 65 *Editorial Address :* Alpha House, The Avenue, Rowington, Warwickshire, England SPRING, 1972

FLOW DIAGRAMS—2

In issue No. 64, we saw that a flow diagram was followed rather like a simple "race" game such as ludo or snakes and ladders, and learned the terms "connector box", "instruction box" and "decision box". Even if you missed that article, you will probably be able to follow these instructions using tables :—



The final result you get should be a good approximation to—

$$\cfrac{1}{\cfrac{1}{\cfrac{1}{n+1}+1}+1}+\cdots$$

What do you notice about your result?

E.G.

WHO'S ZOO

A farmer has his own zoo collection. When the census-taker asked him how many birds and how many beasts he had, he replied, "Well, I have 36 heads and 100 feet altogether". How many birds and how many beasts were there?

P.J.G.

THE COMMON MARKET

Pierre, who was born in Brighton, works in Paris and, when at work, travels approximately 28km per day. In his travels, he does not pass any streets, roads, automobiles, trees, fields, or houses. He does not walk, fly, run, travel on any animal, ride in any vehicle that runs on tyred wheels and is rarely seen on a boat, except when he returns home. He is not alone in his travels. How can this be? What do you think his friends in Paris call him?

P.J.G.

A SQUARE PAD

Jimmy's father bought a plate with the house number written upon it. He knew that the number of the house was a perfect square but the postman pointed out to him that it showed the wrong number. What was the house number?

P.J.G.

SUBSTITUTION

```
THIS
  IS
VERY
----
EASY
```

The problem on the left is an addition sum. You must replace each letter by a digit 0—9 and the same digit must be used to represent the same letter. Put digits for the letters to make a valid addition sum.

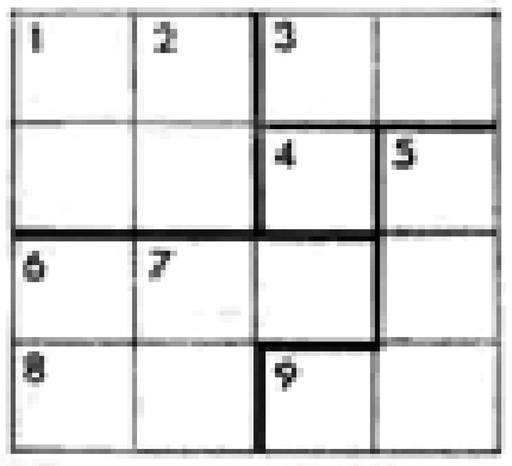
P.J.G.

CONDENSED

The nectar gathered by bees consists of about 70% water. The honey the bees produce from it contains about 17% water. How much nectar is required to make one pound of honey?

A.W.B.

MODERN CROSS-FIGURE



Clues Across

1. Product of $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$
3. Co-ordinates of $(-3,-2)$ after a rotation of 180° about (0,0).
4. Number of axes of symmetry of a square.
6. $3\frac{1}{2}$ in denary, written in binary.
8. The function f is defined as $f: x \to x^2+1$, calculate $f[f(2)]$.
9. Area between the x-axis and the curve given by

x	0	2	4	6	8	10
y	3	5	7	5	3	2

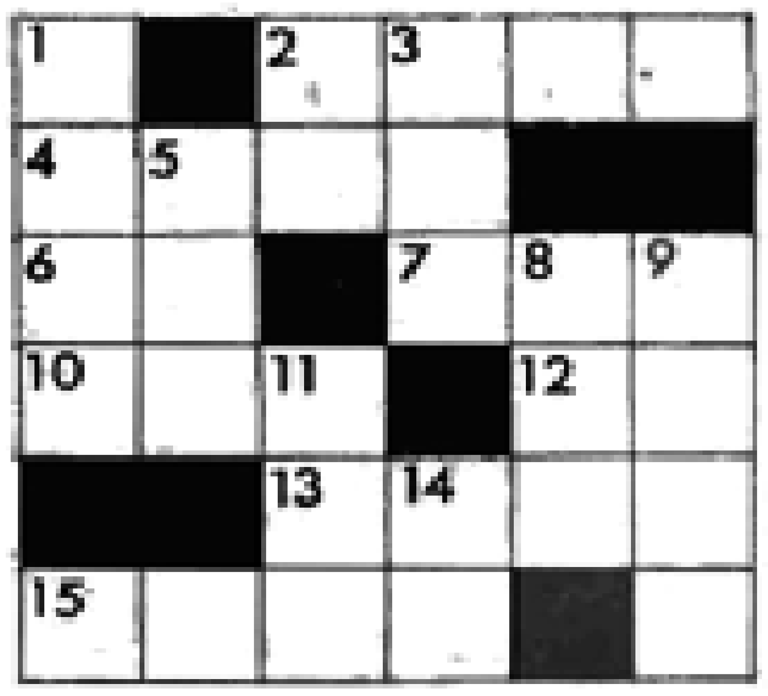
calculated by the trapezium rule.

