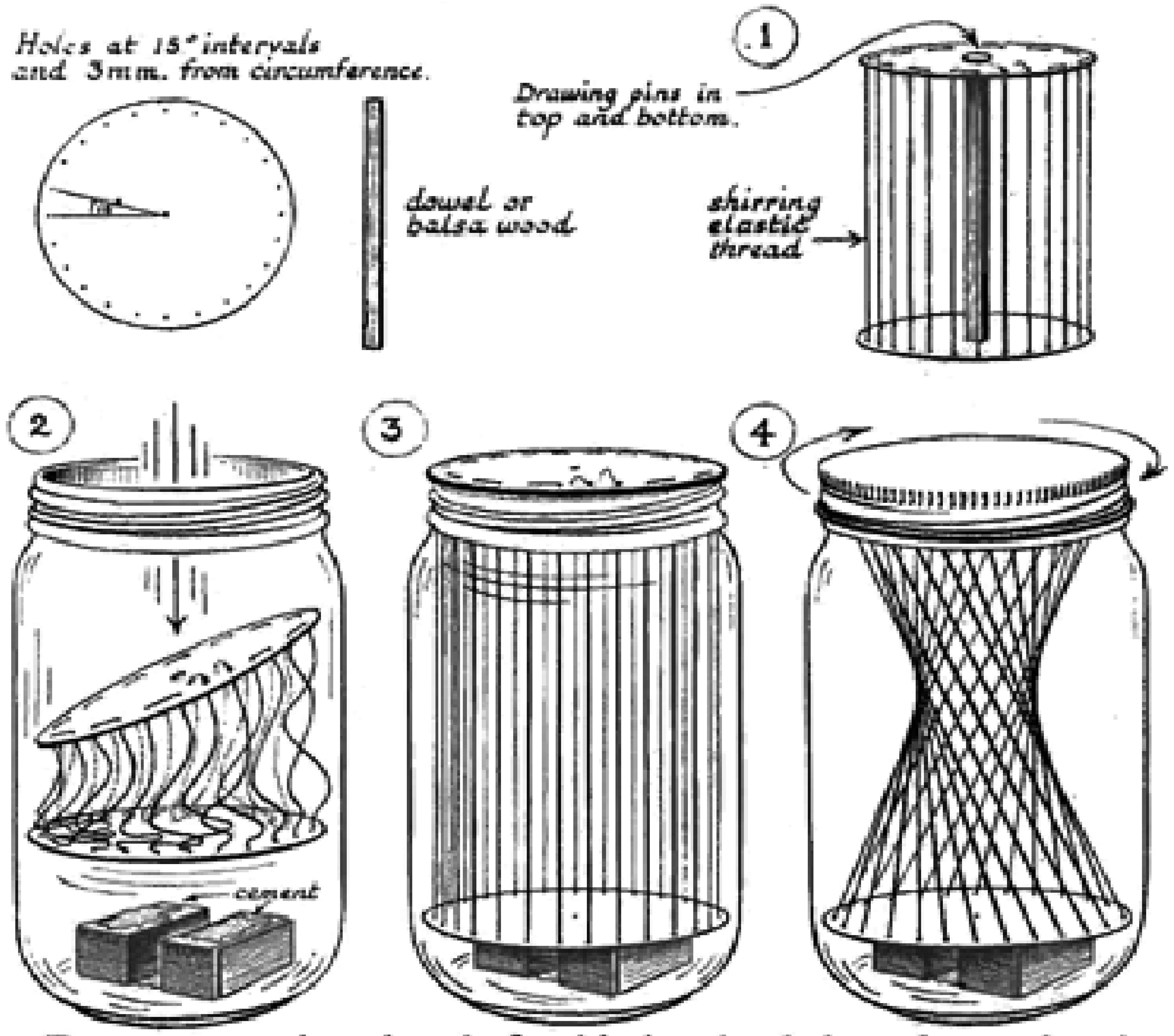
BOTTLING IT UP



Everyone must have heard of a ship in a bottle but who can imagine a mathematical model in a jar? The main requirements for this model are a cylindrical jar, some stiff card, shirring elastic thread and a piece of dowel.

Two circular discs are cut from the card: the diameters should allow one disc to fit closely within the jar and the other to just overlap the neck. With a needle, pierce holes at 15 degree intervals around each disc and at about 3mm from the circumference.

As most jars taper slightly at the base, some packing is usually needed to raise the level of the lower disc. A dissected match box stuck to the inner base with compact adhesive is suitable for this purpose.

Cut a piece of dowel (e.g. rigid strip of balsa wood) which is of a length equal to the distance from the neck of the jar to the top of the packing. With drawing pins, attach the centres of the discs respectively to each end of the dowel. Thread the shirring elastic between the two discs as shown in the diagram to form a cylindrical ruled surface.

