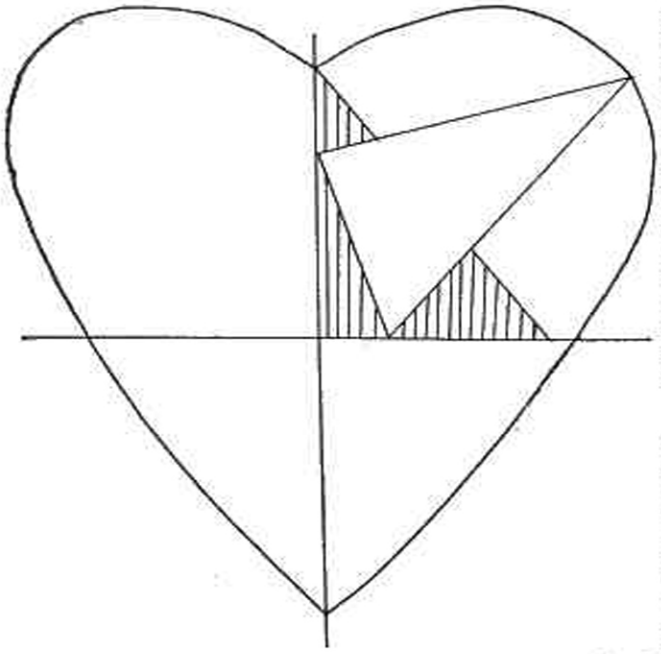
WITH LOVE FROM ME TO YOU



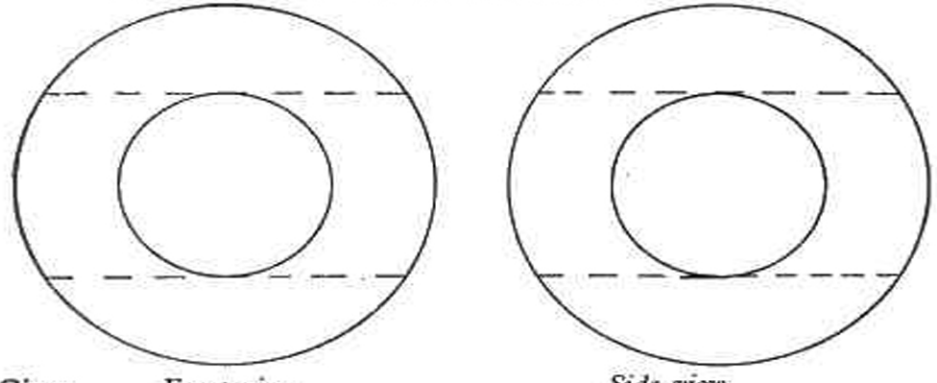
When anything moves, any point on it traces out a path which may be called a graph or a locus. It is usually invisible but can be represented in a diagram. A triangle is often connected with love and the traditional symbol of love is traced out when a right-angled triangle moves in a certain way.

On a fairly large sheet of paper, draw axes at right angles to each other. Cut out a right-angled triangle from cardboard—a 3, 4, 5 triangle is a suitable shape—lay it with the right angled vertex and the vertex where the shorter side meets the hypotenuse on the two axes and trace out the locus of the third vertex.

Every curve can be represented by an equation and the heart-shaped locus of the eternal triangle can be written $y=\frac{ax}{b}\pm\sqrt{(b^2-x^2)}$, where a is the length of the shorter side and b is the length of the longer side of the original triangle.

J.G.

POINTS OF VIEW (*Adapted from Le Facteur X*)



Front view *Side view*

Given

Required :—the plan view of the body.

J.F.H.

332

99771 52515 64682 33580 31653 63270 64184 48135

MATHEMATICAL PIE

No. 42 *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England **MAY, 1964**

THE WANKEL ENGINE

If a circle of radius a rolls without slipping round a fixed circle of radius $2a$, a point on the rim of the rolling circle follows a curve called an epicycloid. Exactly the same curve can be generated by rolling a hollow circle of radius $3a$ round a fixed circle of radius $2a$, like a large ring rolling round a slender finger. This alternative construction is shown by the dotted lines in Fig. 1.

