GENAILLE'S RODS

In issue No. 57, Summer 1969, we referred to Napier's Rods or Bones, which were an aid to multiplication in the early 17th century before people learnt their tables by heart. There have been other aids of a similar nature from time to time but one of the most ingenious is that invented by a French Engineer named GENAILLE about 80 years ago.

The diagram on the front page of this issue shows Genaille's idea (although SHUTL seems to have got at it somehow) so open PIE so that you can see both the front and back pages at the same time. Each of the eleven columns headed Index and 0,1,2, . . 9 represents a separate stick or card. In order to multiply 568 by 6, we select the Index stick and sticks 5,6 and 8 which are then placed side by side, see figure 2 which illustrates a portion of these four sticks.

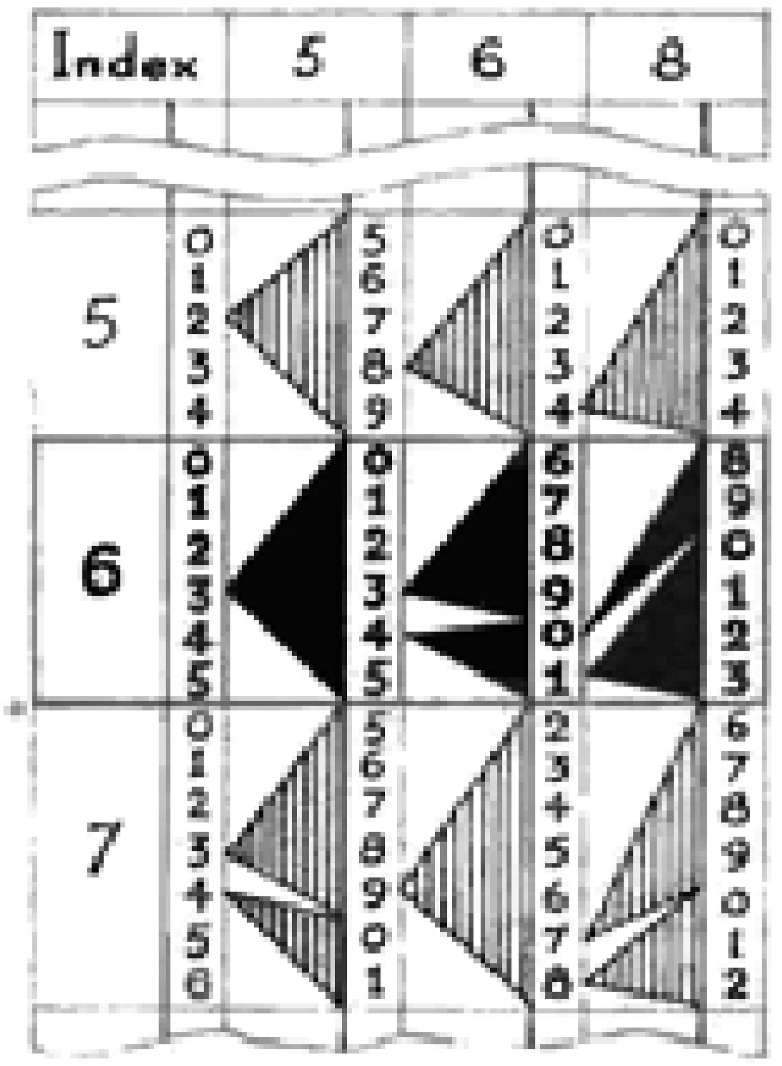


Fig. 2

To read off the answer, we look first at the top figure of the 6×8 rectangle, which is 8. This is the unit digit of our answer. The black triangle leads us from there to the tens digit which is 0. The lower black triangle then leads us to 4, the hundreds digit and the final black triangle leads to 3 on the index stick which is the thousands figure so that $568 \times 6 = 3408$. To follow how the sticks deal with the carrying figure, each stick should be placed against the index stick.

In order to understand how the sticks work, you need to think of what you normally do when you carry out this multiplication. You first say $6 \times 8 = 48$, 8 down and carry 4. Then $6 \times 6 = 36$ which together with the carried 4 gives 40. Put down 0 and carry 4. $6 \times 5 = 30$ and the 4 gives 34. How has Genaille adapted his sticks to handle the carry figures? If we look at the part of the 6-stick which we have used, we find that the top figure is 6 corresponding to carry 0. If we were carrying 1 from the column to the right this right-hand column would carry a triangle pointing to the 7 (i.e., $6+1$) on the 6-stick. If the carry were 2, the triangle would point to 8 and so on.

