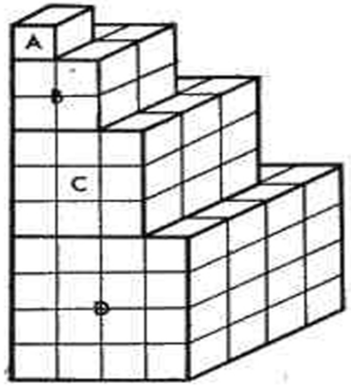
MATHEMATICAL PIE

No. 41 — *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England — FEBRUARY, 1964

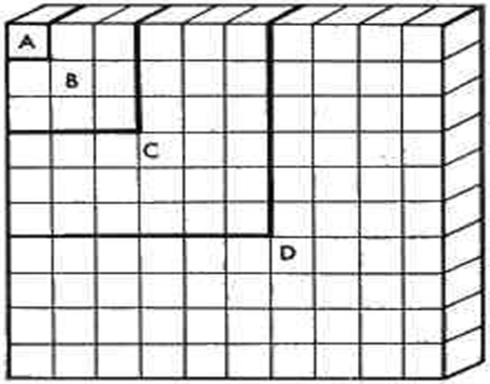
CUBES AND SQUARES

If you have patience and a plentiful supply of sugar cubes, it is easy to build up the structure shown in Figure 1. It consists as you can see of a succession of cubes, the top one having one unit side, the next one two units, the succeeding one three units and the lowest one four units in each of its sides, or edges.



The number of unit cubes (lumps of sugar) contained in each of the successive cubes increases rapidly. In the bottom one these are $4^3 = 64$ units. If the structure were continued until a cube of side 10 units formed the ' foundation ' cube, this one would contain 1,000 unit cubes ; the number in the whole structure would then be very large, more than 3,000. Figure 1, which reminds us perhaps of some modern architecture, is now to be demolished, but not by an earthquake. It is rearranged systematically, to form a kind of pavement, one unit thick on the ground. Figure 2.

The first cube (1 unit edge) is placed in the position *A*. The 8 elements of the next cube are arranged round *A* and will form a square *B* of 3 units side, for $1 + 8 = 9 = 3^2$. The single cubes of the next cube (27 of them) are placed as at *C*, and you will see from Figure 2 that they can be laid to complete another square of side 6, for $1 + 8 + 27 = 36 = 6^2$. Similarly the last cube, containing 64 units, placed at *D* will increase the existing square to another with side 10, for $1 + 8 + 27 + 64 = 100 = 10^2$.



Now when we rearrange the cubes in each instance, they form a square. Thus $1^3 + 2^3 = (1 + 2)^2$; $1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$; $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$. It appears that this is a property of numbers. You can test it quite easily. You will find that $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 225$.

Assuming the rule suggested by the foregoing, what is the sum of the cubes of the numbers from 1 to 10? Generalise this result for the numbers from 1 to *n*.

J.G.

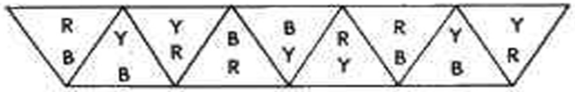
39049 75008 65627 10953 59194 65897 51413 10348

FLEXAGONS

(*Submitted by Miss S. Wallis, Newcastle upon Tyne*).

Flexagons are figures made by folding a strip of equilateral triangles into a hexagon and joining the ends in such a way that if the triangles are coloured in a certain manner, the model can be continuously folded and unfolded to exhibit these different colours in turn.

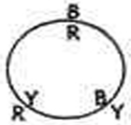
The simplest flexagon is constructed from nine triangles giving eighteen faces, six of which are coloured green, six red and six yellow. A colouring chart for the strip is shown.



