join the mathematical pie

No. 63 *Editorial Address:* 100, Burman Road, Shirley, Solihull, Warwickshire, England SUMMER, 1971

A ROUND TRIP

Diagram 1.

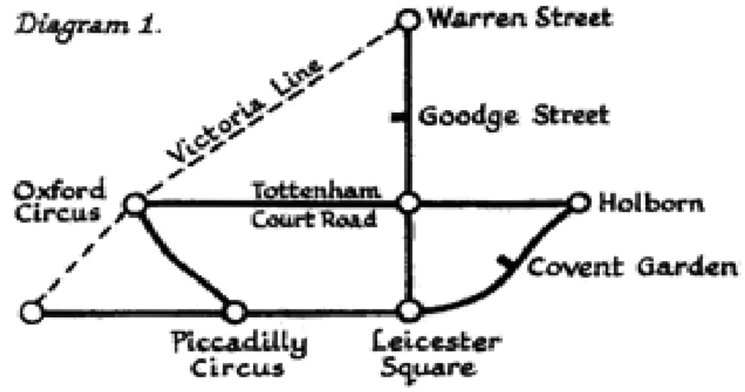
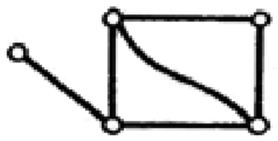


Diagram 1 shows part of the underground system in Central London. Is it possible to completely journey over the section shown without travelling more than once between any two stations? Could this be done before the Victoria Line was opened?

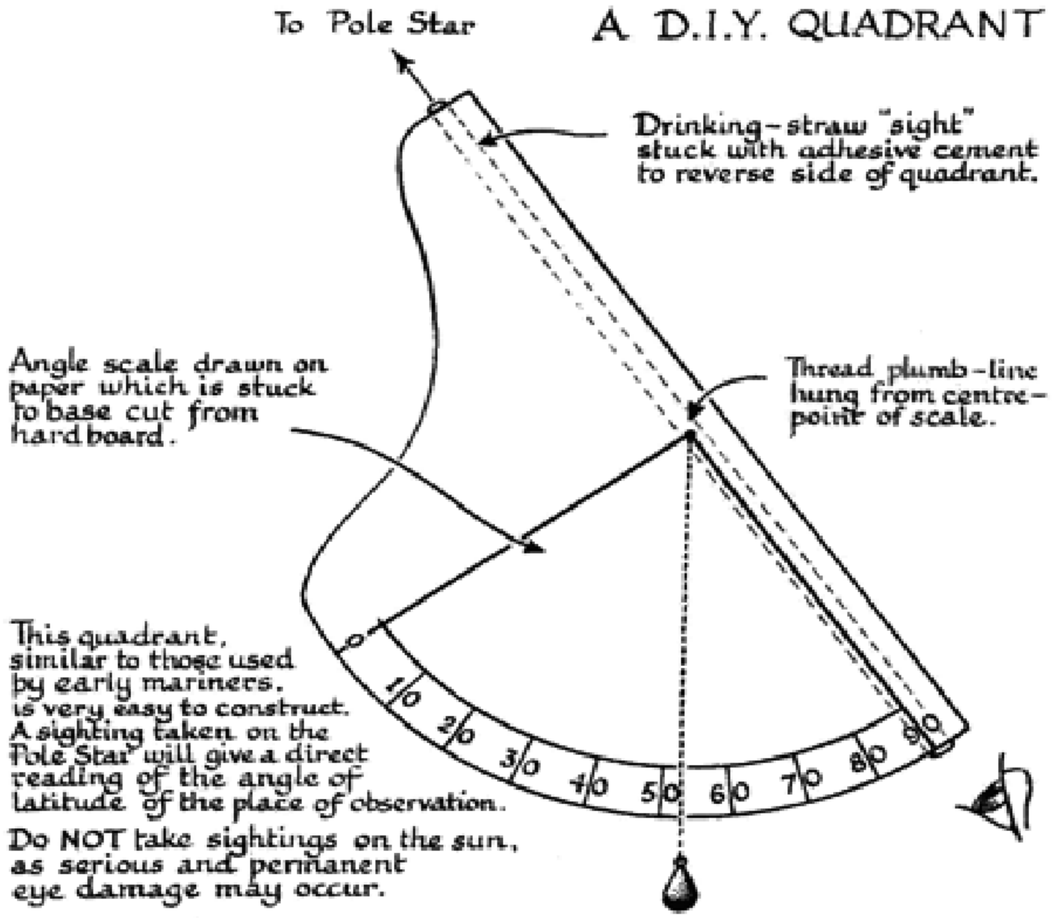
Diagram 2.