Clues Down

1. 131 in base six written in denary.
2. Column vector which results from applying a translation $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ to $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$
5. Probability of throwing three successive heads with an unbiased coin.
6. If y is proportional to x^2 and $y=3$ when $x=1$, find y when $x=2$.
7. (Digit sum of 1 down)+(digit sum of 2 down) written in base 9.

P.J.G.

JUNIOR CROSS FIGURE No. 57



Clues Across

2. Next term in the sequence 1, 11, 121, . . .
4. Only palindromic year of this century.
6. Unlucky Friday.
7. Yards in a quarter mile race if 6 runners compete? N.B. 3 feet make 1 yard.
10. No. of days in 1972.
12. 3x, see 8 down.
13. Next two terms in the sequence (4^2-3^2), (5^2-4^2), (6^2-5^2), . . .
15. First twenties prime number followed by twice that number.

Clues Down

1. First two 2-digit primes.
2. Prime number with digit sum of 10, whose digits are both squares.
3. π to three significant figures.
5. $10^3-(3^2-1)^2$.
8. x^2+x-1, when x is 21.
9. Square of a palindromic No. some of whose factors is a palindromic prime.
11. "Anagram" of 8 down.
14. First square whose digit sum is also a square.

P.J.G.

SOLUTIONS TO PROBLEMS IN ISSUE No. 64

TEST YOUR BRAIN POWER

There are only ten different digits and all are used, for example in the powers of x or y.

ON THE SURFACE

The radius of the circle is $8\sqrt{2}/3$ inches and the radius of the largest circle is 6 inches when the radius of the compasses is $6\sqrt{2}$ inches.

A CUTTING PROBLEM

The maximum number of pieces is (a) 2, (b) 4, (c) 7.

CHANGING THE PROBLEM

Delete a 1, three 7's and two 9's to leave $11+9=20$.

GRAND REUNION

The big celebration will take place after 420 days. 420 is the L.C.M. of 1,2,3,4,5,6 and 7.

A CHAIN LINK

If the three links of one piece were cut and used to join the remaining pieces, the cost would be 9p.

SENIOR CROSS FIGURE No. 60

Clues Across: 1. 5000211; 3. 73; 6. 68; 7. 81; 8. 17; 9. 14; 10. 72; 11. 11; 12. 21; 13. 1614186.

Clues Down: 1. 5178111; 2. 1087216; 4. 31416; 5. 14114; 6. 61728.

JUNIOR CROSS FIGURE No. 56

Clues Across: 1. 23; 3. 85; 4. 81; 5. 649; 7. 11; 9. 97; 11. 390; 12. 72.

Clues Down: 1. 254; 2. 111; 3. 868; 6. 999; 8. 121; 10. 707; 11. 36.

AN OLD PROBLEM

The inclination of the face is very nearly 52°.

B.A.

MORE MATHEMATICAL SHORTHAND — PART 2

It may be that for some particular reason all the letters of the alphabet cannot be used in coding our messages. For example, it might be thought that O, I and Z could be too easily confused with the numerals 0, 1 and 2, so it is often necessary to state which letters may be used. To do this our mathematician will use the letter U, and will write, for example —

$U =$ (a,e,i,o,u) if only the vowels are permitted, or

$U =$ (a,b,c..........x,y,z) if the whole alphabet is available.