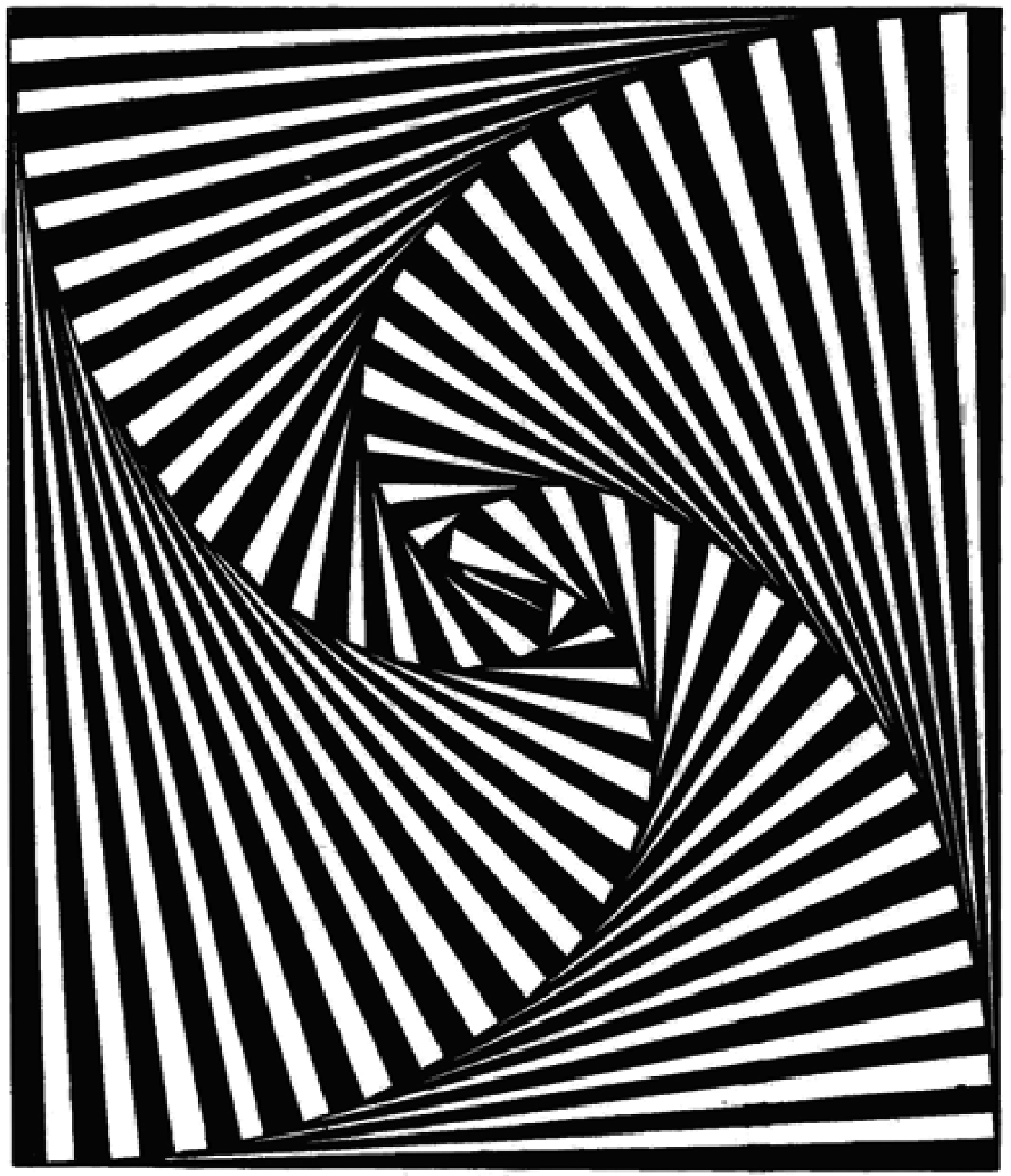
Remove the dowel and push both discs into the jar, firmly attaching the lower one to the top of the packing with quick-drying adhesive cement. When the cement has set, the upper disc can be removed from the jar and placed over the neck, thus exposing the ruled surface generators. By screwing the lid on the jar, the upper disc can be rotated relative to the lower, generating a hyperboloid of revolution.

Readers will probably think of further bottling possibilities, perhaps incorporating a battery and bulb for display. D.I.B.

join the mathematical pie

No. 64 — *Editorial Address:* Alpha House, The Avenue, Rowington, Warwickshire, England — AUTUMN, 1971

PATCHWORK PATTERNS No. 4



TEST YOUR BRAIN POWER

The expansion of $(x+y)^5$ is $x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$.