Draw several diagrams similar to diagram 2, calling the "stations" nodes and the "lines" branches. A condition is that a node must appear at the end or intersection of a branch. From each diagram and the Underground map record the frequency of nodes which have an odd number of branches and observe whether the network can be completely drawn without continuing over the same line more than once. For example, you will notice that the diagram shown has four odd nodes (1, 3, 3 and 3 branches) and that it cannot be drawn as required.

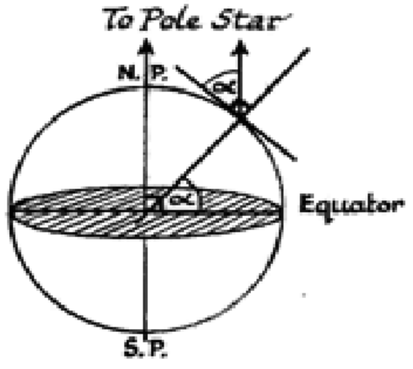
Try to find a rule which determines drawings that can be drawn in this way and give an explanation. E.G.

A D.I.Y. QUADRANT



This quadrant, similar to those used by early mariners, is very easy to construct. A sighting taken on the Pole Star will give a direct reading of the angle of latitude of the place of observation.

Do NOT take sightings on the sun, as serious and permanent eye damage may occur.



The diagram shows how the angle of elevation (α) of the Pole Star gives the angle of latitude.

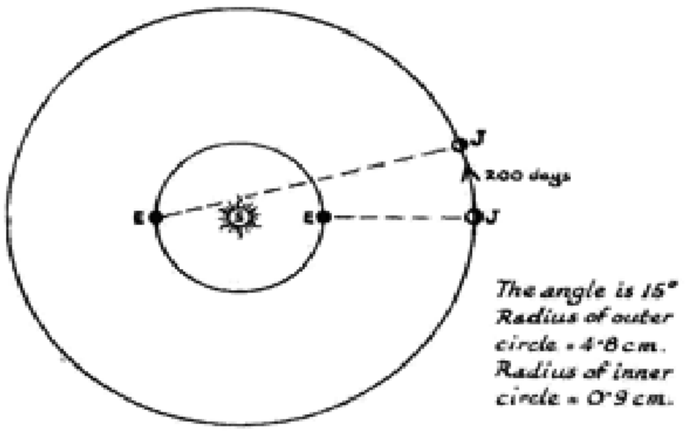
A RADICAL PROBLEM

Use your tables to evaluate $\sqrt{(12)}$, $\sqrt{(12+\sqrt{(12)})}$, $\sqrt{(12+\sqrt{(12+\sqrt{(12)})})}$.

R.H.C.

STAMP COLLECTORS' CORNER No. 25

Jupiter, 770 million Km from the Sun, takes 12 years to complete its orbit; the Earth moving faster on an inside track laps it every 400 days. If we could arrange a beacon on Jupiter flashing at regular intervals, the flash of light would take several minutes to travel to Earth. During the time when the distance between the planets is increasing, each flash will have a slightly longer distance to travel than the previous one and, therefore, will take slightly longer on the journey. To an observer on Earth, the time intervals between the flashes would appear to be longer. After the position of greatest distance had been passed, the successive flashes would have shorter distances to travel so that the observer would think that the flashes had speeded up.



Why are Jupiter's moons 16½ minutes slow every 200 days?

No Beacons have been observed on Jupiter but even Galileo's telescope showed the four largest of Jupiter's moons. The fastest of these appears to orbit in an average time of 40 hours 27 minutes. During the 200 days when the distance between Earth and Jupiter is increasing this satellite appears to be losing time until it becomes 16½ minutes "late", then for the next 200 days, it seems to speed up and restores its average.