Many of the sticks have two triangles per section, e.g., 6×6 which has triangles pointing to the 3 and the 4 on the 5 stick. $6 \times 6 = 36$, so if we are carrying 0,1,2,3 from the previous stick, we shall have only 3 to carry to the next column but if the carrying figure is 4 5,6,7,8 or 9 we need to carry 4 to the left-hand stick.

Once you have learnt how to use these Genaille sticks, you might like to make a set of them to handle multiplication in bases other than ten.

Copies of the front page block will be available for a few pence, printed on thin card.

R.M.S.

mathematical pie

No. 60 | *Editorial Address :* 100, Burman Road, Shirley, Solihull, Warwickshire, England | **SUMMER, 1970**

GENAILLE STICKS

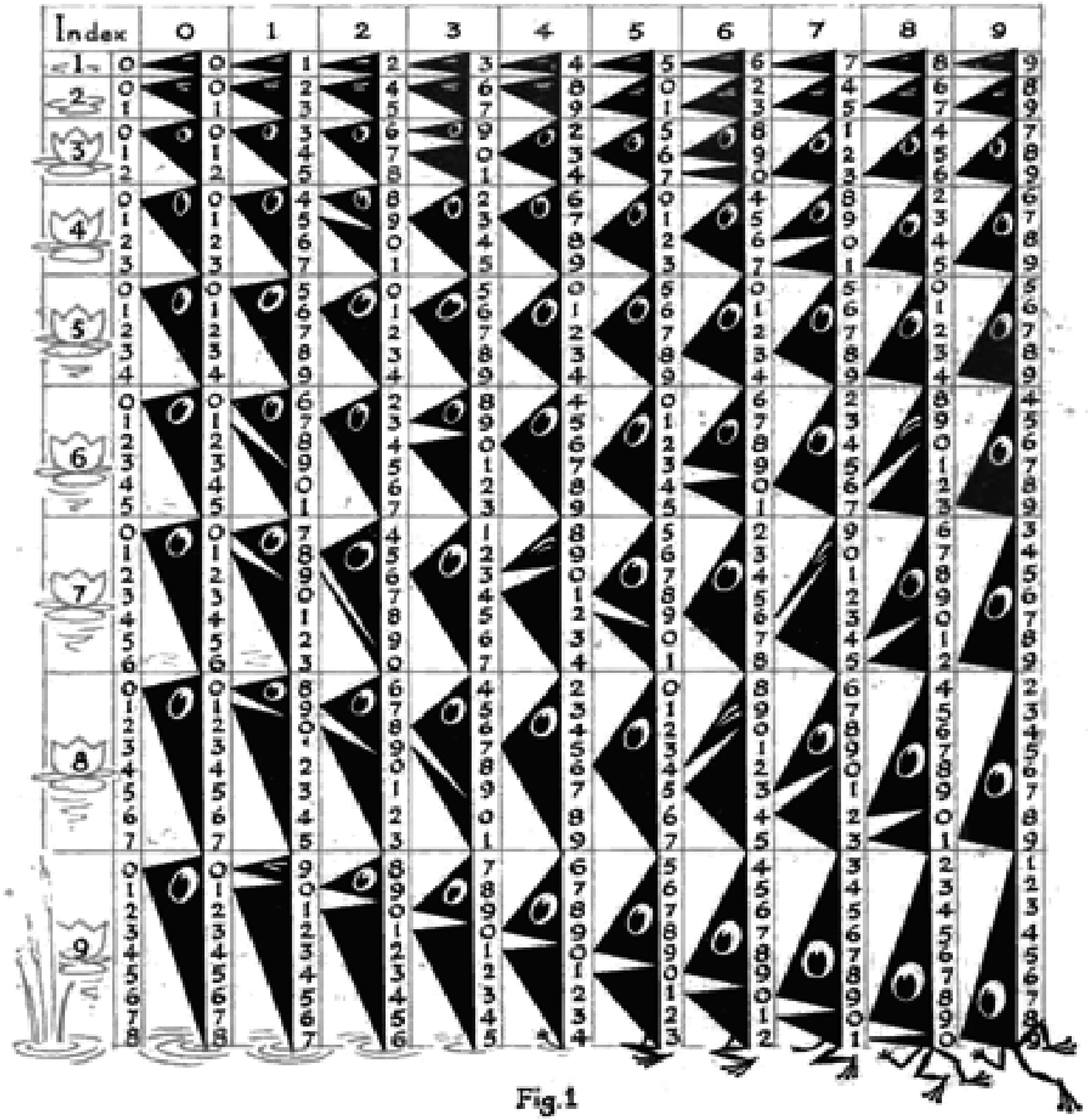
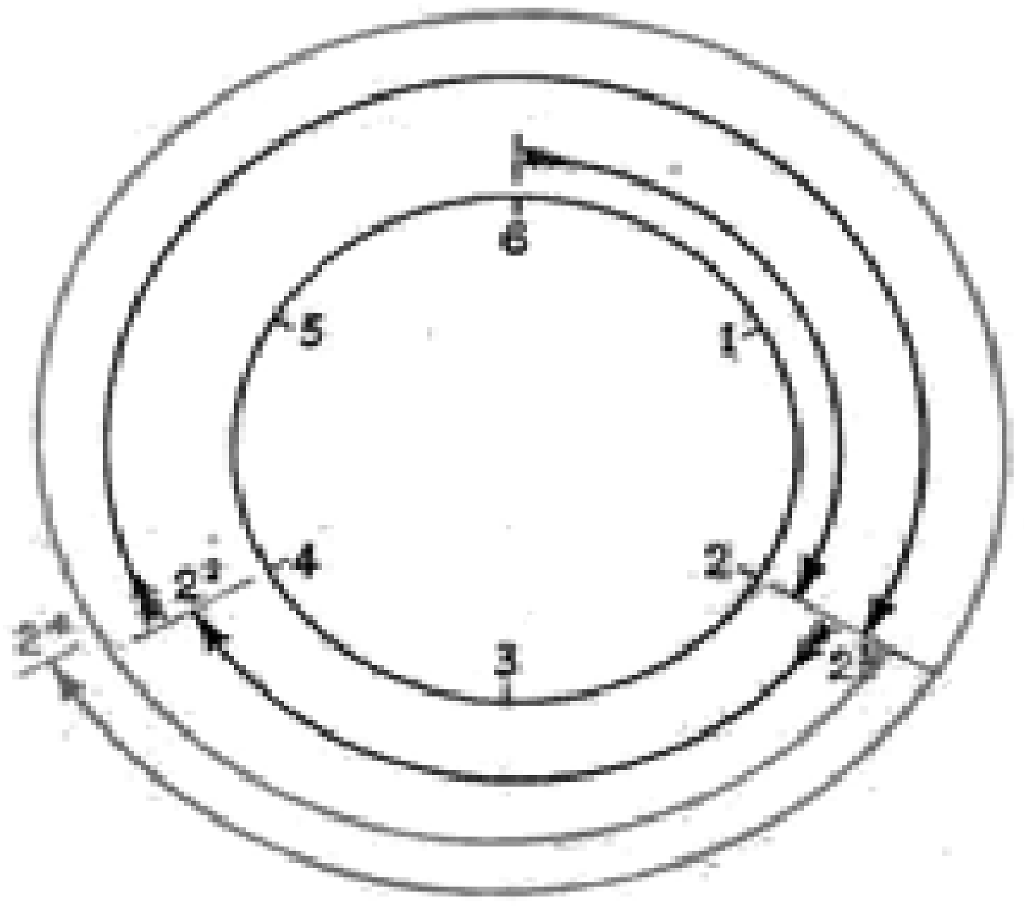