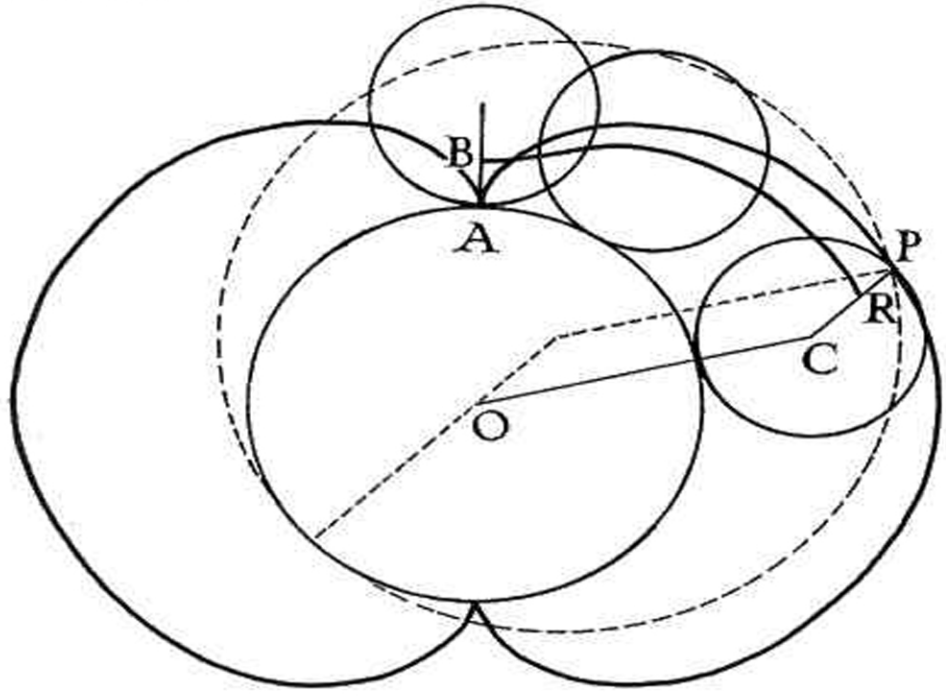


Figure 1

Any student of *A* level mathematics can prove the peculiar property of this particular curve which is that an equilateral triangle of side $3\sqrt{3}a$ can be rotated with all three vertices on the curve whilst its centroid moves round a circle of radius a. Although the triangle is inscribed in the curve, we cannot say that it moves round inside the curve because its sides cut across the cusps (pointed parts) of the curve, as in Fig. 2.

If we look again at the rolling circle and consider a point which is not on the rim, but at some smaller distance—say b—from its centre, we find

325

05527 23231 43898 04477 85279 06769 03700 23206

that it moves round a curve which has two 'dimples' instead of two cusps. If the point is sufficiently near the centre of the circle the dimples become very small and we obtain a curve—an epitrochoid—inside which the triangle can turn without its sides cutting across the curve.

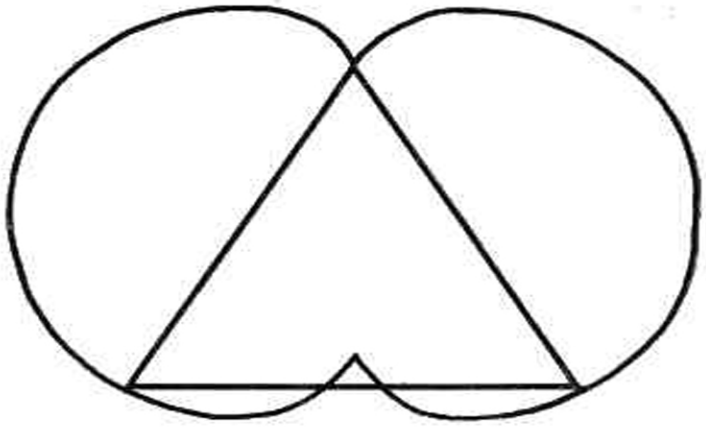


Figure 2

A chamber with a cross-section of this shape and a rotor having a cross-section that is triangular, with curved sides, Fig. 3, are used in the revolutionary Wankel engine which has been successful in experimental cars and may soon be in full scale production.