Ole Roemer (1644-1710) realised the reason for this and used his observations of Jupiter's satellites to make the first estimate of the speed of light in 1675. By drawing a diagram, readers should be able to explain the 16½ minutes difference.

C.V.G.

HOW MANY CIGARETTES?

Old Gasper Fagg and Nick O' Teen are a couple of tramps who have never read the Medical Reports on the dangers of smoking. They both have a habit of collecting cigarette ends and using the tobacco from them to roll their own "Specials". They each reckon that it takes seven cigarette ends to make a "Special".

One particular day, Nick collected 54 stubs. How many "Specials" was he able to smoke that day? Old Gasper who also had 54 stubs that day knew a trick or two more than Nick and managed to make one more "Special" than Nick. How did he do it? I must add that Old Gasper has many faults but he is scrupulously honest with his friends.

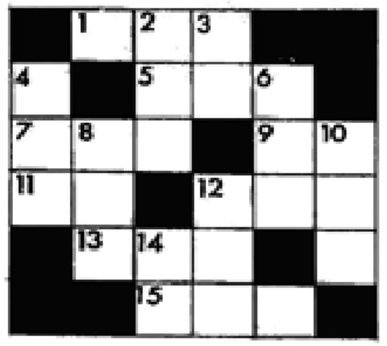
R.M.S.

BODMAS

Evaluate $\dfrac{5\div5+5\times5+5\div5}{5-5\div5+5\times5\div5}$

R.H.C.

JUNIOR CROSS FIGURE No. 55



(Ignore decimal points)

CLUES ACROSS

1. Degrees in an interior angle of dodecagon.
5. x^2, if $2x+1=29$.
7. $(n+1)(n+2)$ when $n = 11$.
9. xy, when $x+y=10$ and $2x - y = 2$.
11. Number of weeks in a year.
12. Smallest, non-zero, angle which CANNOT occur in a triangle.
13. Sum of the angles of a pentagon, in degrees.
15. Square of a prime number.

CLUES DOWN

2. The sum of the first four 3-digit odd integers.
3. (Last single digit prime)^2.
4. The principal which would yield £8.05 at 3½% after 2 years.
6. 2π correct to 3 significant figures.
8. $5^4 - 4^3 - 3^3 - 2^3 - 1^3$.
10. 4 times the first 3-digit prime.
12. Surface area, in square inches, of a cuboid 6" × 4" × 3".
14. The product of consecutive integers.

P.J.G.

SOLUTIONS TO PROBLEMS IN ISSUE No. 62

SENIOR CROSS FIGURE No. 58

The clue to 1 down should have read $(1+8)^n$.

Clues Across: 1. 74; 2. 21; 4. 20; 6. 88; 7. 2828; 9. 135; 11. 184; 13. 5588; 15. 27; 16. 64; 17. 65; 18. 46.

Clues Down: 1. 70; 3. 1888; 4. 231; 5. 18; 7. 255; 8. 218; 10. 3576; 12. 454; 14. 81; 16. 66.

JUNIOR CROSS FIGURE No. 54

Clues Across: 1. 24; 3. 312; 5. 625; 6. 162; 9. 36; 10. 99; 12. 123; 13. 798.

Clues Down: 1. 26; 2. 420; 4. 156; 6. 161; 7. 2636; 8. 297; 11. 99.

A GALAXY OF TALENT—The number of runs scored was 289. 100 to a Bellatrixter is 201 to an Aligoliwog. We must solve the equation $x^2 = 2y^2 + 1$. By trial and error, the possible values for x and y are (1,0) (3,2) (17,12) (99,70) etc. The only pair that fits the conditions is (17,12), so a Bellatrixter has 17 eyes and an Aligoliwog has 12 tentacles.

MISSING FIGURES—The problem is 37195 divided by 215 equals 173.

INCOMPLETE CHANGE—I can have four 10p pieces and one 5p piece only.

ADDITIVE POWER—The number is 1.

THE POWER GAME—The answer is 4.

AN ACADEMIC DILEMMA—September 2nd, 1752, was followed by September 14th when the change from the Julian calendar to the Gregorian.