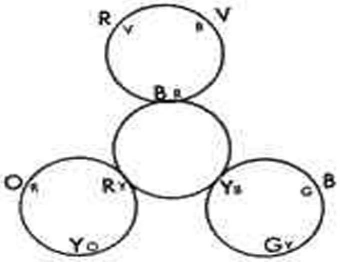
Y=yellow
B=blue
R=red

Front face colour at the top

The strip is folded by giving it three half twists, hiding all faces of one of the colours and the two ends are stuck together. A plan to show the colour changes when the flexagon is operated is given, and as each operation can be reversed, the circular plan can be followed in either direction.



The second flexagon is made from eighteen triangles, coloured with the three primary colours, red, yellow and blue, and in addition three secondary colours, green, violet and orange. When the strip has been coloured, it is given nine half twists to hide all the secondary colours so that it then looks like the first strip shown above, and is completed in the same way. Here the plan of operation has a primary cycle and three secondary cycles branching from it, each containing two of the primary colours with their associated secondary colour. The directions of operation are again shown and it is seen that one cannot from one secondary produce another without first turning back to the central primary cycle.



The third flexagon is a development of the second, as the second is of the first. It consists of 36 triangles, but instead of introducing further colours, the extra triangles have been coloured to produce hexagons of two of the already existing colours in an alternating pattern.

This family of flexagons can be extended, theoretically at least, the difficulty in actual construction and manipulation increases with the size, thin cardboard must be used and a sufficient gap left between each triangle to enable the flexagon to fold.

Note :—there are other types of flexagons which are constructed from triangles not arranged in a straight row so that the plan may take different forms, some of which will be shown in a future issue.

26749 16315 70591 96749 53740 06082 67999 43265

THE HISTORY OF NUMERALS

Primitive people very rarely needed to use large numbers; an analysis of 30 different tribes of Australian Bushmen shews that none of them went beyond 4, and their counting was of the form 1, 2, many; 1, 2, 3, heaps; or 1, 2, 3, 4, lots; so that their methods of writing them would have been very similar to our illustrations in Issue No. 27 and Issue No. 30.

AFRICAN PIGMIES

A, OA, UA, OA-OA, OA-OA-A, OA-OA-OA

A TRIBE IN THE TORRES STRAITS

URAPUN, OKOSA, OKOSA-URAPUN, OKOSA-OKOSA, OKOSA-OKOSA-URAPUN, OKOSA-OKOSA-OKOSA, RAS

Fig. 1

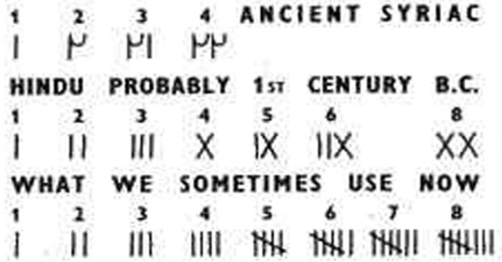


Fig. 2

As races developed their mathematical ideas improvements could not take place without improvements in their written number systems. Such improvements were first reflected in the spoken word as shown in Figure 1 and it will be seen that they are counting in groups of two or three rather as we count in groups of 10.

This counting in groups formed a basis for writing the abstract numerals and some of the earliest examples are shown in Figure 2.

R.H.C.

ENGLISH BY ARITHMETIC

Ever thought of assembling a dictionary?—the easiest way is to call in a computer. But computers, you will say, use numbers, not words—and binary numbers at that. All right then, all we have to do is translate the words which are to go into the dictionary into numbers, and the computer will do the rest, *viz.*: put them into alphabetical order. Actually, we can do this in two stages, first translate words into ordinary (decimal) numbers and then we (or the teleprinter) can easily translate from there into binary numbers.

Suppose we give the letters *A* to *Z* a number apiece starting from 01 and going up to 26.

Then, for *A* we read 01,
for *B* we read 02,

and so on up to *Z* which is 26.

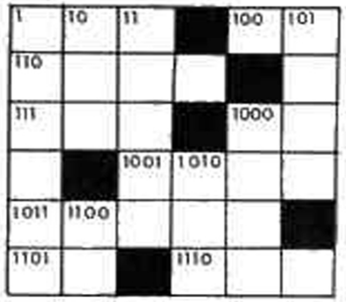
Then the word Art would be written 01 18 20.
01 for *A*, 18 for *R* and 20 for *T*
(translate Maths and 160905).

