## ROCKET PROPULSION

*Submitted by D. Hurden, Esq., Bristol Siddeley Engines Ltd.*

A rocket engine propels an aircraft or missile by converting the chemical energy stored in a fuel into kinetic energy. It does this by burning the fuel at a moderately high pressure in a combustion chamber (which is essentially a cylinder with a hole in one end) and allowing the hot gases so formed to escape as a high-speed jet. According to Newton's Third Law of Motion, the force that accelerates the gases backwards is accompanied by a reaction that accelerates the rocket forward.

This description of the principle on which a rocket engine operates could apply equally well to an aircraft turbo-jet engine, which also converts fuel into a propulsive jet of hot gas. Air is swallowed by a compressor which raises its pressure and feeds it into a combustion chamber in which the oxygen it contains is used to burn a liquid fuel. The hot gases so formed flow through a turbine that drives the compressor and then escape from the jet pipe. The essential difference between such an engine and a rocket is that the turbo-jet relies on oxygen from the surrounding air to burn its fuel whereas the rocket carries its own supply of oxygen with it. A rocket used to propel an aircraft or missile or spacecraft will use either liquefied oxygen or an oxygen-rich liquid like nitric acid to burn liquid fuels such as alcohol or kerosine. The firework rockets launched in great numbers on November 5th burn a solid fuel with oxygen from potassium nitrate.

*Fig. 3*

*Fig. 2*

The fundamental difference between these two kinds of jet propulsion engines is emphasized by the two sketches. In Figure 1 the " engine " is seen to be picking up its " oxygen " from its surroundings before accelerating it aft, while in Figure 2 the " oxygen " is being carried in the vehicle. These pictures also underline two other important facts ; first, any jet engine works on the principle set out in Newton's Third Law and not by pushing against the atmosphere. Secondly, although a turbo-jet will not work in space because there is no oxygen for it to swallow, a rocket will work just as well out of the atmosphere as in it, so some kind of rocket must be used to propel spacecraft.

## LETTERS TO THE EDITOR

From MATHEMATICAL PIE No. 43, page 339, Solutions to " Mathematical Juggler." The *force* on his hand to stop the falling balls would be *greater than the Mass* of one ball so he could not cross the bridge safely. Since when has a force been greater than a mass ????

P. R. Huish, Shirley, Croydon, Surrey.

*Is my face red?*



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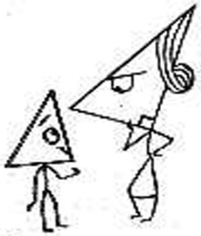
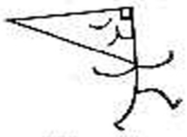
# MATHEMATICAL PIE

**No. 44** *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England **FEBRUARY, 1965**

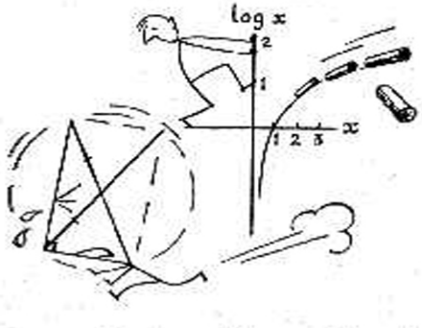

## A STORY COMPLETELY UNCONCERNED WITH THE SINE FORMULA

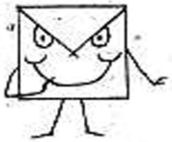
*Submitted by Elizabeth M. Rann, Herts. and Essex High School.*





There was once an acute angled triangle who wanted to become a square. Worried by his problem he consulted his aunt, Right Angled Pythag., who suggested a topological transformation but, unfortunately, she only had the power to change him until he was similar to herself. In this changed form he set off for a walk round the perimeter of the wood near his aunt's house to think, and suddenly he came across a man cutting logs with axes. This man accidentally cut his hypotenuse and he discovered he was a cyclic quadrilateral and an isosceles triangle. But still he was not a square. He caught an aeroplane and flew via the land of the Triangle of Velocity to a point on the longitudinal circle 45°W. There he found a wise old owl who directed him round the earth on a compass bearing of West for 3,960 miles. Sadly, because he was not on the equator this did not get him far ; only a few thousand yards. By now the cyclic quad., because it had had no pies, had the triangle inscribed in it because it had eaten it owing to extreme hunger. Moving millimetre by millimetre the cyclic found that his other corners were right-angled and, because of an unknown theorem, he was a square ! It was tragic that on his way home to his aunt, Right Angled Pythag., he fell down a Three Dimensional Well to the land of Forgotten Facts and dwelt there, forgotten, evermore.