HOW TO MAKE AN ENVELOPE—There are six cyclic quadrilaterals in the figure.

The solution to HIGH FINANCE from issue No. 61 was incorrectly given as £345,678 12s. 9d., but it should have been £234,567 18s. 9d. We have received many letters pointing this out. The first was from R. M. Trebfa of Lichfield who has been sent a book token.

R.A.

continued from page 497

As their quest for exploration increased, ancient mariners appreciated the need for more accurate navigation than the rough guidance given by the sun and familiar stellar formations. About 300 B.C., Pytheas, a Greek astronomer, was possibly the first to accurately calculate latitude. An accurate method of determining longitude was to elude sea-men for another two-thousand years until 1762 when John Harrison constructed a ship's chronometer from which the calculation could be made with an error no greater than eighteen miles.

The invention of the marine timekeeper, essential for the economic and safe navigation of the faster steam ships which were soon to follow was analagous to that of radio broadcasting on which the successful growth of air transport was later to depend. From that 17th-century observatory in a south-east London park, the Greenwich Time Signal was to give the whole world its time—and its bearings.

By enabling the positions of aircraft and ships to be shown on a cathode-ray screen, radar has made a valuable contribution to safer navigation as the sea-ways and air-ways become increasingly congested in the mobile modern world. A further recent development, the Decca Airborne Equipment Mark 15 consists of a receiver, associated computer, zone identification meter with a flight log and a controller, all operating in conjunction with ground based transmitter chains. Under this system, the pilot obtains his position directly on a flight log display head.

As man continues to travel faster, further and more frequently, his ability " to recover the place first assigned " will depend considerably on his success in continuing to perfect the art of navigation.

D.I.B.

Acknowledgements and References: The National Maritime Museum and The Greenwich Observatory; The Science Museum; " Science for the Citizen " by Lancelot Hogben.

WRAP UP!

The solution to the problem of wrapping a parcel was incorrect. The error arose because we neglected the two pieces of string down the ends of the parcel. Several letters have been received on this topic ; the first was from M. Hichens of Hartlepool, who receives a book token. The maximum volume is $1\frac{1}{2}$ cubic feet, and the dimensions are 2 ft. by 1 ft. by 8 in. An analytical solution involves partial differential equations which are beyond most of our readers but a graphical solution would also lead to the answer.

LET'S FACE IT

Can you name a solid that has exactly :—

(a) one face, (b) two faces, (c) three faces, (d) four faces, (e) five faces, (f) six faces ?

R.H.C.

PYTHAGORAS AT HOME

A rectangular room has a perimeter of 28m and its width is three-quarters of its length. What is the length of the diagonal of the room?

D.I.B.

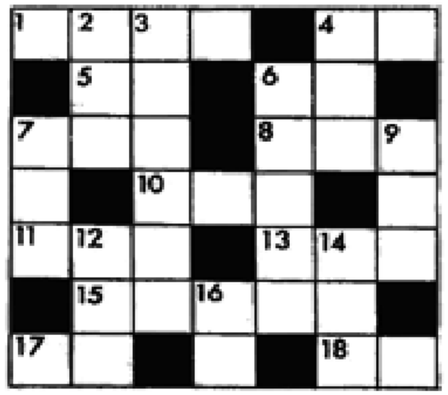
CUT UP A TRIANGLE

Draw a triangle ABC. Divide the triangle into THREE cyclic quadrilaterals.

In how many ways can this be done?

R.H.C.

SENIOR CROSS FIGURE No. 59



CLUES ACROSS

1. Radius of circle of latitude 60 °N, when radius of earth is 3,960 miles.
4. Product of odd primes.
5. Smallest prime whose digit sum gives a 2 digit square number.
6. Geometric mean of 9 & 16.
7. Prime power of 2 minus a single digit prime, the answer being a cube.
8. 6!+5!+4!
10. 10th term of A.P., first term 10, common difference 107.
11. ab^{2a}, where a and b are both prime numbers.
13. An even number raised to the power equal to the number itself.
15. Time taken, in hours, to move at 0.1 knots from (10°E, 10°22'N) to (170°W, 70°54'N) via the North Pole.
17. Last two digits of the next year in which Xmas Day and New Year's Day fall in the same year.
18. Number of spots on a 10-faced die.