The expression uses 14 digits including the indices. How many different digits would be used in writing out the expansion of $(x+y)^{100}$?

C.V.G.

NUMBER PATTERNS

$3\times37=111$ and $1+1+1=3$
$6\times37=222$ and $2+2+2=6$
$9\times37=333$ and $3+3+3=9$

How far does this pattern extend?

Similarly

$3\times34=102$ and $1+0+2=3$
$6\times34=204$ and $2+0+4=6$

Can you find any other numbers apart from 37 and 34 that do this?

Now try

$4\times28=112$ and $1+1+2=4$
$8\times28=224$ and $2+2+4=8$

Does this work for any number other than 28?

R.H.C.

SUBSTITUTION

$$\begin{array}{r} M\ A\ T\ H \\ +\,E\ M\ A\ T \\ \hline I\ C\ A\ L \end{array}$$

Find eight digits to replace the eight letters used to make the addition correct.

D.I.B.

ON THE SURFACE

P is a point on a sphere of radius 6 inches. A pair of compasses is opened to a radius of 4 inches and a circle is drawn on the sphere with centre P. What is the radius of the circle?

What is the radius of the largest circle that can be drawn on the sphere?

B.A.

A CUTTING PROBLEM

What is the maximum number of pieces into which an annulus (i.e. the area between two concentric circles) can be cut with (a) 1 cut, (b) 2 cuts, (c) 3 cuts?

Can you find a general formula for this operation?

E.G.

CHANGING THE PROBLEM

$$\begin{array}{r} 1\ 1\ 1 \\ 7\ 7\ 7 \\ 9\ 9\ 9 \\ \hline \end{array}$$

Strike out six of the digits in the addition on the left so that the remaining numbers will have a sum of 20.

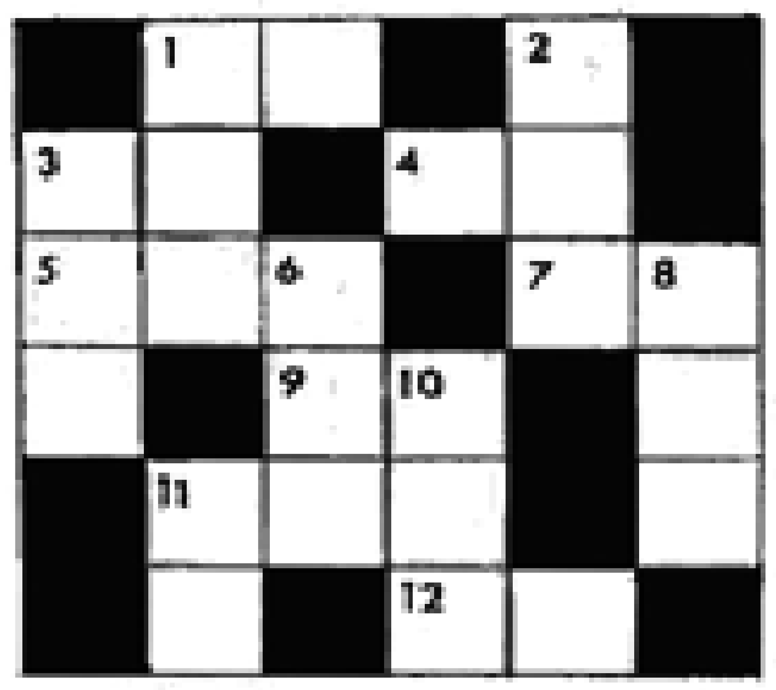
R.H.C.

MAGICAL SEVENS

Using four sevens and any operation sign, make up all the numbers from 1 to 15.

R.H.C.

JUNIOR CROSS FIGURE No. 56



Ignore decimal points.
Work to the required accuracy.

CLUES ACROSS

1. First 2-digit prime whose digits are prime.
3. Change from £1 after buying 3 pints of milk at 5p per pint.
4. A square number.
5. $54+5^2-50$.
7. Square root of 8 down.
9. Last 2-digit prime.
11. Inches in 1 metre.
12. $2x^2+3x+7$ when x is 5.

CLUES DOWN

1. CCLIV.
2. One ninth as a decimal.
3. Minimum product of the number of days in 2 consecutive months in 1971.
6. Emergency call?
8. Palindromic square of a prime number.
10. Palindromic jumbo jet?
11. Exterior angle of a regular ten-sided polygon.

P.J.G.

AN OLD PROBLEM

Herodotus said that the area of an inclined face of a square based pyramid was equal to that of a square described on its altitude. What value does this condition give for the angle which the plane face makes with the base?

R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 63

How many cigarettes? Nick smoked 8 specials by using the stubs from his specials to make another special.