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## WHO IS FROM WHERE?

Three men, let us call them $A$, $B$, and $C$, went out to dinner in Paris. One was English, one Scottish, and one Welsh. The waiter was asked if he could guess their respective nationalities. He said that $A$ was English, $B$ was not English, and $C$ was not Welsh.

Only one of his answers was correct. What were the nationalities of $A$, $B$, and $C$? J.G.

## JIG-SAW PUZZLE

*Submitted by Canon D. B. Eperson, Christ Church College, Canterbury.*

Any trapezium, when cut into two pieces along the line joining the mid-points of its two non-parallel sides, can be made into a parallelogram in two different ways. Can you find them? Answers on page 346.

Any trapezium can also be cut into two pieces that can be made into a triangle. How should the cut be made?

In how many ways can this be done, each providing a triangle of a different shape?

## PYTHAGOREAN TRIADS

It is a common experience when using the theorem of Pythagoras to find that when the hypotenuse is calculated, its length is found to be irrational, it is the square root of a number which is not a perfect square. To find the sides of certain right angled triangles if all the sides are to be rational, the following rule is worthy of note :—

(1) Write down any fraction, using any two numbers whatever,

*e.g.,* $\frac{9}{17}$. Invert it and double; $\frac{17}{9} \times \frac{2}{1} = \frac{34}{9}$

(2) Add 2 to each: $\frac{9}{17} + 2 = \frac{43}{17}$. $\frac{34}{9} + 2 = \frac{52}{9}$

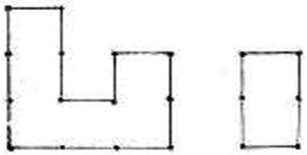
(3) Multiply each by the L.C.M. of the numerator and denominator of the original fraction.

$$\frac{43}{17} \times 17 \times 9 = 387; \quad \frac{52}{9} \times 9 \times 17 = 884.$$

The resulting numbers (387 and 884) when used as the shorter sides of the right angled triangle will always yield a hypotenuse which is rational. The sum of the squares of 387 and 884 is 931225. The square root is exactly 965.

The numbers were purposely made large, try the method with smaller numbers and prove that the result is always true. J.G.

## CAN YOU MATCH THIS?



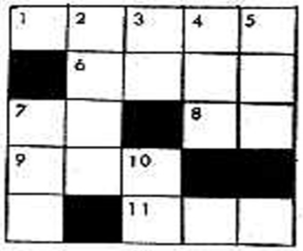
Here you see twenty matches arranged in two groups so that the larger encloses three times the area of the smaller. Can you now transfer *one* match from the larger group to the smaller and rearrange them so that the thirteen matches again enclose an area three times as large as that enclosed by the seven matches. If it is any help to you, twelve of the matches are not touched at all and, of course, none is bent or broken. R.M.S.



40430 27253 86076 48236 34143 34623 51897 57664

## JUNIOR CROSS FIGURE No. 37

*Submitted by Peter Packard, Felixstowe Grammar School.*





CLUES ACROSS

1. Product of the first three perfect numbers.
6. sine 83° 30′.
7. $3x + 2y = 22$
   $x - y = -1$. Find $xy$.
8. Multiply the square of the second prime number by the third prime number.
9. Product of two consecutive prime numbers.
11. Square of 0.9.

CLUES DOWN

2. cosine 67° 1′.
3. $x(y + 1) + 3y$. See 7 across.
4. The first three terms of an A.P. of which the first three terms add up to 9.
5. If 540 half-pennies are put end to end, how far short are they of half a mile (in yards).
7. Exact square root of 47089.
10. Two sides of a right-angled triangle are 40 and 50 units. Find the number of units in the third side.

## DICTIONARY MATHS.

(*genuine extracts*)

GEOMETER 1. A student of geometry. 2. A kind of moth, the caterpillars of which appear to measure out the ground as they move by drawing up and extending the body in loops.

BUN Small round sweet spongy cake with convex top and too few currants.