CLUES DOWN

2. If π is taken as 3.14, calculate $\pi^2 - \pi$ ÷integral part of π.
3. Three prime numbers, all ending in the same digit, the first having 2 consecutive digits, the 3 being in A.P. with a negative common difference.
4. Coefficient of x^5 in $(1+x)^9$.
6. $\log \sqrt[5]{(10^{5\pi})}$.
7. Sum of the remaining 2 terms of a Pythagorean triad whose smallest number is 11.
9. Third perfect number (excluding unity).
12. " Anagram " of 13 across.
14. Palindromic number.
16. Largest two digit factorial.

P.J.G.

LETTERS TO THE EDITOR

Sevenoaks, Kent.

Dear Sir,

I am highly intrigued to see your reference to my contribution to " Beyond the Looking Glass " maths in Mathematical Pie of Spring, 1971.

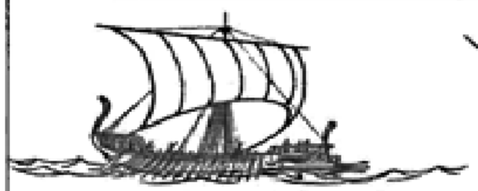
It does not seem logical, however, to give indices negative signs, for an index is not an integer ; it is an operator. And operators are not necessarily reversed in sign in a world where quantities are so reversed.

Yours sincerely,

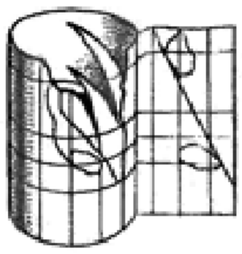
R. L. WORRALL.

ED. NOTE.—Operators and signed numbers are shown by the same symbols. The same treatment can be given to operators as to signed numbers because it was illogical to treat integers in this way in the first instance.

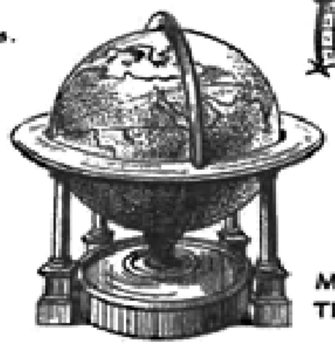
HISTORY OF NAVIGATION



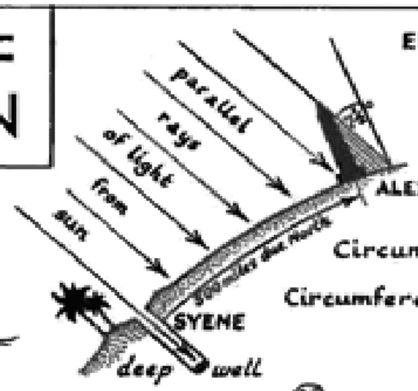
When out of sight of land, ancient mariners learned to guide themselves roughly by sun, stars or regular winds.



MERCATOR'S PROJECTION BY GERHARD MERCATOR 1512—1594



MERCATOR'S TERRESTIAL GLOBE 1541



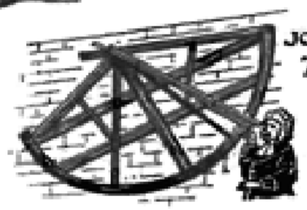
ERATOSTHENES' measurement of the earth c. 250 B.C.

Circumference x $\frac{7\frac{1}{2}}{360}$ = 500

Circumference = 24 000 miles

Finding longitude by CROSS-STAFF and LUNAR DISTANCE TABLES. PETER APIAN 1545

GUNTER'S QUADRANT c. 1650

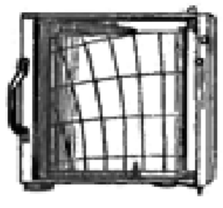


JOHN FLAMSTEED'S 7 FT. MURAL ARC which measured zenith distances of stars and marked first Greenwich Meridian 1689

ROYAL OBSERVATORY — Established at Greenwich in 1675 by Charles II to perfect art of navigation.