Now let us return for a moment to our riddle-me-re. Suppose the second line had read —

" My second in apples but never in pears." (See issue No. 50).

This means that of the letters available for coding (in this case the whole alphabet) those in D, where $D =$ (p,e,a,r,s) are excluded for this line, and our mathematician calls the letters he can still use D'. So in this case $D' =$ (b,c,d,f,g......n,o,q,t,u . . . z). How then do we write the possible letters for this new line? Well, they will appear in $C =$ (a,p,l,e,s) and also in D', and we write this, you will remember, as $C \cap D'$. This doesn't leave much choice, and you will quickly see that $C \cap D' =$ (l).

Of course, we now have many more ways of writing our coded words. For example, what would this be when it was unscrambled?

$U =$ (e,p,i,g,r,a,m)
$A =$ (r,i,p,e)
$B =$ (m,a,r,g)
$A \cap B' =$?

B' is the set of letters in U which are **not** in B, *i.e.*, e,p, and i. Therefore $A \cap B' =$ (e,p,i) or the word pie.

Here then is a second message for you to decode.

1. $O =$(a,e,i,o,u,y)
   $P =$(o,q,s,u,w,y)
   $O \cap P =$?
2. $Q =$(l,o,u,d)
   $R =$(l,u,s,h)
   $Q \cap R =$?
3. $U =$(m,n,o,p,q......z)
   $S =$(m,p,s,v,y)
   $T =$ (o,w,n)
   $S' \cap T =$?
4. $U =$(a,b,c,d,e,f,g,h,i)
   $V =$(h,a,i,g)
   $W =$(c,d,f)
   $(V \cup W)' =$?
5. $X =$(a,e,i,o,u)
   $Y =$(e,v,a,s,i,o,n)
   $Z =$(v,a,s,t)
   $X \cap (Y \cap Z) =$?
6. $U =$(a,i,o,u,c,d,e,f,g,n)
   $A =$(a,f)
   $B =$(e,u)
   $A' \cap B' =$?
7. $U =$(a,b,c,d,e,m,n,o,p,q,r,s,t)
   $C =$(b,c,d,e,m,n,o,p,q)
   $D =$(b,c,d,n,o,p,q,s,t)
   $(C \cap D)'$?

S.T.P.

QUOTE

Behind the artisan is the Chemist, behind the Chemist a Physicist, behind the Physicist a Mathematician. W. H. WHITE

The early study of Euclid made me a hater of geometry,....and yet, in spite of this repugnance, which had become a second nature in me, whenever I went far enough into any mathematical question, I found I touched, at last, a geometrical bottom.

I should rejoice to see....Euclid honourably shelved or buried "deeper than did ever plummet sound " out of the schoolboy's reach.

J. J. SYLVESTER (1814–1897)

POLLUTION PROBLEM

Two similar tanks stand side by side and contain the same volume of liquid which are different. An attendant removes one gallon of the liquid from one tank but falls and pours it into the other tank. To rectify his mistake, the attendant removes a gallon of the mixture from the second tank and puts it into the first tank. Which of the two tanks is the more polluted? Would it make any difference whether the liquids were thoroughly mixed or not?

R.H.C.

SENIOR CROSS FIGURE No. 61

1		2	■	3	4	
	■	5		■		■
■	6		■	7		8
9	■	10			■	
11	12		■	13		■
■		■	14		■	15
16			■	17		

Ignore decimal points in answers.