Fig. 1

CLOCK ARITHMETIC No. 4

Here we go round the Mulberry bush

In issues No. 34, 35 and 37, we talked about the arithmetic of a clock face. We took a clock face numbered from 1 to 6 only, then 2^2 means $2+2=4$, 2^3 means twice 2^2 or $(2+2)+(2+2)=4+4=8$ which brings us to the position number 2, see the diagram below.



2^4 would then mean 8 twice over or $8+8$ which is represented by an *additional rotation* equal to that which has been done already from the start, round once and finish at B. This additional rotation is shown in *red*, bringing us finally to C so that $2^4=4$. Continuing the idea, $2^5=16+16$ which brings us to 2. i.e., 2^5(clock 6)$=2$.

Suppose we now build up a table of answers ; 2^1(clock 6)$=2$, 2^2(clock 6)$=4$, 2^3(clock 6)$=2$, 2^4(clock 6)$=4$, 2^5(clock 6)$=2$, 2^6(clock 6)$=$?

Similarly 3^2(clock 6)$=3$, 3^3(clock 6)$=3$, 3^4(clock 6)$=3$. Now construct a table of various powers of the numbers 1,2,3,4,5 and 6 (clock 6) as below :

		NUMBERS					
CLOCK 6		1	2	3	4	5	6
	1	1	2	3	4	5	6
POWERS	2	1	4	3	4	1	6
	3	1	2	3	4	5	6
	4	1	4	3	4	1	6
	5	1	2	3	4	5	6
	6	1	4	3	4	1	6

You will notice that each vertical line eventually repeats but the important repetition concerns the horizontal lines. The first horizontal line is 1,2,3,4,5,6 and this occurs again in the 3rd, 5th lines, etc. Similarly, line 2 is repeated in lines 4, 6, etc.