SPIRAL 1. Forming a curve that winds continually about a centre from which it constantly recedes. 2. Winding constantly about a centre while undergoing continual change of plane.

(Mathematicians may not agree with some of these !). J.F.H.

## DOUBLE OR WIN

Three men play a game with a rule that the loser is to double the money of the other two. After three games each has lost one game and each ends with 24/-. How much had each at the start of the game? R.H.C.

## SOLUTIONS TO PROBLEMS IN ISSUE No. 43

A DOG'S LIFE

The statements $A$, $C$, $E$, $H$ are true.

DIVIDE AND RULE

The triangle must be one with angles 36°, 72°, 72° or 45°, 45°, 90°.

FOOTBALL RESULTS

The club won 20 games, drew 8 games and lost 6 games.

FOR BIRD WATCHERS

The four birds must be at the vertices of a regular tetrahedron.

SENIOR CROSS FIGURE No. 41

CLUES ACROSS: (1) 11, (3) 101, (5) 1111, (7) 201, (8) 11, (10) 1210, (12) 110, (13) 11.
CLUES DOWN: (1) 122, (2) 1102, (3) 111, (4) 11, (6) 1101, (9) 111, (10) 11, (11) 10.

QUICKIE CROSS FIGURE

10 Across should read $3 \times 2$. 1 Down should read $134 \times 13$.

CLUES ACROSS: (1) 3202, (4) 41, (10) 11, (11) 2031.
CLUES DOWN: (1) 3402, (2) 21, (3) 2011, (10) 13.

B.A.



11872 67675 62204 15420 51618 41634 84756 51699

Bugia in N. Africa and it was through this connection with commerce that Leonardo became acquainted with the various arithmetical systems in use around the Mediterranean. Convinced of the superiority of the decimal system which we still use today he wrote the *Liber Abaci* to make it more widely known. His other works include algebra, practical surveying, trigonometry and a rather remarkable series: 0, 1, 1, 2, 3, 5, 8, 13, 21, . . . .in which each term is the sum of the previous two. This series has a habit of cropping up in all sorts of strange places.

Towards the end of our period we have *Oresme* (c.1360) who used a form of coordinate geometry, *Peurbach* (1423–1461) and his pupil *Regiomontanus* (1436–1476) who published a table of sines and the latter a Trigonometry far in advance of anything done before. Indeed the end of the 15th century was remarkable for the number of text-books published on various branches of Mathematics.

R.M.S.

## CHARLIE COOK INVERTS

Solve $$\frac{1}{x-9} + \frac{3}{x-10} = \frac{6}{x-8} + \frac{2}{x-6}$$

Invert $$\frac{x-9}{1} + \frac{x-10}{3} = \frac{x-8}{6} + \frac{x-6}{2}$$

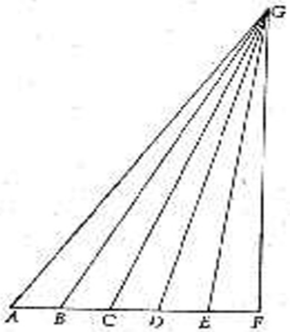
Multiply by 6, $6(x-9)+2(x-10)=x-8+3(x-6)$

$$x = 12.$$

Answer checked by substitution.

J.G.

## DOWN MEXICO WAY 1968




Six runners, each capable of running 10 yards per second, are arranged along a straight line $AF$ at equal intervals of 30 yards. Their goal is a point $G$ situated 200 yards from $F$, and $\angle AFG$ is a right angle. The runners do not start simultaneously, but at time intervals of 1 second; $B$ starts one second after $A$; $C$ one second after $B$, and so on. Which of the runners will arrive first at $G$ and in what order will they finish? Where are the runners after 5 secs?

J.G.

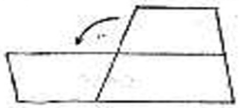
## CORNERING

(1) A square of 10-inch side is made into a regular octagon by cutting off four corners. Where should the cuts be made?

(2) The same square is made into an equilateral triangle by cutting off three corners. Where should the cuts be made?

What is the length of each side of the triangle?

J.G.