He had 6 stubs left.

Old Gasper smoked 9 specials; 8 as Nick, he then borrowed a stub from Nick to add to his stubs to make a ninth special and returned the stub after smoking his last special. He was scrupulously honest.

PYTHAGORAS AT HOME

The room is 8m by 6m, so the diagonal is 10m.

CUT UP A TRIANGLE

Take any point inside the triangle and draw perpendiculars to the three sides. There are infinitely many ways.

SENIOR CROSS FIGURE No. 59

Clues Across: 1. 1980; 4. 15; 5. 79; 6. 12; 7. 125; 8. 864; 10. 973; 11. 162; 13. 256; 15. 59240; 17. 72; 18. 55.

Clues Down: 2. 972; 3. 895929; 4. 126; 6. 18324; 7. 121; 9. 496; 12. 652; 16. 24.

LET'S FACE IT

a, b, c, are impossible with plane faces, d is a tetrahedron, e is a rectangular based pyramid, f could be a cuboid or a pyramid with a pentagonal base.

BODMAS

The fraction is 27/11.

JUNIOR CROSS FIGURE No. 55

Clues Across: 1. 144; 5. 196; 7. 156; 9.24; 11. 52; 12. 180; 13. 540; 15. 289.

Clues Down: 2. 416; 3. 49; 4. 115; 6. 628; 8. 523; 10. 404; 12. 108; 14. 42.

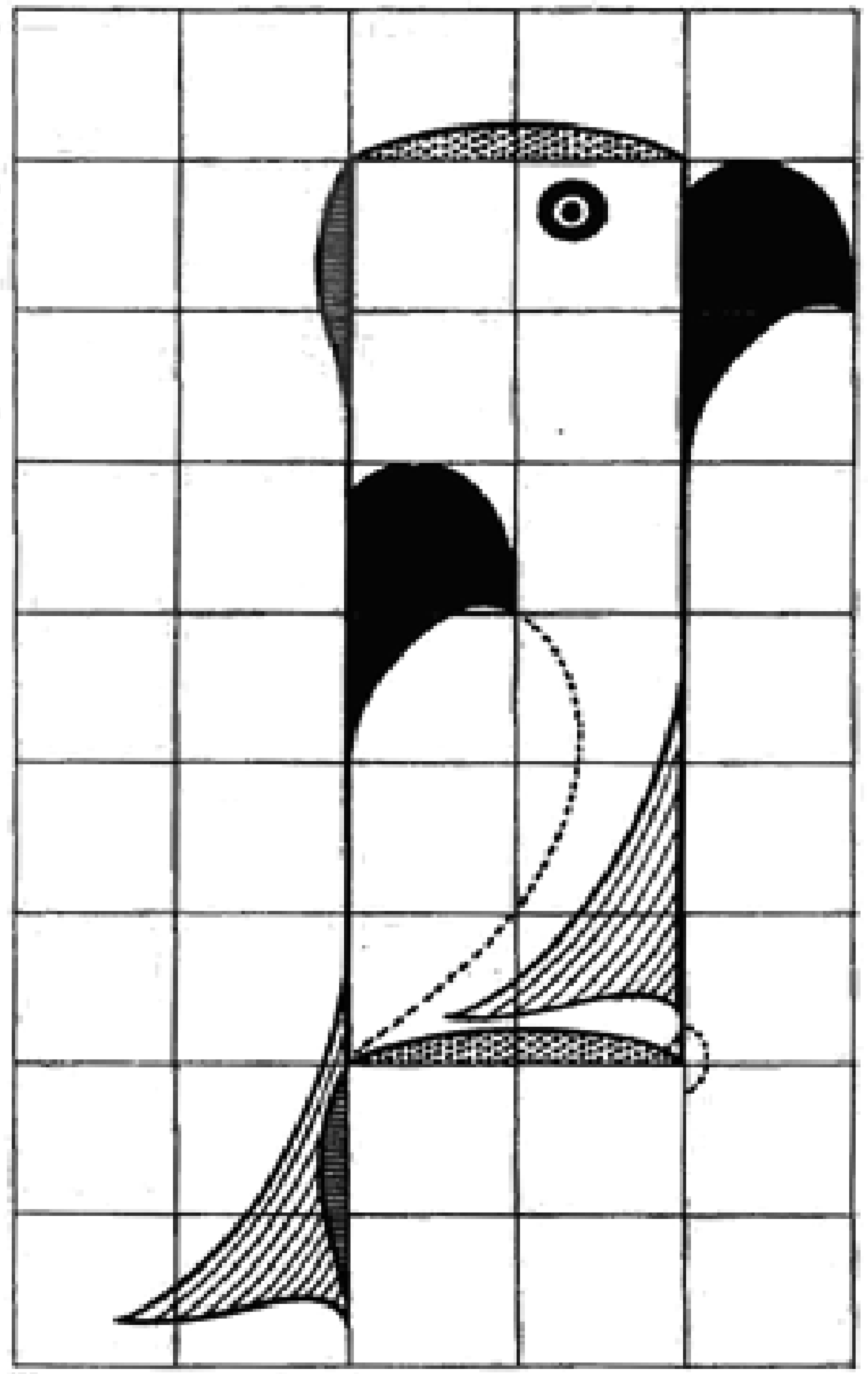
The clue to 1 Across should have read " Degrees in an interior angle of a decagon.