The three spaces between the rotor and the chamber take the place of the cylinders of the piston engine. In the piston engine the four ' strokes ' of the cycle take place in each cylinder in turn. Air-petrol mixture is drawn in, induction, through the inlet valve, compressed, ignited by the spark and expanded, then exhausted, but in the Wankel engine these four strokes take place in different parts of the chamber.

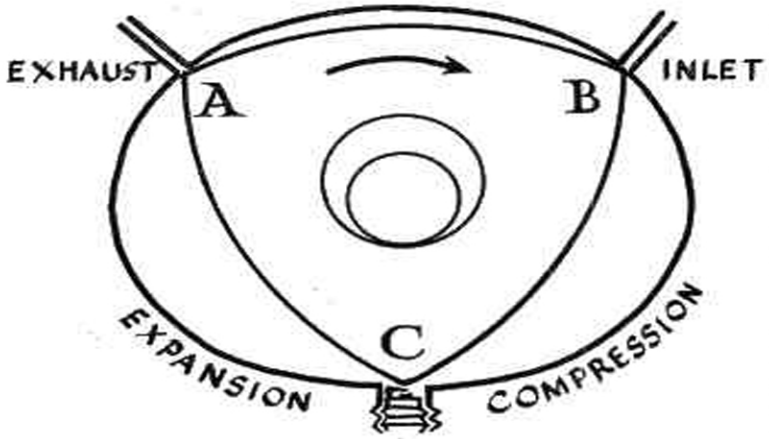


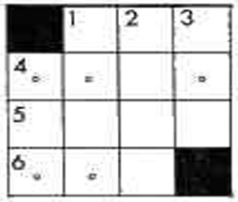
Figure 3

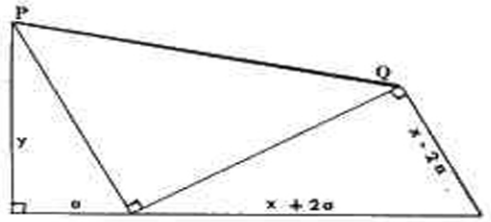
Trace the outline of the rotor and place your tracing over the chamber in Fig. 3.

In the diagram the space on AB is at the start of the cycle. As the rotor turns the space increases and air-petrol mixture is drawn through the inlet port. When AB is vertical A passes over the inlet, which is then connected to the next space, and compression begins. When AB is horizontal at the bottom of the chamber, the mixture is fully compressed, then ignited by the spark, it expands and drives the rotor until B passes over the exhaust port. As AB turns into the horizontal the space decreases, the spent gas is expelled through the exhaust port and the cycle is completed. Each space passes through the complete

20700 35037 59717 95349 43860 43758 61175 41117

JUNIOR CROSS DIAGRAM No. 2





HINT : Terms are usually in alphabetical order. Small circles indicate ± squared terms.

CLUES ACROSS :

1. $(c-d)-4(a+b)-2(a+b)-(c-5d)$.
4. $(y+a-b)(y-a+b)$.
5. Area of a rectangle ; adjacent sides $2(a+b)$ and $2(x+y)$.
6. $(a-bc)^2-b^2(c-1)(c+1)$.

CLUES DOWN :

1. Numerator when $\frac{b^2-a^2}{a}+4y-6$ is written as a single fraction.
2. Multiply the difference of $(a+2x)$ and $(ac+3)$ by $2b$ and rearrange.
3. Divide $6dx^2-x(b^2x-4bxy)$ by $(-x^2)$.
4. PQ^2 in the figure. J.G.

" Who's the Square on the Hypotenuse ? "

DROP A BRICK ?

(*Adapted from Le Facteur X*)

Johnny was playing with his model boat in the bath. As cargo for the boat, he had a half-brick. After a time, he took the piece of brick out of the boat and put it at the bottom of the bath. Does the level of the water in the bath rise, fall, or remain the same? J.F.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 41

FIVERS

11,617 in the decimal scale equals 332,132 in the scale of five. Hence two received £1 each, three £5, one £25, two £125, three £625, and three £3,125. Number of relatives is 14.

SENIOR ALGEBRAIC CROSS DIAGRAM

CLUES ACROSS : (1) ax^a, $2bx$, c ; (4) a^2x, ab, $3c^2$, a^2 ; (6) $3c^3$, b^3, a^3 ; (8) c, c^4, c^9, c^8 ; (9) b, $4ac$.