Unfortunately, words may be of different lengths—2, 3, 4, 5, 6 or more letters, and these would lead to 4, 6, 8, 10 and 12 digit numbers which would be difficult to compare.

However, there is an easy way round this. All we have to do is put a decimal point at the beginning of each word. Thus, Art now becomes .011820.

BINARY CROSS FIGURE

(*Submitted by R. G. Everett, Lincoln*).



CLUES ACROSS:

1. 1011011 ÷ 1101.
100. $\sqrt{1001}$.
110. 11 ! + 11.
111. 100011 ÷ 111.
1000. 110010 ÷ 11001.
1001. (10 × 11 !) + 1.
1011. $\dfrac{101\,! \div 101}{110}$
1101. $\sqrt{\dfrac{100\,!}{11\,!}}$
1110. $\sqrt{11001} - 1$.

CLUES DOWN:

1. £1 — 10011s. — 100d. ÷ 1000. Answer in pence.
10. Maximum value of $(10 - x)(10 + x)$.
11. $\dfrac{1,000,000 - 1}{11} + 1$.
101. $\sqrt{10101001}$
1000. (H.C.F. of 110, 1010, 1100) × 101
1010. $\sqrt[11]{1111101}$
1100. $\sqrt[110]{\dfrac{101\,!}{10} + 100}$

The clues are given in the binary notation. Answers to be given in the binary notation.

SOLUTIONS TO PROBLEMS IN ISSUE No. 40

THE VENUS CLOCK

The time shown on the clock was 2.15 a.m.

SALUTE TO GHANA

The graph produced the Ghana star.

SIMPLE SOLITAIRE

Label the pegs B1, B2, B3, B4, B5, W1, W2, W3, W4, W5 from the centre. Move B1, jump over B1 with W1 and advance B2; jump over B2 with W1, jump over B1 with W2 and advance W3, and so on. The number of moves required is 35. If n pegs of each colour and $(2n+1)$ holes are used, the number of moves is $n(n+2)$.

SENIOR CROSS FIGURE No. 40

CLUES ACROSS: (1) 60, (2) 112, (5) 66, (7) 216, (8) 678, (11) 132, (14) 78, (15) 30 (17) 217, (18) 224, (19) 15, (21) 521, (22) 81, (23) 61, (25) 24, (26) 101.

CLUES DOWN: (1) 606, (2) 12, (3) 13, (4) 26, (6) 612, (9) 771, (10) 8875, (11) 1321, (12) 302, (13) 121, (16) 441, (20) 56, (22) 84, (24) 11, (25) 21.

JUNIOR ALGEBRAIC CROSS DIAGRAM

CLUES ACROSS: (1) $8a$, $6b$, $10c$, (4) $4b$, $8ab$, $4a^2b$, (5) a^2, b^2, c^2, (7) $3bx$, a^2, $4ab$.
CLUES DOWN: (1) $8a$, $4b$, (2) $6b$, $8ab$, a^2, $3bx$, (3) $10c$, $4a^2b$, b^2, a^2, (6) c^2, $4ab$.

SCENE AT WIMBLEDON

Three of the men form the vertices of a triangle and the fourth must stand at the point of intersection of the altitudes of the triangle; the net may be drawn anywhere so as to pair off the players.

SPECIALLY FOR RED INDIANS

To test the statement given, find the maximum number of hairs on a person's head. This will be less than 10,000,000; Suppose it is 1,000,000. Take 1,000,001 people in London, then these may have a different number of hairs on their heads from 0 to 1,000,000. The next person considered must also have 0 to 1,000,000 hairs on his head and therefore the same as one of the first million and one considered.

