EASY TO TWELFTH MAN

If you have been given a function of x such as $x^3 - 5x^2+11x - 2$ and asked to obtain its value when different numbers are substituted for x, I expect you will do it as follows $x=2$

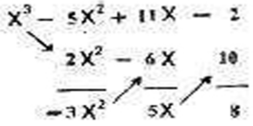
$$\begin{aligned}&(2)^3-5(2)^2+11(2)-2\\&=8-5(4)\quad 22-2\\&=8-20+22-2\\&=8\end{aligned}$$

Have you tried it this way $x^3-5x^2+11x-2x=2$, the first term $x^3=xx^2-2x^2$

the second term becomes $2x^2-5x^2=-3x^2=-3xx=-6x$
the third term becomes $-6x+11x=5x=10$
the constant term becomes $10-2=8$, which is the answer.

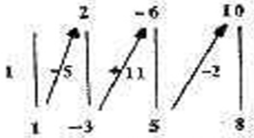
It would be unfair to say that this is a longer method than the first one because it has been set out carefully so that you can follow all the steps. As soon as you have grasped the whole process this method of substitution can be abbreviated something like this

when $x=2$



or better still we can dispense with the x labels

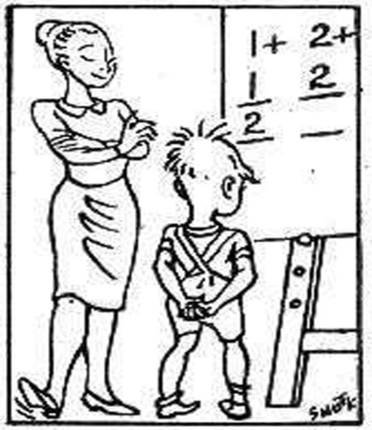
when $x=2$



This dispensing with the x labels is no worse than what we do with our ordinary numerals where 549 really means $5(10)^2+4(10)+9(1)$.

Why not try this method of substitution next time you have to obtain various values of a function before you go on to drawing a graph. R.H.C.

"Yoo-Hoo, Ivy!"



"Er, now let me make a rough guess."

37182 94127 03389 31378 51786 09040 70866 71149

MATHEMATICAL PIE

No. 45 Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England MAY, 1965

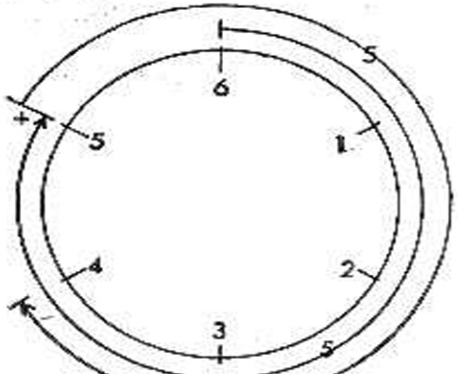
CLOCK ARITHMETIC No. 4

Squares and Square Roots

In issue No. 37, you were shown how to make out a multiplication table for a circular face marked out with the scale of 6. This revealed that in modulo 6

$$2\times5=5+5 \qquad 3\times4=4+4+4 \qquad 4\times5=(5+5)+(5+5)$$
$$=4 \qquad\qquad =2+4 \qquad\qquad = 4+4$$
$$\qquad\qquad =6 \qquad\qquad = 2$$

and from these and similar results, the full multiplication table was constructed.



x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	2	4	6
3	3	6	3	6	3	6
4	4	2	6	b 4	2	6
5	5	4	3	2	a 1	6
6	6	6	6	6	6	6

Now examine the table and pick out those numbers which are the product of equal numbers, *e.g.*, in square (a) we learn that $5\times5=1$, or using symbols $5^2=1$. Similarly in square (b) we discover that $4^2=4$. Listing all the results from an examination of the diagonal of the table produces $1^2=1$, $2^2=4$, $3^2=3$, $4^2=4$, $5^2=1$, $6^2=6$. In normal arithmetic, the fact that $8^2=64$ leads us to invent new symbols and words for an inverse process namely that $\sqrt{64}=8$. Similarly $\sqrt{81}=9$. Using this same idea of an inverse root process for clock arithmetic in modulo 6 gives

$$\sqrt{1}=1 \text{ or } 5,\ \sqrt{3}=3,\ \sqrt{4}=2 \text{ or } 4, \text{ and } \sqrt{6}=6.$$

These latest results raise some interesting issues. Why should $\sqrt{1}$ and $\sqrt{4}$ have two answers whilst $\sqrt{3}$ and $\sqrt{6}$ have only one? What has happened to $\sqrt{2}$ and $\sqrt{5}$ which are so far missing? When you have done your homework properly, you will be led to the conclusion that 1 has *six* square roots, all of them different ! ! ! R.H.C.