CLUES DOWN : (1) ax^2, a^2x ; (2) $2bx$, ab ; (3) c, 3^2c, $3c^2$, c^4 ; (5) a^3, b^3, c^3 ; (7) a^2, c^2 ; (8) c, $4ac$.

OBSTRUCTION

See next issue.

DARTS

The first man cannot win as 301 is an odd number. He could win if the scale were to an odd base *e.g.*, 5.

T.V. TIMES

$\pi \times 625$ is more accurately 1963.5.

BINARY CROSS FIGURE

CLUES ACROSS : (1) 111, (100) 11, (110) 1001, (111) 101, (1000) 10, (1001) 1101, (1011) 11001, (1101) 10, (1110) 100.

CLUES DOWN : (1) 111011, (10) 100, (11) 10110, (101) 1101, (1000) 1010, (1010) 101, (1100) 10.

ALGEBRA CORNER

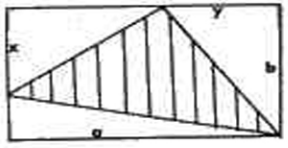
3. The addition signs in the second expression should be multiplication signs. B.A.

42267 62164 94318 60332 24008 78900 25483 29237

is a member of two great families of curves : the caustic curves (formed by reflection) and the epicycloids (formed by circles rolling on circles).

Jean's son, Daniel, developed the theory of Wave Motion from the vibration of stretched strings. This is applicable to many problems from musical instruments to electron microscopes and the design of television aerials. On his work on Fluid Dynamics is based the design of the modern jet engine and indeed the aircraft itself owes much to Bernoulli's Theorem. We shall meet Daniel Bernoulli again in our next Time Chart. R.M.S.

ALGEBRA CORNER No. 2



1. What profit % is made by buying articles at x for y pence and selling at y for x pence ?
2. What is the area of the shaded triangle ? The answer contains 3 terms.

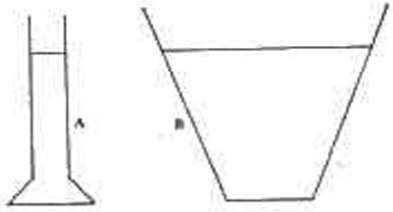
$y^2 - 4ax$	$a + bx$	$a - b$
$h^2 - ab$	$ax^2 + 2hxy + by^2$	$\frac{c}{ab}$
$g^2 - bc$	$a + b$	$ax + hy + g$

≡

4	17	1
29	637	1
1	9	80

3. The diagrams show an algebraic cross diagram and its solution. Find the values of a, b, c, x, y, g and h.
4. In a race of x yards, A beats B by b yards and C by c yards. If B and C only ran the race, by how many yards would B beat C? J.G.

A WEIGHTY PROBLEM



The diagram shews two vessels filled to the same depth with water. The area of the base of vessel A is the same as that of vessel B and vessel B holds 100 times as much water as vessel A. Which base has the greater load on it? R.H.C.

MATHEMATICAL JUGGLER

A man carrying 3 bowling balls came to a bridge that would only carry his weight and one ball at a time, so he decided to juggle the balls as he crossed so that 2 are always in the air. Smart?

THE AVERAGE FAMILY

The average family is composed of 1 underpaid male, 1 overworked female, and 2·2 underfed children. (*From Moroney : Facts from Figures*)

FROM A MATCHBOX

The Right Angle for tackling problems is the TRY ANGLE.

330

26193 36416 92936 97783 37178 40550 29073 62691

cycle once in every revolution of the rotor.

To ensure smooth operation of the rotor it has on one side a circular recess with internal teeth which engage with a stationary gear of two-thirds the diameter, see Fig. 4. On the other side the 'centre' of the rotor, which moves twice round the circumference of a small circle in every revolution of the rotor, is connected by a crank to the output shaft. C.V.G.