A RADICAL PROBLEM

The values were 3.464, 3.932, 3.991. If the process is continued the value approaches 4.

B.A.

TESSELLATIONS

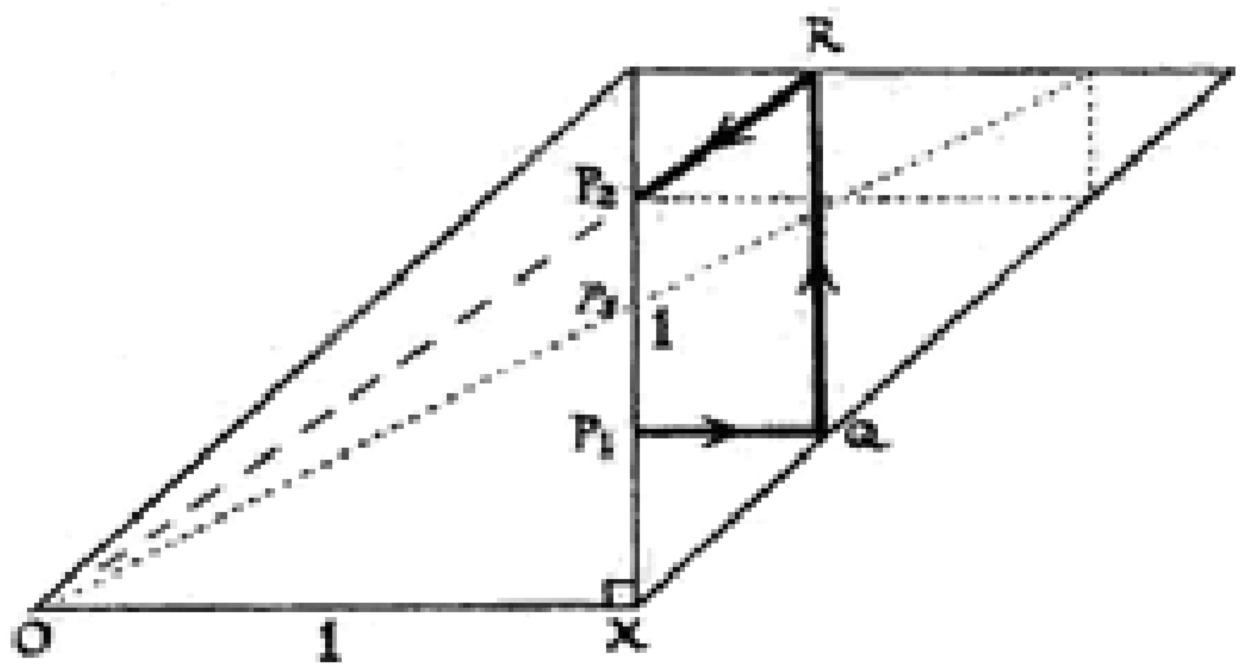


It is well known that all triangles will form a tessellation; all quadrilaterals and some hexagons too, see issue No. 27. Other figures will also fill an area. The figure pattern on the front page is a curve of pursuit formed by four points chasing the one in the next vertex. These figures will tessellate and the parrot is also a shape that will form a tessellation. The basic shape is the rectangle but there have been some minor alterations which are clearly shown.

If anyone has other interesting shapes that tessellate, the editor will be pleased to receive them and send a book token to the originator.

E.G.

THE ABSOLUTE LIMIT



Draw a parallelogram as shown. Take any point P_1 on the vertical line. Draw P_1Q horizontally and QR vertically. Draw from R towards O to find a new point on the vertical line, P_2. Repeat the process taking P_2 as your new starting point.

What will be the length of XP_n if n is a very large positive, whole number?

E.G.

GRAND REUNION

Seven good friends dine in the same restaurant. All are eating there today; however each one does not eat in this restaurant every day. The first eats every day, the second every other day, the third every third day, the fourth every fourth day, the fifth every fifth day, the sixth every sixth day, and the seventh every seventh day.