91041 40792 88621 50784 24516 70908 70006 99282

TWO FOR A PENNY

1. The scale of an ordnance survey map is 1 mile to 1 inch. A halfpenny is placed on the map, how many acres of ground would be represented by the area covered by the coin?
2. Three halfpennies are placed flat on a table so that each touches the other two. What is the area of the space left between them?
3. Six halfpennies are arranged with their centres on a circle so that each touches the two adjacent ones. What is the radius of the circle which will just enclose the six coins?

J.G.

ALGEBRA CORNER No. 3

1. Square $x + y + z$.
2. If the length of a diagonal of a rectangular solid is 6 inches, and the sum of the areas of all the faces is 64 square inches, what is the sum of all the edges of the solid?
3. If $a^2 = bc$ and $b^2 = ca$, does $c^2 = ab$?
4. The number of inches in a yards b feet is equal to one sixth of the number of pence in £a . . bs. If a and b are whole numbers, what are their values?
5. The parallel sides of an isosceles trapezium are 2 inches and $2n^2$ inches. The slant sides are $(n^2 + 1)$ inches. What is the area?
6. What is the angle in degrees between the hands of a watch when the time shown is x minutes past y?

J.G.

AT THE CROSS WAYS

Two lines cross at an angle α. A point V is fixed in position with respect to the lines. Find a construction for drawing another line through V and cutting the first two lines at P and Q in such a way that PQ is a given length l.

J.F.H.

A PARADOX

"He put 2 and 2 together and made 5," is a phrase that we often hear. Mr. A. R. Pargeter, Blundell's School, suggests $\sqrt{(.2)^{-2}} = 5$. Thus two 2's can make 5.

Can you find other examples of this kind?

THE ROARING FORTIES?

FORTY	Each letter represents
TEN	a different figure in
TEN	the addition sum.
SIXTY	

J.F.H.

12066 04183 71806 53556 72525 32567 53286 12916

JUNIOR CROSS FIGURE No. 38

Ignore decimal points and work to the appropriate number of significant figures.

1		2	3	■
	■	4		5
6	7			
8			■	
■	9			

Clues Across

1. Area of a square of side 47.2 units.
4. Third side of a triangle, with hypotenuse 10 units and one side 3.4 units, squared.
6. One tenth of 9!
8. Number of litres in one pint.
9. Diagonal of a square of side 1 unit.

Clues Down

1. Number of grams in one ounce.
2. The length of a diagonal of a square of side 2 units.
3. The length of a rectangle whose diagonals are 10 units and breadth 4.6 units.
5. $1^6 + 2^6 + 3^6 + 4^6 + 4$.
7. The breadth of a rectangle whose diagonals are 1 unit and length is $\frac{3}{4}$ unit.

B.A.

SOLUTIONS TO PROBLEMS IN ISSUE No. 44

WHO IS FROM WHERE?

A is Welsh, B is English, C is Scottish.

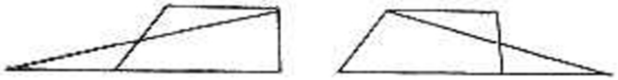
A PROGRESSIVE DATE

There are 51 dates per century which are in geometric progression.

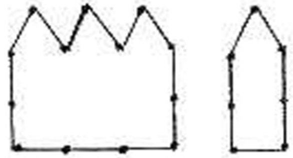
A SUPERIOR ADDRESS

The first 3 is 29,999,997,000 more than the second 3. The first 3 is 10,000,000 times as big as the second 3.

JIG-SAW PUZZLE.



CAN YOU MATCH THIS?



SENIOR CROSS FIGURE No. 42

Clues Across: (1) 480249; (5) 24; (6) 8415; (9) 27; (10) 54; (11) 1000; (12) 42; (13) 102564.
Clues Down: (1) 428571; (2) 2250; (3) 44; (4) 937024; (7) 44; (8) 1122; (9) 20; (11) 122; (12) 40.