Now repeat this process in a different clock base, say clock 5. $2^5=32=2$ (clock 5). Carry on and construct a similar table to that shown above and see if any pattern appears as above.

Generalise the theory, if possible.

R.H.C.

MATHEMATICAL INN SIGNS

THE PARA-BOLA

THIS TAKES THE CAKE

This cake must be suspicious
Of being found delicious
When we meet a team for tea.
For we're back at the beginning
And he just sits there grinning
Aren't I a crafty little me.

Thompson, 14 Park View,
Liverpool L22 2AP

This verse was inspired by the article in issue No. 58.

SOLUTIONS TO PROBLEMS IN ISSUE No. 59

In the article Look behind the Mirror No. 2, a misprint was overlooked ; in the centre column of base ten working a 5 was printed as a 3. Although great care is taken with the issue, misprints are sometimes missed. The editor will send a book token to the first reader who recognises a misprint and writes in to suggest what the original should have been or how the misprint can be corrected.

LOTTERY—The probability is 89 to 245.

ROUND TRIP—A polygon with diagonals can be traced out without lifting the pencil and without following the same line more than once provided the polygon has an odd number of sides.

GREENGROCERY—The price was 6d. each and 2 shillings per dozen. A sold 1 doz. and 9 singles ; B sold 2 dozen and 5 singles, C sold 3 doz. and 1 single, costing 6/6d. at each shop.

THE NEW PAPER SIZES—One of the editorial board was rather amused by the suggestions that a ratio of $\sqrt{2}:1$ was a completely rational system.

SPOT THE BALL—The spots can be placed on the sphere in groups of 4, 6, 8, 20 and 32 ; the vertices of the regular polyhedra.

SENIOR CROSS FIGURE No. 55
Across : 1. 339 ; 3. 417 ; 5. 825 ; 7. 96 ; 9. 38 ; 10. 12 ; 11. 25 ; 12. 12 ; 14. 45 ; 16. 132 ; 18. 180 ; 19. 470.
Down : 1. 319 ; 2. 98 ; 3. 45 ; 4.738 ; 6. 25 ; 8. 612 ; 9. 354 ; 12. 111 ; 13. 23 ; 15. 500 ; 16. 10 ; 17. 24.

ASK DAD—The width of the lawnmower determines the length that must be cut to cover an acre. The length is the distance that must be worked.
Going metric will probably lead to a unit of kilometres per hectare.

JUNIOR CROSS FIGURE No. 51
Across : 1. 12 ; 3. 612 ; 6. 52 ; 7. 503 ; 8. 236 ; 11. 608 ; 13.146 ; 15. 71 ; 17. 162 ; 18. 55.
Down : 1. 15 ; 2. 222 ; 3. 6560 ; 4. 10 ; 5. 235 ; 9. 3662 ; 10. 111 ; 12. 875 ; 14. 46 ; 16. 15.

THE TOWER OF HANOI—If the blocks are numbered 1, 2, 3—starting from the bottom, then the number of moves for the *n*th block is 2^{n-1}. Thus for five blocks the total number of moves is

$$2^4+2^3+2^2+2^1+2^0 = 11111 \text{ (base 2)} = 2^5-1 \text{ (base 10)} = 31.$$

In general, the number of moves for a tower with n blocks is 2^n-1.
The distance travelled by each block is always made up from the sequence . . . $+2+1+2+1+1+2+1+1$ . . . starting either with a 2 or with two 1's.
The moves are 2, 2, 6, 10, 22 . . . or 1, 3, 5, 11 . . . for each block No. 1, No. 2, etc., which are related to the Fibonacci series 1, 1, 2, 3, 5, 8.

LETTUCE COS—Using similar triangles and the cosine ratio, the trisector is $\frac{2ab\,(a-5_3b)}{a^2-3b^2}$.

FOOTSTEPS OF TIME—It takes an infinite time to reach the point $(-1, 0)$.

B.A.