When the friends again all appear in this restaurant on the same day, they will have a big celebration. In how many days from today will this celebration take place?

P.J.G.

QUOTE

Mass times acceleration equals force which means distance. This is Newton's first law of motion. (B.B.C. Sportsview introducing an item on body-building drugs used by athletes).

S.T.P.

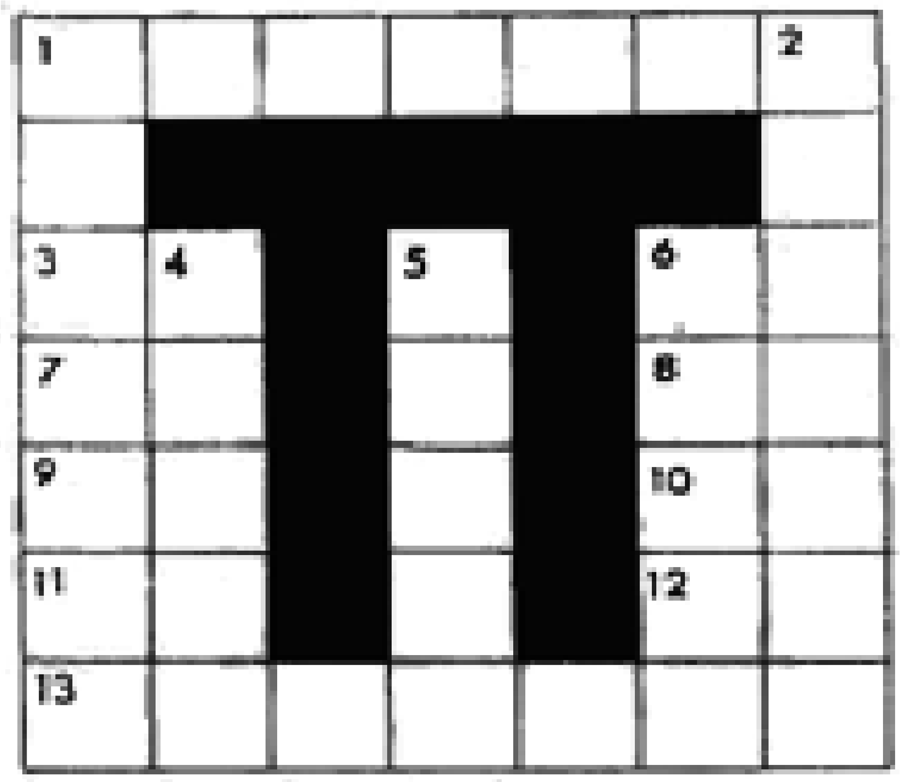
A CHAIN LINK

A man went to a machine shop to have a broken chain repaired. He had five separate pieces of chain to be fastened together and each piece had 3 links attached. The rate for the job is 1p to cut a link and 2p to weld a link. The machinist claimed that he would charge 12p for refastening the chain.

The man argued that the job could be done for 9p if it were done most efficiently. How did the machinist think he would do the job and what had the man in mind?

P.J.G.

SENIOR CROSS FIGURE No. 60



Use $\pi = 3{\cdot}142$ unless otherwise stated. Ignore decimal points.