DOWN MEXICO WAY, 1968

The order of finishing is D, C, E, B and A and F tie last.
After 5 seconds, A, B, C, D, and E have travelled 50, 40, 30, 20 10 yards and F is about to start.

CORNERING

(1) The cuts should be made $5(2 - \sqrt{2})$ inches from the vertices.
(2) The cuts are made from one vertex to the points on the opposite sides $10(\sqrt{3} - 1)$ from the opposite vertex.
The length of the sides of the triangle is $20\sqrt{(2 - \sqrt{3})}$.

JUNIOR CROSS FIGURE No. 37

Clues Across: (1) 83328; (6) 9936; (7) 20; (8) 45; (9) 143; (11) 081.
Clues Down: (2) 3904; (3) 39; (4) 234; (5) 865; (7) 217; (10) 30.

DOUBLE OR WIN

The first loser had 39/-, the second had 21/-, and the third had 12/-.

B.A.

01729 67026 64076 55904 29090 45681 50652 65305

PROBLEM CORNER

Printer's Error

In setting up $2^5.9^2$ the printer put 2592, which surprisingly enough is the same. Can you find another number of four digits which has this peculiar property? R.M.S.

Enclosed

What is the greatest area that I can enclose using 400 yards of wire netting?

What is the greatest rectangular area that can be enclosed by the netting? R.M.S.

It is not cricket

The factorial sign (!) is a very convenient way of writing some products, *e.g.*, 5! means $1 \times 2 \times 3 \times 4 \times 5$ and 10! means $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$ which equals 3,628,800. You are surprised how large it is? Hence the (!) sign.

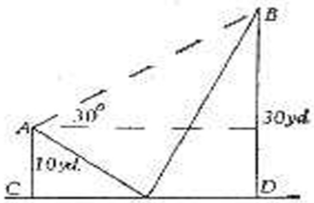
A newspaper headline once said, "Bradman out for 12!".

I doubt if the famous Australian cricketer made 579,001,600 runs in the whole of his career, let alone in one innings!

What is the largest number you can make using three matches? (Hint: use one of them upside down for the factorial sign (!)). It is well over Bradman's "score." R.M.S.

Handicapped

At our School Sports, the Junior girls had to run from a point A, touch a wall CD and then run on to the point B. One bright lass scored an unfair advantage by working out exactly the quickest route to take from A to B fulfilling the condition. How far did she run? R.M.S.



FIFTH COLUMN SOLUTION (from page 351)

Try plotting the five points. They lie on a straight line. Can you find the equation of the line?

Factorise the expressions on the left hand side of the equations,

$$(x - y)\ (2x + y - 4) = 0 \text{ and } (2x + y - 4)\ (3x - y - 2) = 0$$

Hence the first equation represents the pair of straight lines

$$x - y = 0 \text{ and } 2x + y - 4 = 0$$

and the second equation represents the pair of straight lines

$$2x + y - 4 = 0 \text{ and } 3x - y - 2 = 0$$

so that any point which lies on $2x + y - 4 = 0$ will satisfy both equations.

R.M.S.

57710 28402 79980 66365 82548 89264 88025 45661

FIFTH COLUMN

(Adapted from Mathematics Student Journal)

You may have met the fact that the graphs of two quadratic equations can intersect in at the most 4 points.

For example, the graphs of the two equations

$$2x^2 - y^2 - xy - 4x + 4y = 0$$

$$\text{and} \quad 6x^2 - y^2 + xy - 16x + 2y + 8 = 0$$

meet in the four points (–1, 6), 1,1), (0, 4), (2, 0); this is a fact that can be verified by substitution.

Your teacher will probably pat you on the back for a good effort, so then floor him with the fact that (3, –2) also fits both equations.

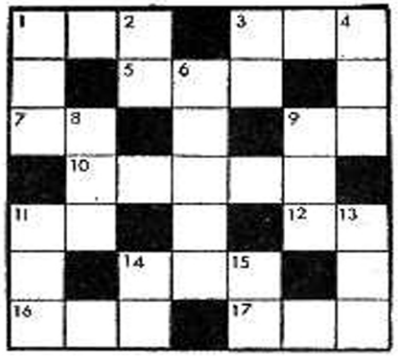
Can you see why? Turn to page 354 for the explanation. R.M.S.