Continued from page 473

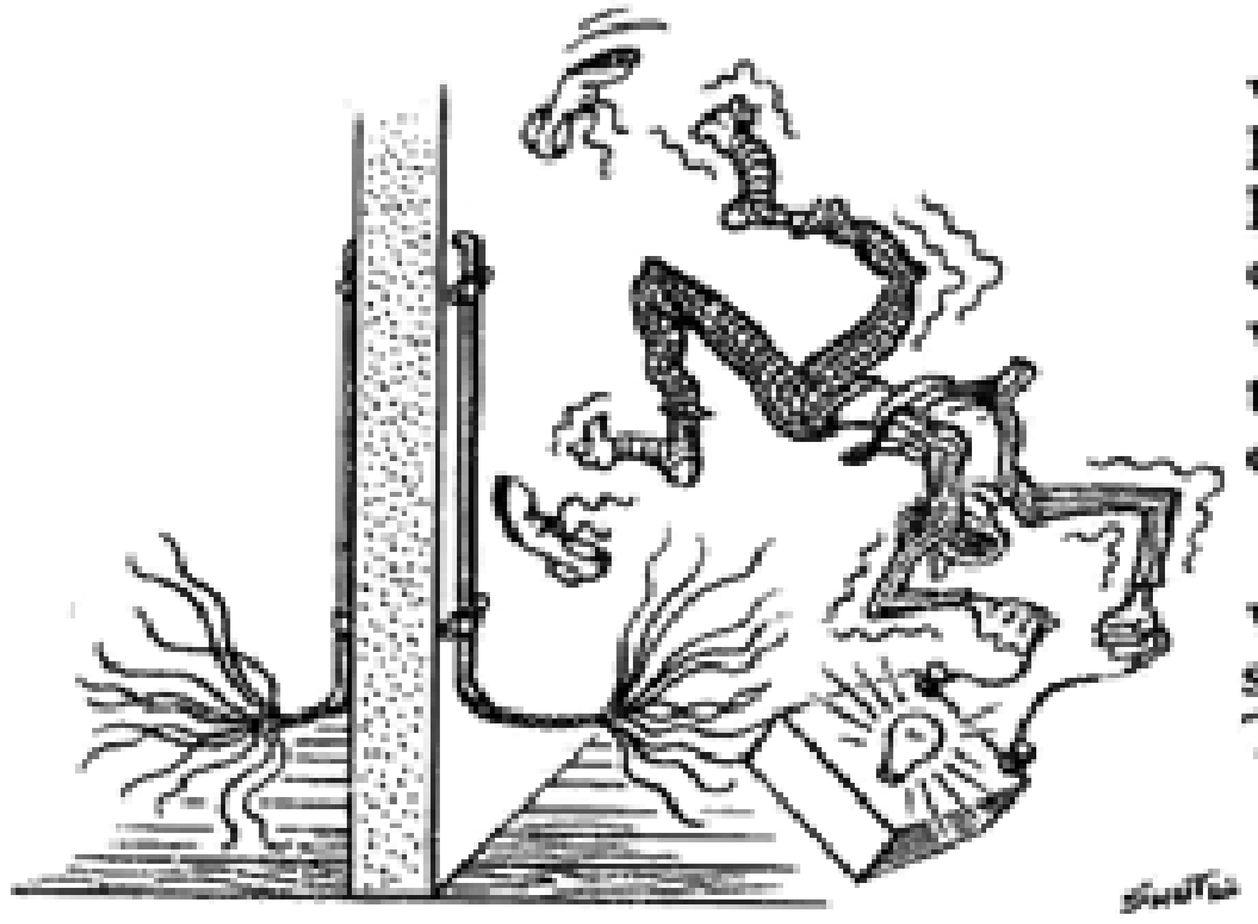
recommended for adoption in the Système International d'Unités (S.I.) have been reduced to : length—metre ; mass—kilogramme ; time—second ; electric current—Ampère ; temperature—degree Kelvin ; luminous intensity—Candela.

Acknowledgements and references : Metric Display—Science Museum ; Historical Metrology by A. E. Berriman ; History of Mathematics Vol. II by D. E. Smith ; Encyclopaedia Britannica.

D.I.B.

PAIRING UP

Bill, our electrician is, unfortunately, colour blind so that the other day when he was faced with 12 wires disappearing into a pipe through a wall and reappearing in the next room, he couldn't use the usual colour coding of the wires to sort them out.