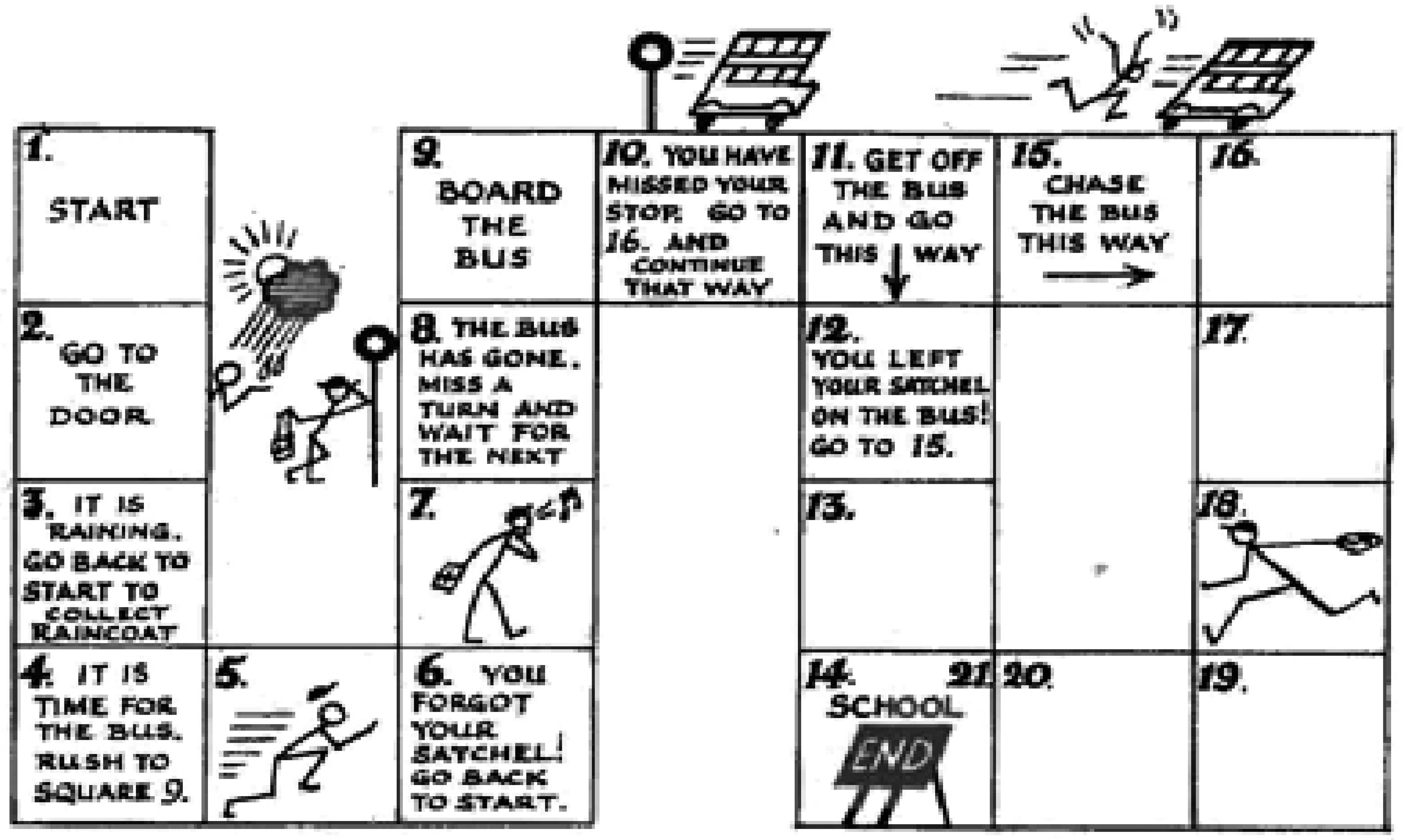
Clues Across

1. Cube of the sum of the first 18 positive integers.
3. Area between $y = 3x^2 + 2x + 3$, x-axis, $x = 4$, and $x = 5$.
6. Next term in the sequence 8, 12, 20, 28, 44, 52.
7. $3^{12} \div 6^8 \times 4^4$.
8. Number of dots visible on a die if the 4 is face downwards.
9. They come in all weights in nature, but mathematical ones are this weight only.
10. Half a " large " French square number.
11. Minimum value of $2x^2 - 8x + 19$.
12. Product of odd primes, the digit sum of the answer being one of the primes used.
13. Roots of $x^2 - 18x + 30 = 0$ correct to two decimal places the larger root first.

Clues Down

1. Fourth, fifth, and sixth terms of the series whose rth term is $3(r^2+1)$.
2. $\pi^2 + 1$ correct to seven significant figures.
4. π to 4 decimal places.
5. $\log_{10}\sqrt{(665)}$, use 4 figure logs.
6. First two perfect numbers, excluding 1, separated by their arithmetic mean.

P.J.G.