SENIOR CROSS FIGURE No. 43

Ignore decimal points and work to the appropriate number of significant figures.

$x + y = p, \quad y + p = q + 1$

$x + p = q, \quad x + y = q - 2$



Clues Across

1. $\dfrac{pq}{x}$
3. $\dfrac{xp + q}{q - y}$
5. $\dfrac{y}{q}$
7. $x(3q + p)$
9. $(q - x)^2$
10. One tenth of 9!
11. $(p + x)(p + q)$
12. $\dfrac{p^3 + p}{x}$
14. $\dfrac{1}{x + p}$
16. $\dfrac{p}{q}$
17. $x^2 y^2 + 2pxy + p^2$

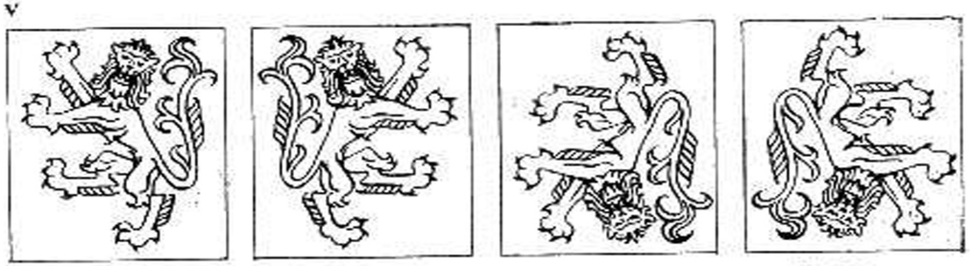
Clues Down

1. pqy
2. xy^3
3. q^2
4. $\dfrac{xy(p + x)}{y + p}$
6. $\sqrt{8}$
8. Consecutive figures.
9. $\dfrac{x}{q}$
11. $\dfrac{xy}{q}$
13. $q^3 + (y + p)^2 + y$
14. xq
15. $p^2 + xy$

B.A.

42487 76182 58297 65157 95984 70356 22262 93486

ABOUT TURNS



O Lion H V. Lion H. Lion R. Lion

On the little sketch of the heraldic lion (rampant gardant argent) are drawn two axes labelled OH and OV and you must imagine a third axis OR at right angles to the paper. If you will copy the little lion and the axes on tracing paper, you can perform three simple operations. First place your tracing so that it fits exactly on the printed sketch, then turn it through 180° about OV as if you were turning the page of a book. You now have a lion facing to the right. The letter **V** can be used to describe this operation. Heavy type has been used to emphasize that **V** represents an operation, not a number as in ordinary algebra.

If the operation **V** is performed twice the lion is back where he started. It is convenient to introduce another symbol, **I**, (I for identical) which means that the picture of the lion is unchanged. Using this symbol we can write $\mathbf{V}.\mathbf{V}$ Lion $= \mathbf{I}.$ Lion or $\mathbf{V}^2.$ Lion $= \mathbf{I}.$ Lion. A similar statement would be true of a unicorn or of any picture. This can be expressed by writing $\mathbf{V}^2 = \mathbf{I}$, which is a very compact way of saying that the result of turning a picture twice through 180° about the axis OV is to return it to its original position.

If the symbol **H** is used to represent the operation of turning the picture through 180° about OH and **R** to represent turning through 180° about OR, we have $\mathbf{H}^2 = \mathbf{I}$ and $\mathbf{R}^2 = \mathbf{I}$. Here is the first oddity which shows that this algebra is not quite the same as ordinary algebra, $\mathbf{V}^2 = \mathbf{H}^2 = \mathbf{R}^2$ but **V**, **H** and **R** are all different. The same operation performed twice gives **I**. What happens if the operation **V** is performed and then the operation **H**? Experiment shows that the result is the same as performing the operation **R**, that is $\mathbf{H}.(\mathbf{V}.$ Lion$) = \mathbf{R}.$ Lion or $\mathbf{H}.\mathbf{V} = \mathbf{R}$.

Just as we can write out a multiplication table for numbers, so we can write out a table for operators. When two operations are performed in succession, the one performed first is printed last.