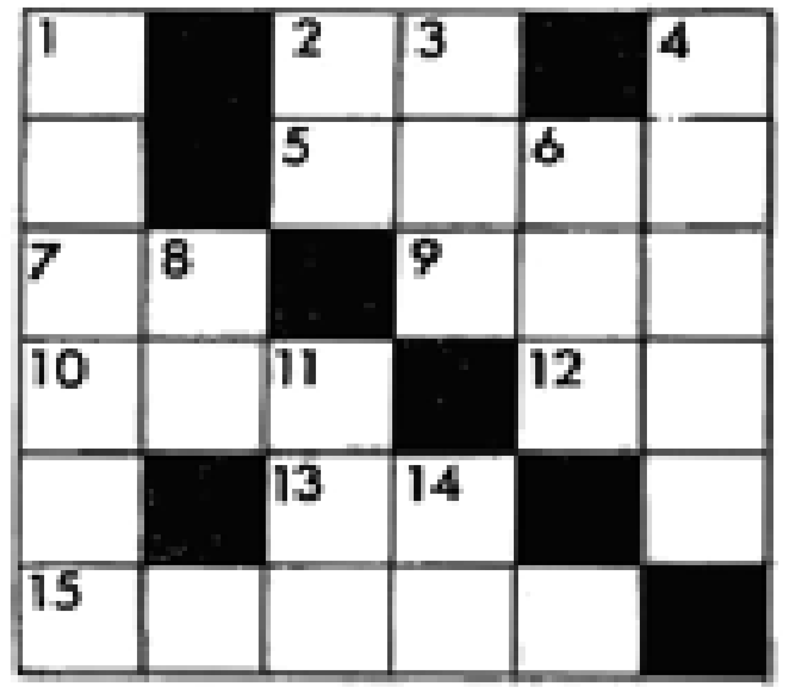


He was able to connect the wires in pairs at either end and he had a "continuity" tester with him—that is a gadget consisting of a battery, a bulb and two leads wired up so that the bulb lights if the two leads are connected to the ends of a continuous circuit.

How did Bill identify the 12 wires at both ends? There are several ways of doing it. (Book Token for neatest solution).

R.M.S.

JUNIOR CROSS FIGURE No. 52



Ignore decimal points. Write answers correct to the appropriate accuracy.

Clues Across

2. Area of a triangle with sides, 6 cm., 5 cm., $\sqrt{61}$ cm.
5. π.
7. The seventh prime number.
9. The volume of a cube of side 61 cm.
10. The roots of $x^2 - 12x + 11 = 0$.
12. The only digit missing from the solutions is zero.
13. Take 12 from 5 across.
15. The cube of a prime number.

Clues Down

1. A power of two plus ten to the power of two.
2. The sixth prime number.
3. The cube of a cube.
4. Another power of two.
6. The side of a square of area 18 square units.
8. More than a life's span—just.
11. (x,y) if $2x + y = 33$, $x + 2y = 27$.
14. All square at the seventh.

Check : The sum of the digits in the solutions is 105.

B.A.

A BASE QUESTION

Find a three digit number in base ten which has the digits reversed when written in base nine.

Let the three digits be a, b and c, then

$$a\,b\,c_{ten} = c\,b\,a_{nine}$$
$$10^2a + 10\,b + c = 9^2c + 9\,b + a$$
$$99a + b - 80c = 0$$
$$99\,a + b = 80\,c.$$

Test various possible values of a and c, and try to find a value for b within the number systems being considered.

$a = 5$ and $c = 6$ gives $495 + ? = 480$, impossible.
$a = 4$ and $c = 5$ gives $396 + ? = 400$, leads to $b = 4$.
Thus $445_{ten} = 544_{nine}$.

Now consider a similar problem in other number bases. Can you find a three digit number such that its digits are reversed when changing from base nine to base eight, base eight to base seven, etc. ?

B.A.

SENIOR CROSS FIGURE No. 56

Ignore decimal points. Write answers correct to the appropriate accuracy.