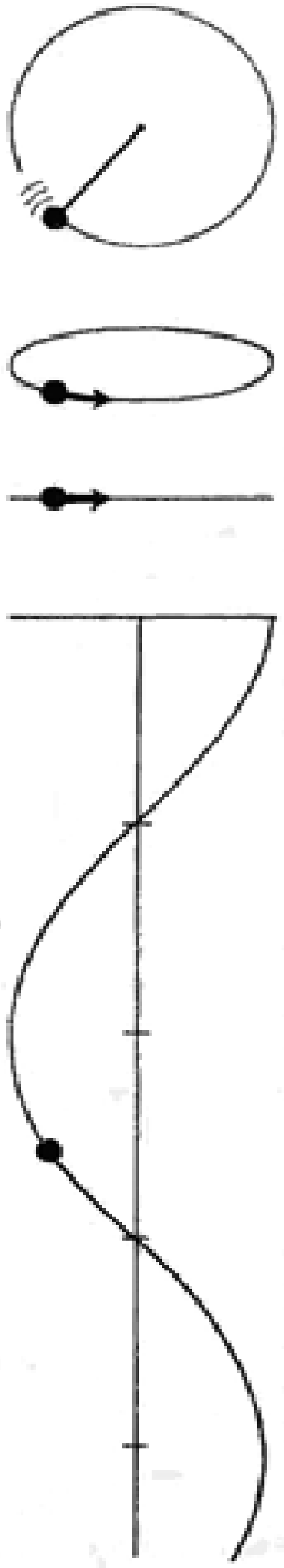
CLUES ACROSS

1. sin 60°.
3. Product of the roots of the equation $2x^2 - 17x + 35 = 0$.
5. Square root of $9x^2 + 12x + 2$ when $x = 21$.
6. $81^{\frac{1}{4}} \times 16^{0\cdot25} \times 256^{0\cdot125}$.
7. The principal which produces an interest of £10-20 in 8 months at 10% per annum.
10. Time difference in minutes between two places whose latitude differ by 156°.
11. Area between $y = x^3 + 3x^2 - 2x + \frac{3}{20}$, x-axis, y-axis and $x = 5$.
13. Largest 2-digit prime whose digit sum is a square number.
14. Arithmetic mean of 8 down and 13 across.
16. Maximum value of $x^3 + 6x^2 - 15x + 71$.
17. Coefficient of x^5 in the expansion of $(1+x)^{10}$.

CLUES DOWN

1. Sum of the first 18 terms of an A.P. whose 3rd term is $1\frac{14}{17}$ and 11th term is $5\frac{2}{17}$.
2. Probability of obtaining at least one score greater than 4 in two consecutive throws of a die (correct to 5 decimal places).
4. " Anagram " of 11 across such that $\frac{1}{29}$th of the answer is a square number.
7. $\log_{10} 10\pi$.
8. $\log_{10}(7!)$ to 1 decimal place.
9. 7π if π has its usual fractional approximation.
12. If $y \propto x^4$ and $y = 112$ when $x = 2$, calculate y when $x = 3$.
15. Cube root of 1728.

P.J.G



If you have ever swung a " conker " round in a horizontal circle you may have noticed that from a viewpoint on the same level as the circle, the conker seems to move back and forth along a straight line as suggested in the diagrams alongside. The speed seems to be greater near the centre of the line than it is near either end. This type of " flattened circular motion " is called Simple Harmonic Motion, often abbreviated to S.H.M., and it is typical of the types of movement called oscillations.

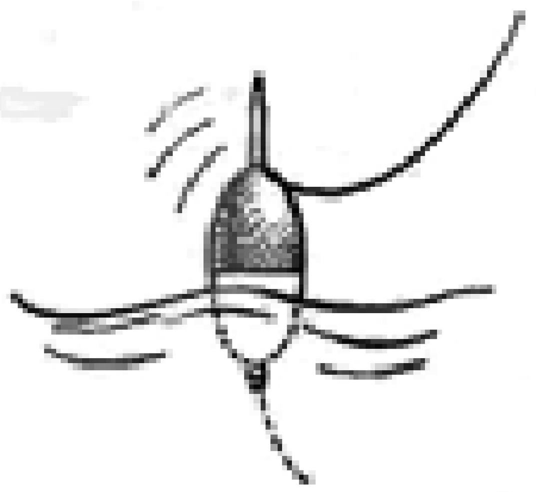
The variation in distance from the mid-point varies as shown by the graph which you may know is called a cosine curve. Bobbing floats and masses hanging on springs oscillate in this way, see below, and the variation in strength of the electric current supplied by some types of generator follows the same type of pattern.