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Table 1.

	I	V	H	R
I	I	V	H	R
V	V	I		
H	H	R	I	
R	R			I

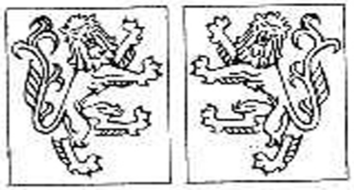
Table 2.

Perform the operations **R.V**, **V.R** and so on and complete the second table.

In *Mathematical Pie* No. 38 and No. 39 all possible types of repeating patterns were listed. Using operational algebra, properties of these patterns can be deduced. To describe all the possible patterns more operators are required but we can make a start with **I**, **V**, **H** and **R**.

Consider pattern A of two rampant lions face to face. It consists of the original Lion and **V**. Lion and can be described by

$$A = \text{Lion} + \mathbf{V}.\ \text{Lion} = (\mathbf{I} + \mathbf{V}).\ \text{Lion}.$$



Pattern A

If we apply the operation **V** to A, we have

$$\begin{aligned} \mathbf{V}.A &= \mathbf{V}.(\mathbf{I} + \mathbf{V}).\ \text{Lion} \\ &= (\mathbf{V}.\mathbf{I} + \mathbf{V}^2).\ \text{Lion} \\ &= (\mathbf{V} + \mathbf{I}).\ \text{Lion} \\ &= \mathbf{I}.A \end{aligned}$$

In words, this means that the pattern A is symmetrical about OV. The four lions of pattern B are made by adding pattern A to its reflection in OH.

Pattern B

$$\begin{aligned} B &= (\mathbf{I} + \mathbf{H}).A \\ &= (\mathbf{I} + \mathbf{H}).(\mathbf{I} + \mathbf{V}).\ \text{Lion} \\ &= (\mathbf{I}^2 + \mathbf{I}.\mathbf{V} + \mathbf{H}.\mathbf{I} + \mathbf{H}.\mathbf{V})\ \text{Lion} \\ &= (\mathbf{I} + \mathbf{V} + \mathbf{H} + \mathbf{R}).\ \text{Lion} \end{aligned}$$

This result expresses in symbols that the pattern B consists of four lions in different positions. To test that our operational algebra works $\mathbf{V}.B = \mathbf{V}(\mathbf{I} + \mathbf{V} + \mathbf{H} + \mathbf{R}).$ Lion $= (\mathbf{V}.\mathbf{I} + \mathbf{V}^2 + \mathbf{V}.\mathbf{H} + \mathbf{V}.\mathbf{R}).$ Lion $= (\mathbf{V} + \mathbf{I} + \mathbf{R} + \mathbf{H}).$ Lion $= B$

Similarly $\mathbf{H}.B = B$

Now $\mathbf{R}.B = \mathbf{H}.\mathbf{V}.B$ (because $\mathbf{R} = \mathbf{H}.\mathbf{V}$)

$= \mathbf{H}.B$ (because $\mathbf{V}.B = B$)

$= B$ (because $\mathbf{H}.B = B$)

This is a formal proof of the theorem that a pattern with two perpendicular axes of symmetry has also rotational symmetry. Perhaps this is using a sledge hammer to crack a nut but it would be very difficult to prove that there are only seven types of linear pattern and seventeen types of plane pattern without using operational algebra. C.V.G.

SOLUTIONS TO SENIOR CROSS FIGURE No. 43

In the scale of 26, A=1, B=2, C=3 . . . X=24, Y=25, Z=0. The solutions are given in the scale of 26 using these letters.

ACROSS: 1. FS, 3. PI, 5. PM, 7. BZ, 9. Y, 10. BAQR, 11. CF, 12. BM, 14. EM, 16. AAL, 17. DQ.

DOWN: 1. DA, 2. BB, 3. AW, 4. TE, 6. AOAR, 8. IZ, 9. KZ, 11. AFY, 13. Uy, 14. N, 15. AE.

SOLUTIONS TO JUNIOR CROSS FIGURE No. 38

ACROSS: 1. CGQ, 4. AHZ, 6. BAQR, 8. VF, 9. BBJ

DOWN: 1. DHZ, 2. AOAR, 3. ADH, 5. GFF, 7. YU. B.A.