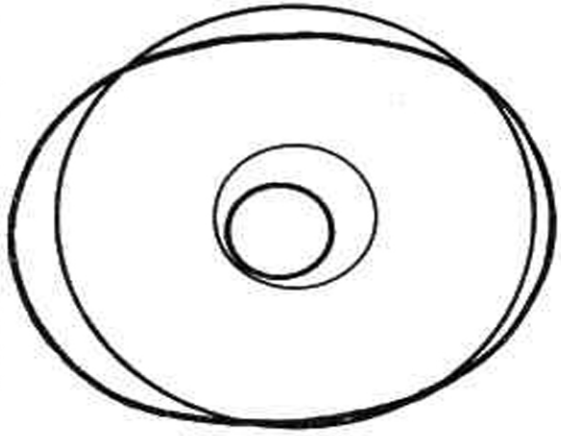


Figure 4

"I make it 22 revs per minute"

FUN WITH NUMBERS No. 7

(*Suggested by Miss Margaret M. Gow, Sedgley Park College, Prestwich, Manchester*).

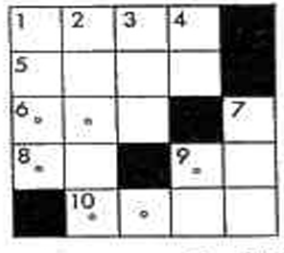
$4^2 = 16$ and $17^2 - 15^2 = (2 \times 4)^2$

$5^2 = 25$ and $26^2 - 24^2 = (2 \times 5)^2$

Generalise these statements and verify that the generalised result is true.

SENIOR CROSS DIAGRAM No. 2

HINT for arrangement ; the small circles represent squares.



CLUES ACROSS :

1. Divided by $ax^2 + bx + c$, the quotient is x and the remainder is d.
5. $(a + b + c)\ (a^2 + b^2 + c^2 - bc - ca - ab)$.
6. Value of y^2 when $\frac{x}{a} = c$ in figure.
8. $B^2 - AC$ when $B = a + b$; $A = a + 2b + c$ and $C = a$.
9. $(f + 1)^2 - (1 - 2f)$.
10. $(x - g)^2 + (y - f)^2 - (g^2 + f^2)$.

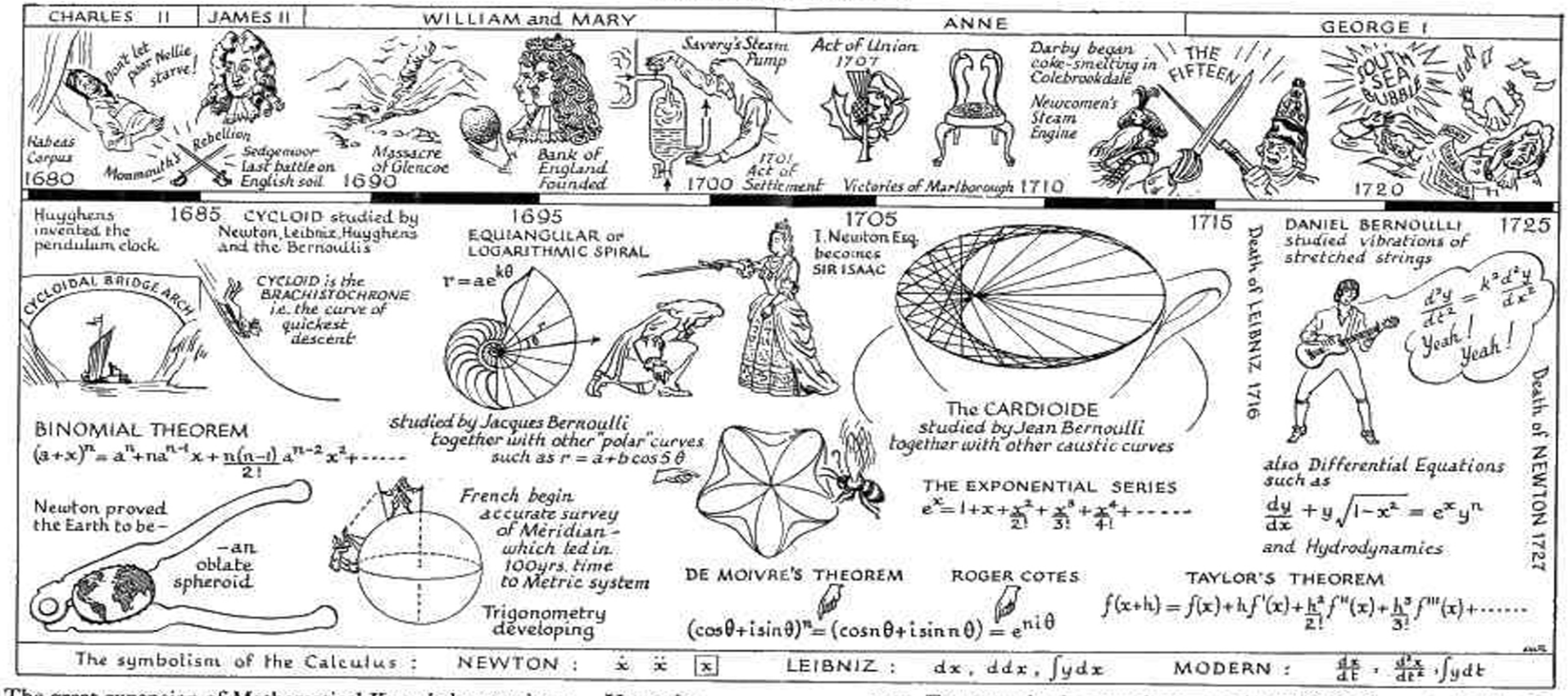
CLUES DOWN :