"GREENWICH MERIDIAN" The prime meridian of the world since 1884.

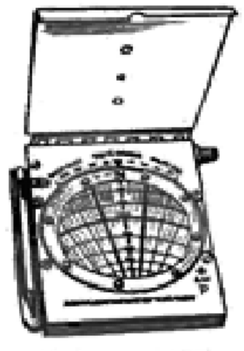
PLAN RANGE FINDER 1939



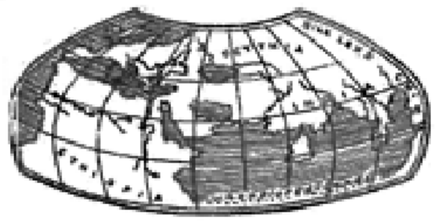
BAKER NAVIGATION MACHINE 1917. Position-line calculation eliminated by use of Mercator's chart on rollers.

BYGRAVE SLIDE RULE for solving spherical triangle without use of mathematical tables 1920

ROYAL AIRCRAFT ESTABLISHMENT BUBBLE SEXTANT using an artificial horizon 1926



AERIAL NAVIGATIONAL COMPUTER 1939—1945

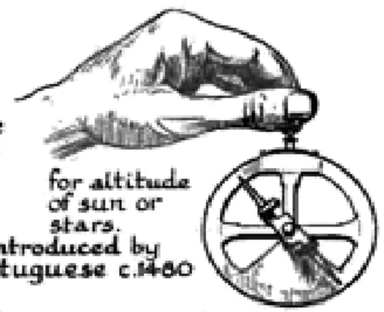


PTOLEMY'S MAP OF THE WORLD c. 200 A.D.

LODESTONE

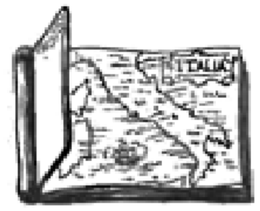
Introduced to Europe during 13c. Magnetic ores were known to Chinese several hundred years B.C.

THE MARINER'S ASTROLABE



for altitude of sun or stars. Introduced by Portuguese c. 1480

MERCATOR'S CELESTIAL GLOBE 1551

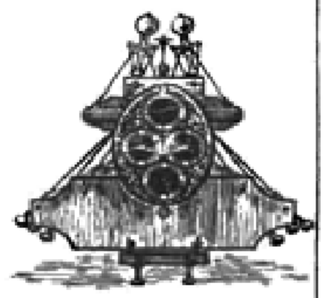


THE FIRST PRINTED ATLAS BY ABRAHAM ORTELIUS

—THE FIRST THEODOLITE BY LEONARD DIGGES 1571

— OCTANT or HADLEY'S QUADRANT 1731

One-eighth circle arc for measuring sun's altitude by double reflection for angles less than 90°



UNITED KINGDOM NATIONAL AIRWAYS SYSTEM — showing Control Areas.

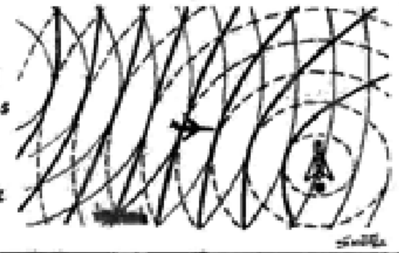
FIRST SEXTANT — by Capt. J. CAMPBELL 1757 Measured angles greater than 90°

JOHN HARRISON'S FIRST MARINE TIMEKEEPER - 1735 A later model won him a £20 000 prize for its accuracy in finding longitude.

DECCA AIRBORNE EQUIPMENT MK. 15 1967

DECCA 'LANES' formed at half wave-length intervals.

HYPERBOLIC POSITION LINE PATTERN formed by lines of zero phase difference between master & slave transmitters.



"The arte of navigation demonstrateth how, by the shortest good way, by the aptest direction, and in the shortest time, a sufficient ship, betwene any two places (in passage navigable) assigned, may be conducted and, in all stormes and natural disturbances chauncying, how to use the best possible means where by to recover the place first assigned." Thus, in 1570, Dr. John Dee, mathematician and astrologer, wrote the first English definition of the art of navigation.

continued on page 498