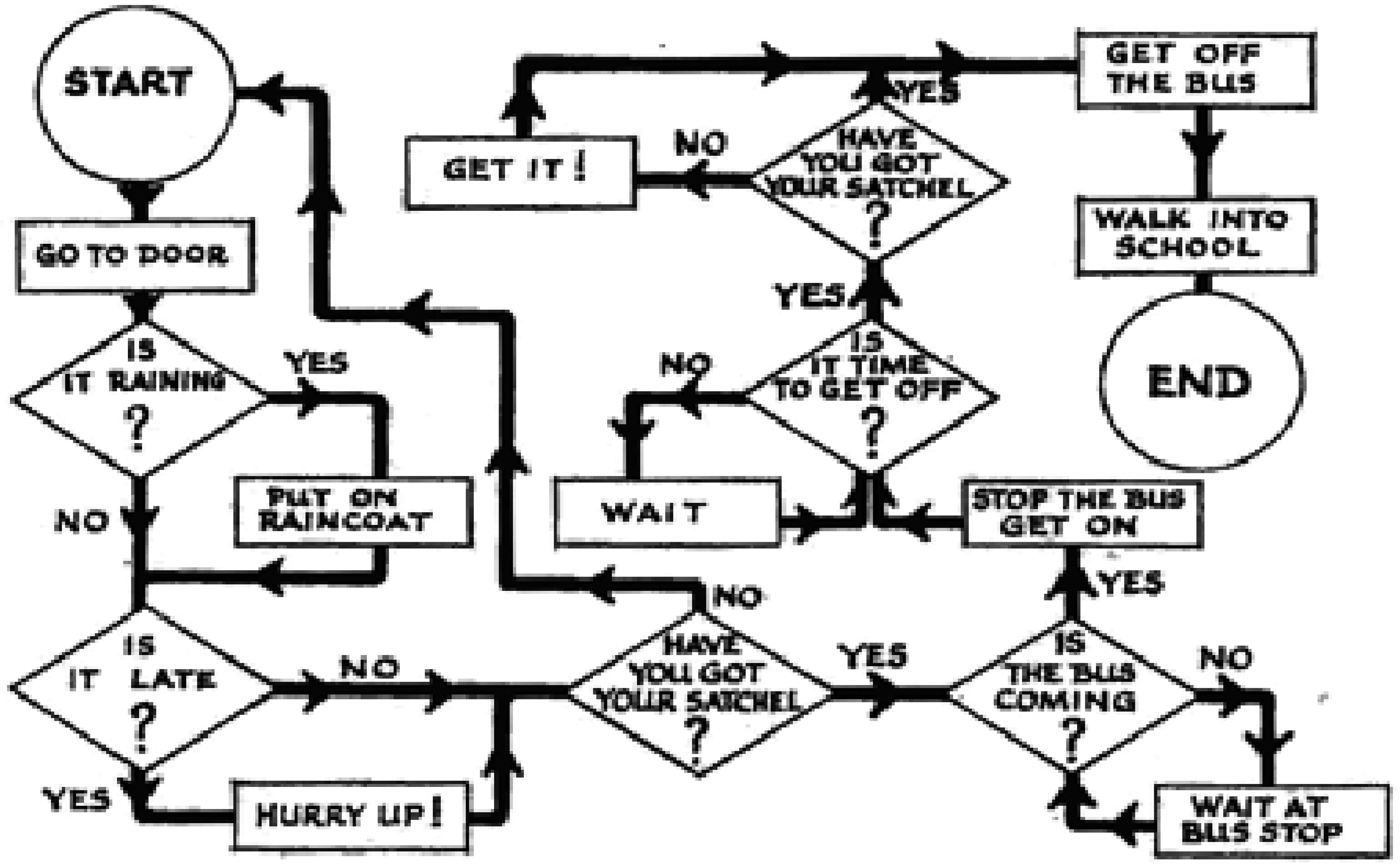
TO PLAY: Each player has a counter. Moves are made by spinning a coin: move one space for 'tails' and two spaces for 'heads'. Follow the instructions when you land on a square which has any.
To finish you must land exactly on the 'school' square.

The illustration shows a simple game " Going to school by 'bus ", which emphasises the number of different things which need to be done in order to complete the journey successfully. It is really quite a feat of memory, yet thousands of pupils do it every weekday quite easily, because they have learned the things they need to do, and they are intelligent enough to make decisions on their own. A robot schoolboy—or schoolgirl—would have more difficulty. A computer " brain " (not a very good name, since it cannot do many of the things a human brain can) can only remember, and follow, any instructions it is given, and this means that every single decision, however trivial, must be included in these instructions.

The computers used in industry need to be told everything to do, too, and this is why there are such people as computer programmers. These people have to think of every step that the computer will need to take to finish a calculation, and write it in such a way that the computer will " understand " it.

The first stage in writing a programme for a computer is to write a " flow diagram ". One is shown . . . . . . This flow diagram, like all others, is followed in the same way as the game, except that instead of numbering all the squares, arrows are put in between them to show which way the instructions " flow ". Also, of course, the squares have changed to circles, rectangles or rhombuses.

Circles are used to connect parts of programmes together and so are called " connector boxes ". Most often these appear only at the beginning and the end of the programme.



Rectangles contain instructions (instruction box), Rhombuses contain questions. The answer to each question fixes which " exit " we take when we have made our decision (decision box). The flow diagram shown has, in places, improved on the game so that anyone following the instructions would not waste so much time; but even so there has not been room to say everything—for instance, I have taken it for granted that—

(i) To catch a 'bus, anyone would go to a 'bus-stop.

(ii) No-one would get off a 'bus while it is moving !

Even these simple ideas would have to be put into the flow diagram for our robot pupil ! Try drawing a flow diagram to explain how to achieve some other everyday tasks, e.g., making a cup of tea, crossing a road, etc., or how to play a game such as snakes and ladders (this is easier since there are fewer things to do). In a later issue we will look at the way in which a flow diagram can help with some numerical calculations.

E.G.