1. Numerator of $\frac{a}{a^2 + b^2} + \frac{1}{x^3 + a^2}$ written as a single fraction.
2. Value of $K + \mathcal{J}$ when $\frac{K}{x^2 + b(b+1)} = b$ and $\frac{x^2 - \mathcal{J}}{a} = c$.
3. $c\ (c^2 + x) - 2bx$.
4. Square root of $d^2 - 6\ abcd + 9a^2b^2c^2$.
7. Area of a border f ft. wide around a rectangle x ft. long and y ft. wide.
9. Added to $x^2 + 2fx + x^2 + g^2$ gives the sum of two squares. J.G.

327

77684 06484 74521 50457 98113 97853 08940 90850

TIME CHART No. 11



The great expansion of Mathematical Knowledge continues. Up to the time of Newton it might have been possible for one man to master all the then known Mathematics but from then on it was an impossibility. Men had to select but many were proficient in several fields. A genius has the ability to seize upon the essential idea in a situation and to express it simply. Following up the consequences of such an idea may provide work for other men for a couple of centuries or so. Newton was one such genius and Leibniz was another.

Now that the Calculus was allied to Algebra, mathematics had a wonderful new tool. It was used to investigate functions of all sorts : the Binomial and Exponential functions in particular. The Binomial function finds uses in approximations and in probability. It may be regarded as a generalisation of Pascal's triangle. (Use it to work out $(1+1)^5$ and compare with Pascal's triangle). The exponential function is sometimes called the law of Organic Growth ; it has the property that it is its own rate of change and e^1 or e itself is a Universal constant like π, appearing in many strange situations. Both the Binomial and Exponential Functions are special cases of Taylor's Theorem. This is as fundamental to the advanced mathematician as the twice times table is to someone playing double or quits.

Many of the developments of this period were due to the work of many

men. For example, the curve known as the CYCLOID was studied by Newton, Liebniz, Huyghens, the Bernoullis and others. This is the curve that would be traced out by a spot of paint on the outside of a cycle tyre as the cycle follows a straight path. Cycloidal arches are sometimes used for bridges giving great strength. Inverted, the cycloid is the curve down which a body will slide under gravity in the least time and in fact the time taken for a body to slide from any point to the lowest point of the inverted arch is always the same.

The Bernoullis appear again and again in the mathematical history of the next 100 years. In 3 generations this family produced eight mathematicians of whom Jacques I, Jean I and Daniel are outstanding. Jacques I worked in many fields : the calculus of variations, probability, the study of Polar Curves, *i.e.*, curves whose equations are best expressed in terms of r, the distance from a given point and θ, the angle turned through from a given direction. Such a curve is the equiangular spiral $r = ae^{k\theta}$, a curve which is beautifully illustrated by the Pearly Nautilus shell and is also found in the arrangements of the florets in the head of the daisy, aster or sunflower.

His brother Jean I wrote on the theory of tides, the mathematical theory of ship sails, the principle of Virtual Work in Mechanics and also studied the CARDIOIDE. You have probably noticed this curve traced out by the reflection of the sun's rays on to the surface of a cup of tea. The cardioide