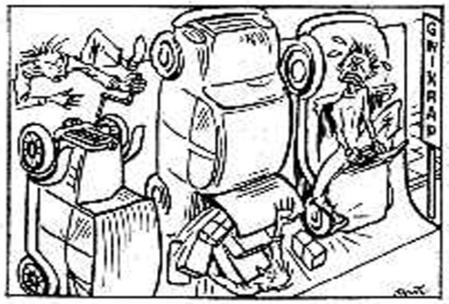


## JIG-SAW PUZZLE ANSWER



55582 15725 03107 12570 12668 30240 29295 25220

## THE GRAVITY OF THE PARKING PROBLEM



In a letter to the Editor of a small newspaper, a citizen suggested that the parking instruction which said 'that cars should be parked at right-angles to the curb' should read 'perpendicular to the curb!'

## A PROGRESSIVE DATE

*Submitted by Mr. B. K. Booty, Wotton-under-Edge.*

On the first day of August, 1964, the day, month, and year (1.8.64) were in Geometric Progression. (G.P., that is, the ratios 1 : 8 and 8 : 64 are equal). Counting the year '01 as 1, etc., how many such dates occur in a century?

## A SUPERIOR ADDRESS

In the number 35,678,243,100, how much greater is the value of the 3 on the left in comparison to the 3 on the right?

R.H.C.

## SENIOR CROSS FIGURE No. 42

*Submitted by Kenneth Turner, Hutcheson's Grammar School, Glasgow.*

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | 2 | 3 | 4 |
| | ■ | ■ | 5 | | |
| 6 | 7 | 8 | | 9 | |
| 10 | | 11 | | | |
| | 12 | | ■ | ■ | |
| 13 | | | | | |

**Ignore decimal points in the answers.**

CLUES ACROSS

1. Product of the squares of the roots of $x^2 - 12x - 693 = 0$.
5. Area of a right angled triangle with hypotenuse 6 units and an angle whose tangent is 2, over 3.
6. The sine of the angle whose cosine is .5402.
9. The smallest number that is three times the sum of its digits.
10. The smallest number which is the sum of its digits plus the number formed by its digits transposed.
11. The sum of the series $1500 - 750 + 375 - \dots$
12. The area between the graphs $y = x^2 - 2x + 6$, $y = 2x - 3x^2 - 2$, $x = 0$, $x = 3$.
13. The smallest number ending in 4, which is multiplied by 4 when this 4 is transposed to the first place.

CLUES DOWN

1. The smallest number ending in 1 which is reduced to $\frac{1}{2}$ of its value when this 1 is transposed to the first place.
2. The sum of the infinite series $2025 - 202\frac{1}{2} + 20\frac{1}{4} + \dots$
3. The circumradius of the triangle with $a = 77.2992$, $\angle B = 28° 12'$, $\angle C = 90° 21'$.
4. The product of the squares of the roots of $x^2 - 77x - 968 = 0$.
7. The side $a$ of a triangle in which $\angle A = 61° 39'$, $\angle B = 30°$, $b = 25$ units.
8. The smallest number which gives remainders of 4, 1 and 2 when divided by 13, 19, and 7 respectively.
9. The value of $x$ to give $x(x^2 - 28x - 80)$, its minimum value.
12. $$\frac{3x(x^2-2x+3)}{5x-9} + \frac{(x^3+3)(x^2+4)}{x+2}$$ when $x = 2$.