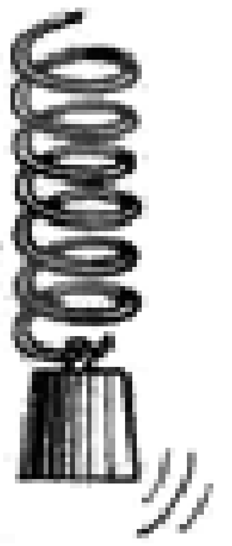
You might like to try the graph paper, shown on the right, based on cosine scales ; draw a square on a piece of plain paper and draw semi-circles on two adjacent sides. Divide the semi-circles into some number of equal parts, the diagram shows six but you might try more. Horizontal and vertical lines drawn as shown produce the graph paper we want. It is best to keep this, and to draw the graphs on tracing paper placed over it.

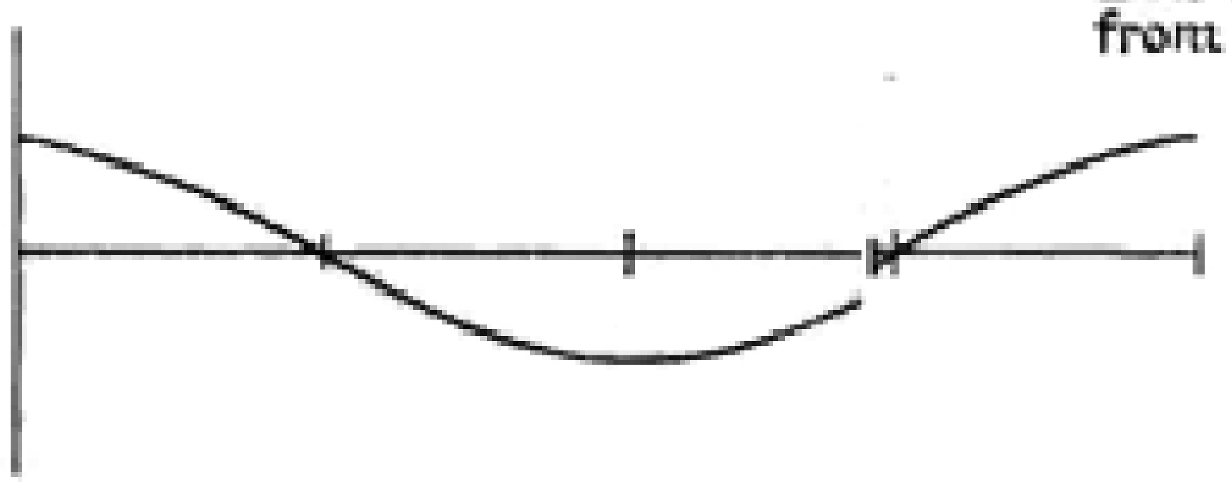
Taking any point on the graph paper we may draw some interesting patterns by following this procedure : Choose a pair of numbers, e.g. 1 and 2. Starting anywhere, move 1 space horizontally and 2 spaces vertically. " Bounce " off the edges if necessary, as shown in the lower diagram opposite.