Clues Across

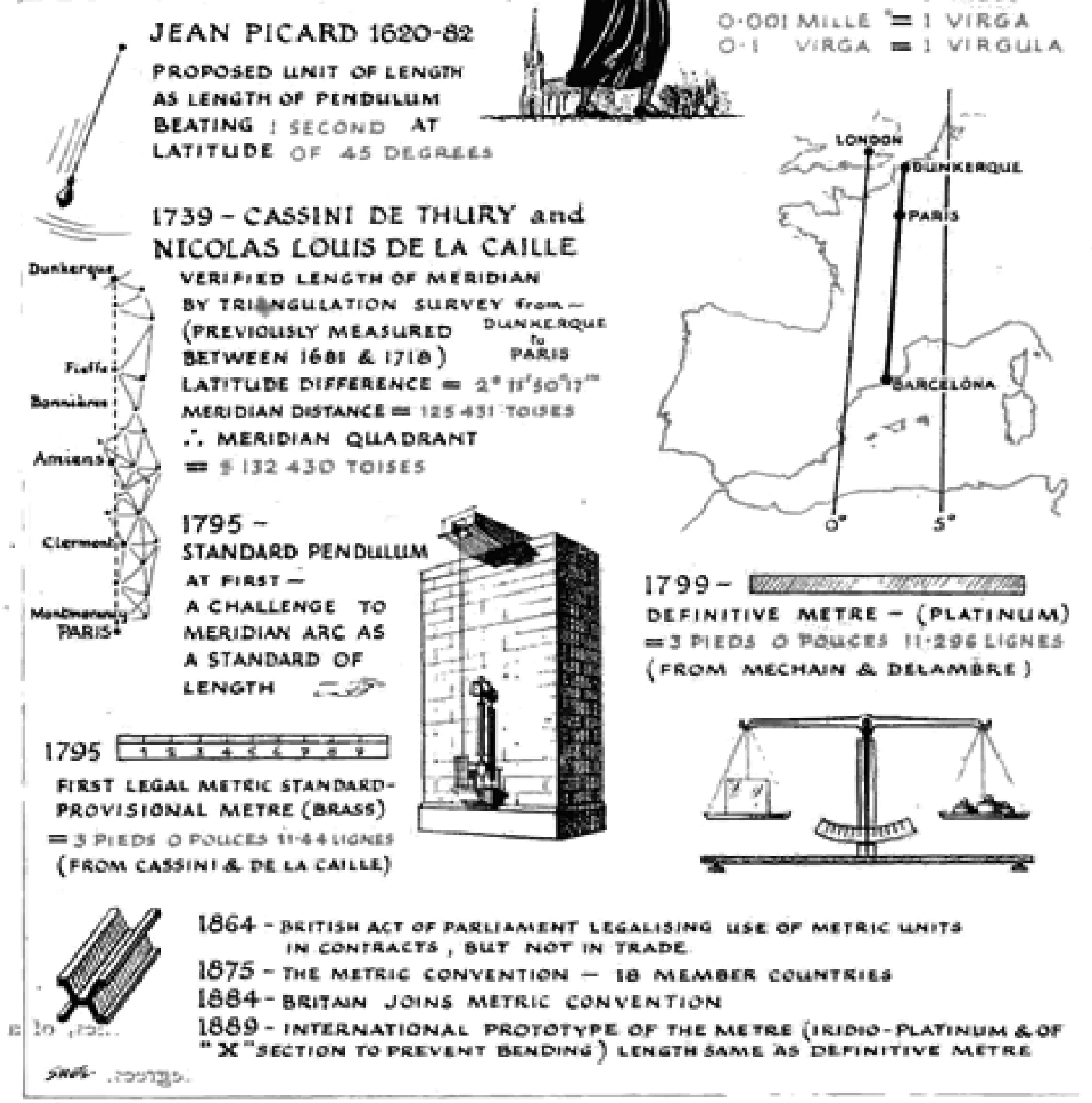
2. One side of a triangle with hypotenuse 6 cm. and the third side 1 cm.
5. The diagonal of a block 1 cm. by 1 cm. by 2 cm.
7. Area, in square inches, of a square of side 2 ft. 4 in.
9. The largest rod that will fit into a box 2 ft. $8\frac{3}{4}$ in. $\times$ 3 ft. $\times$ 7 ft., measured in inches.
11. Interest on £200 invested for 1 year at $4\frac{3}{4}$ per cent.
12. Smallest angle of a 3,4,5 triangle.
13. Number of sides of a regular polygon with angles 165 degrees.
14. Angle subtended by a diameter.
15. Circumference of a circle of diameter 153 cm.
16. A square prime.
18. Area of the annulus between circles of radius 13.15 cm. and 7.55 cm.
19. Roots of $x^2 - 20x + 91 = 0$, larger first.