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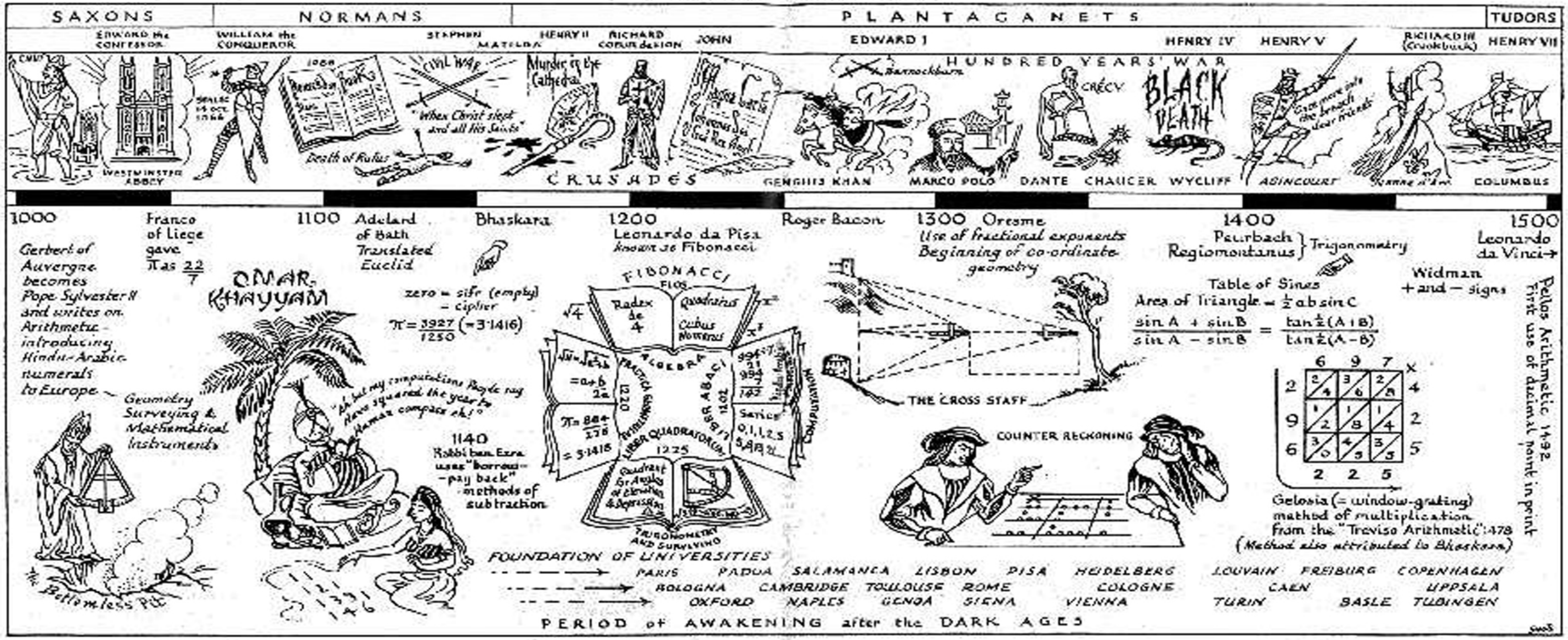
# TIME CHART No 5




The 3rd century A.D. saw the end of the age of Greek Mathematics and from then until the middle of the 11th century there was little mathematical activity in Europe. We now come to the period of awakening after this Dark Age.

There were three factors which accelerated the spread of knowledge in Western Europe at this time, namely, the great expansion of trade between wealthy merchants in many countries, the founding of the great Universities beginning with Paris, Bologna and Oxford and the invention of printing with movable type.

The spread of commerce brought with it a need for greater facility in Arithmetic and we see the development of methods of multiplication and division. Until then all calculation had been done on the counting frame or sand-table and the results recorded in the awkward Roman system of numerals. Now, however, the spread of the Hindu-Arabic system with its place notation and the use of the cypher or zero for the empty column enabled calculation to be done on paper and the counting frame to be discarded like the chrysalis case of a butterfly after it has served its purpose. This was the era of the publication of many books on the Art of Reckoning such as the *Treviso* Arithmetic (1478) and about this time we find the symbols + and — first appearing in print, though they had long been in use to



91642 19399 49072 36234 64684 41173 94032 65918

indicate over or underweight bales of merchandise.

Another consequence of this spread of trade was the need for accurate measurement of land. This led to the science of surveying and the use of instruments such as the quadrant, the cross-staff and the astrolabe and to the development of trigonometry.

*Omar Khayyam* (c.1044–1124) is usually remembered for his poetry, notably the *Rubaiyat* which is well known in its English translation. In addition he was no mean mathematician, writing on Euclid, astronomy and calendar reform as well as a noteworthy book on algebra which contains the triangle of numbers usually attributed to Pascal four centuries later.

Another of the famous names of the era is *Bhaskara* who lived in India in the mid-twelfth century. His most important work is the *Lilavati* (named after his daughter) which deals with the common arithmetical operations, mensuration, commercial rules (*e.g.*, for Interest), proportion, some algebra and a statement of the rules for operating with zero. In other books he deals with negative numbers, surds, simple quadratic equations and Pythagorean numbers ($a$, $b$, $c$ such that $a^2 + b^2 = c^2$).

Probably the most famous mathematician of this time is Leonardo da Pisa, often known as *Fibonacci*. His father was the Customs officer for



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