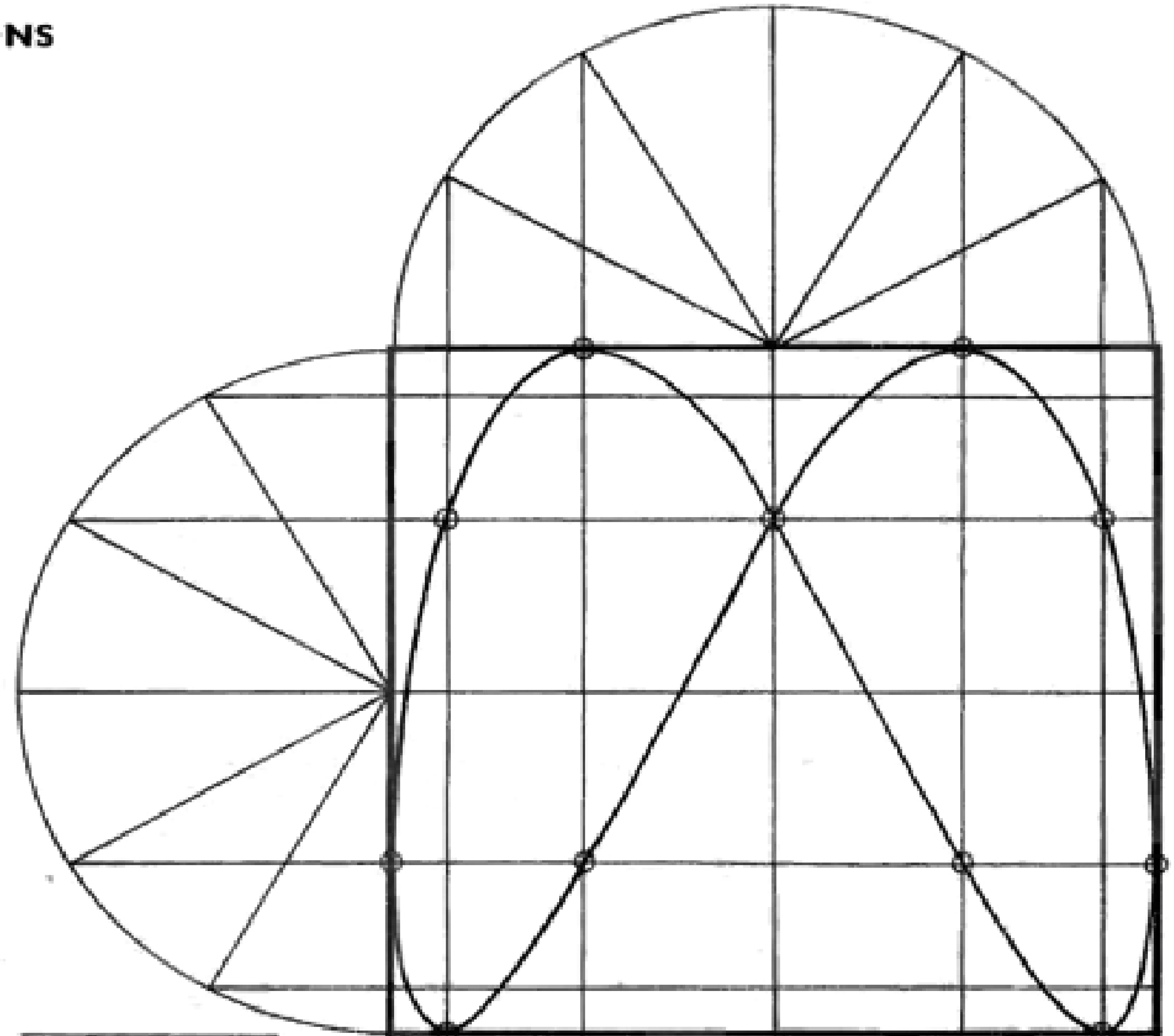
Continue in this fashion until you return to your starting point. Draw a curve through the points in the order they were plotted. The shapes produced are called Lissajous Figures, and can be obtained by feeding two different electric currents into an oscilloscope, which is a simple version of a T.V. screen. Can you find a connection between the shape of the curve and the pair of numbers you choose? Electrical engineers use this idea to find the frequency of oscillation of an electric current.