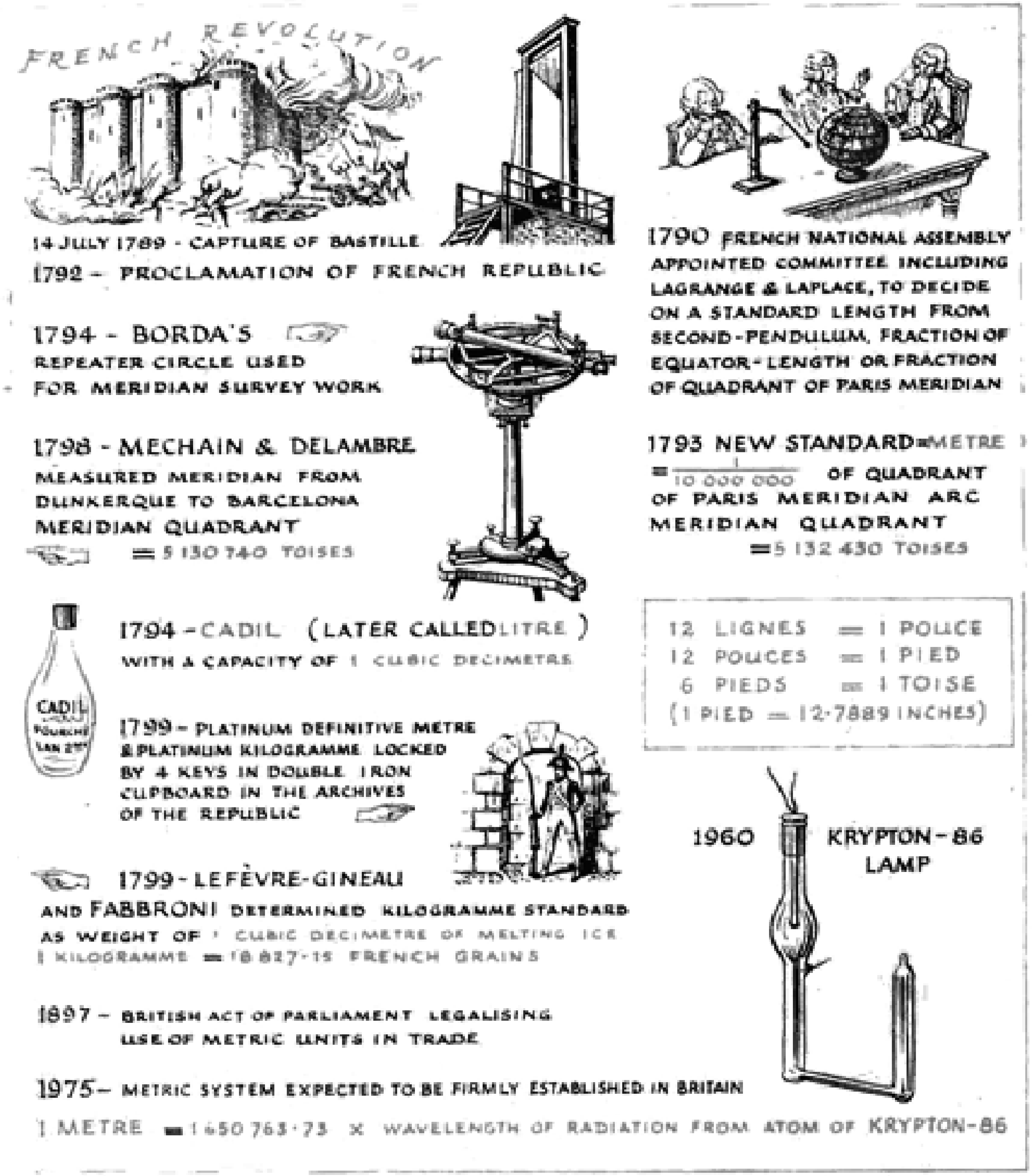
Clues Down

1. The volume of a cylinder 10 cm. long, diameter 5.5 cm.
3. (x,y) if $3x + 2y = 41$, $2x + 3y = 39$.
4. Another square of a prime.
5. The volume, in cubic inches, of a cube of side 2 ft. 5 in.
6. Nearly a radian in degrees.
8. $222^2 - 10(20)^2$.
10. The sum of the angles of a hexagon.
14. The product of the roots of the equation in 19 across.
15. Angle, in degrees, between a diagonal and the shorter side of a rectangle 25 cm. by 26.53 cm.
17. The height, in inches, of a pyramid on a square base of side 14 ft. and a slant height of 8 ft. 9 in.
18. Eleven yards in feet.

B.A.

metric weights and measures





The uncertainties and peculiarities of a multiplicity of different measuring standards in use in trade during the Eighteenth Century, prompted many Frenchmen to propose more straightforward unified systems. Even well before this time, in 1670, Gabriel Mouton devised a set of linear units which closely resembled the system adopted over a century later.

The metre was originally based on a calculation for one ten-millionth of the quadrant of the Paris meridian arc, but as subsequent calculations with improved instruments resulted in differing values for the metre, the 1799 Definitive Metre was eventually fixed as the standard.

At the General Conference of Weights and Measures in 1960, it was decided that a standard bar was not sufficiently precise for modern metrological needs. In its place, the Conference gave a new definition of the metre as 1 650 763.73 × wavelength of radiation from the atom of krypton-86. The necessary apparatus for this measurement can be produced relatively simply and is portable, thus providing a most convenient means of obtaining the precise standard almost anywhere.

Although it is unlikely that units such as the millimetre and gramme will cease to form part of the general metric vocabulary, the main units

Continued on page 474