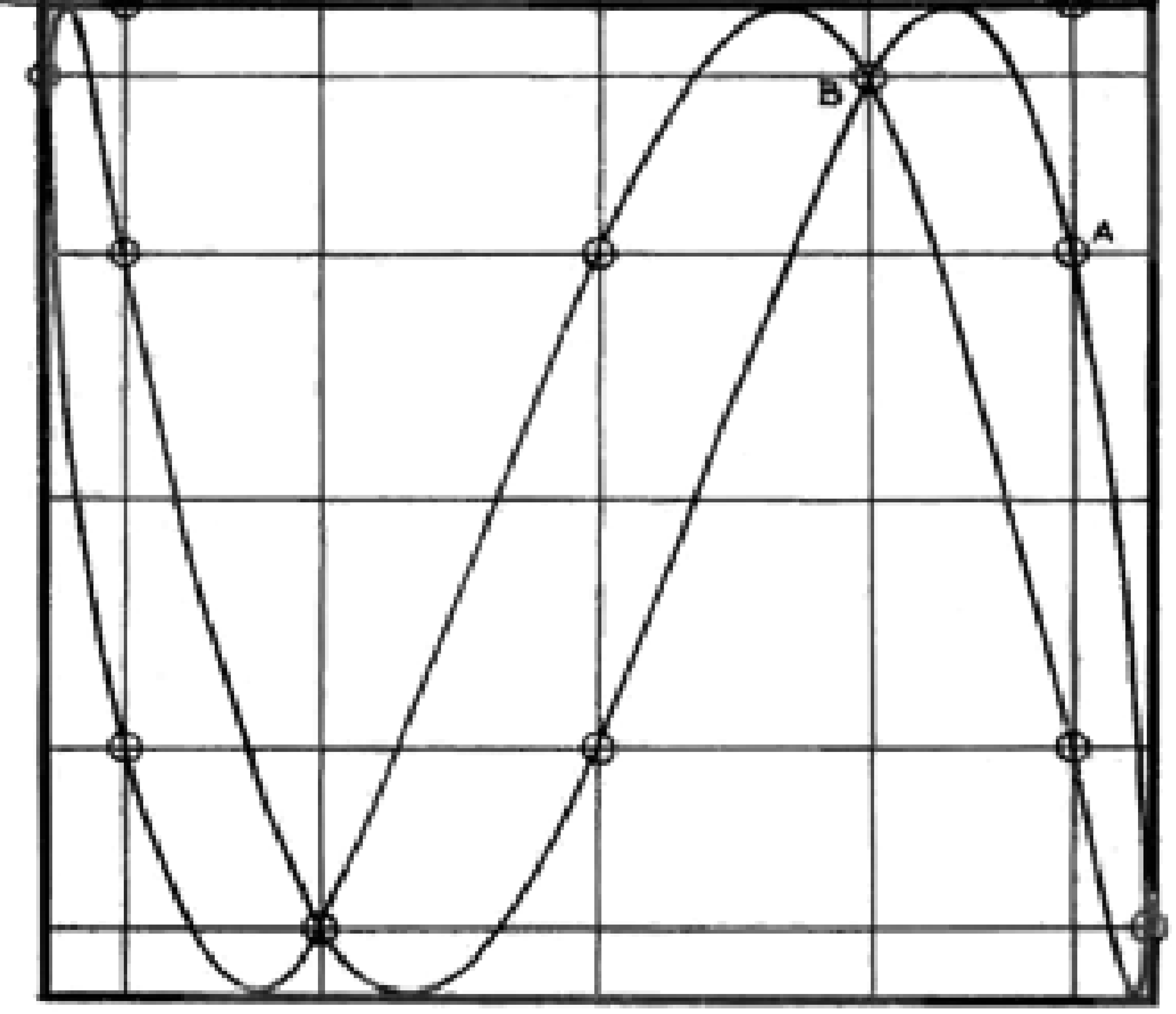
E.G.

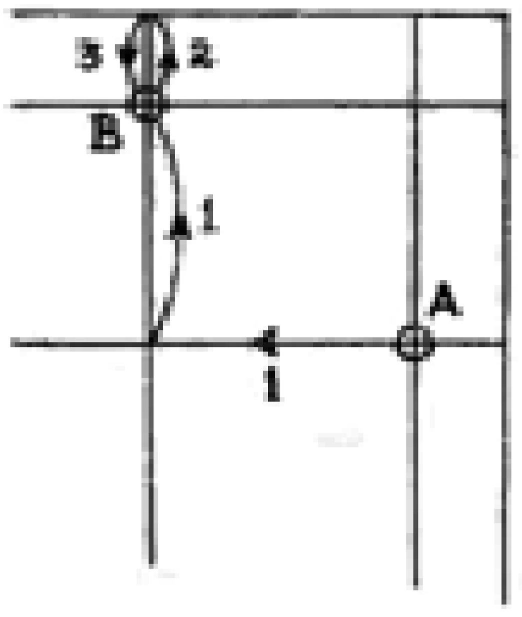












"Bouncing" from the top edge to move 1 horizontally and 3 vertically from A to B.