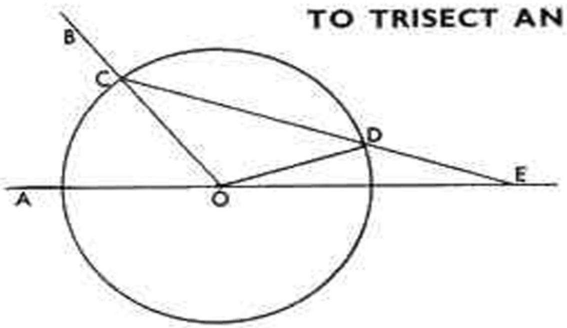
To find how many hairs a person has on his head, take a unit square of scalp, count the number of hairs and multiply this by the number of square units of scalp.

B.A.

Philosophy (Science) is written in that great book,........(the Universe); but we cannot understand it if we do not first learn the language, and comprehend the characters in which it is written. It is written in the mathematical language and its characters are triangles, circles, and other geometrical figures.

Galileo Galilei 1610. J.G.

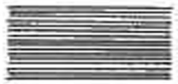
TO TRISECT AN ANGLE



To trisect the angle AOB, draw a circle, centre O to cut OB at C. Produce AO. From C, draw a line CDE so that DE=the radius of the circle. Then $\angle DOE = \frac{1}{3}\ \angle AOB$.

J.F.H.

A CARD TRICK



A pack of cards stacked as a rectangle is displaced as shown.

Therefore the area of a parallelogram is the product of the base and the height. J.G.

OBSTRUCTION

In the course of making a relief map, a student got as far as shown in the Figure. It represents a hill on a plain.

The line AB represents a map coordinate and the student discovered, after he had stuck the model hill in place, that he should have continued the line AB on the other side of the hill. The only ruler that he had was a single-sided straight-edge and, of course, he could not lay this over the hill. Find a method of drawing the required line A^1B^1 without crossing the hill. J.F.H.

DARTS

Two men play darts, one to score 301 all in doubles, and the other to score 1,001 under normal rules. Who has the better chance of winning?

In which scale of notation would it be worthwhile taking a bet? R.H.C.

TV TIMES

In a current advertisement for Pye Electronics, the following statement appears: $\pi \times 625 = 1964$.

How nearly true is this statement? R.H.C.

67286 80336 99136 76363 17847 04533 36146 92473

Now let us see what happens if we translate the, their, there, and put them in alphabetical order, we have

the	.200805.
their	.2008050918
there	.2008051805

Thus, if the words are in alphabetical order, the corresponding numbers are in increasing order of magnitude — and the fact that some words are longer than others doesn't matter any more to the computer. If the computer is asked to put two words in alphabetical order it simply subtracts the first from the second. If the answer is positive then the words are not in alphabetical order; if it is negative then they are.

In this way it can order all the words we feed into it — and out come the words for a dictionary, in alphabetical order. K.A.

ALGEBRA CORNER

1. Verify that the difference between the cost of a lb. b oz. of meat at x shillings y pence per lb. and x lb. y oz. at a shillings b pence per lb. is $ay - bx$ farthings.

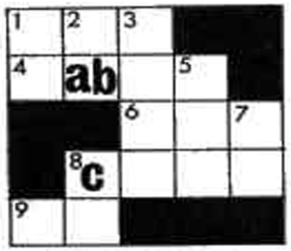
2. Invent a simple algebraic expression which is such that when it is divided by a, the remainder is b, and when it is divided by b, the remainder is c.

3. Simplify $\dfrac{0.a + 0.b + 0.c}{0.abc}$ and $\dfrac{0.a + 0.b + 0.c}{0.abc}$ J.G.

FIVERS

A man dies leaving £11,617 to be divided amongst his relatives. He stipulated that the money was to be distributed in single pounds or whole numbers of pounds which are powers of five. He further stipulated that the numbers of pounds was not to be given to more than four persons. How same sum of money was not to be given to more than four persons. How many relatives had he and how much did each receive? R.M.S.

SENIOR ALGEBRAIC CROSS DIAGRAM



CLUES ACROSS:

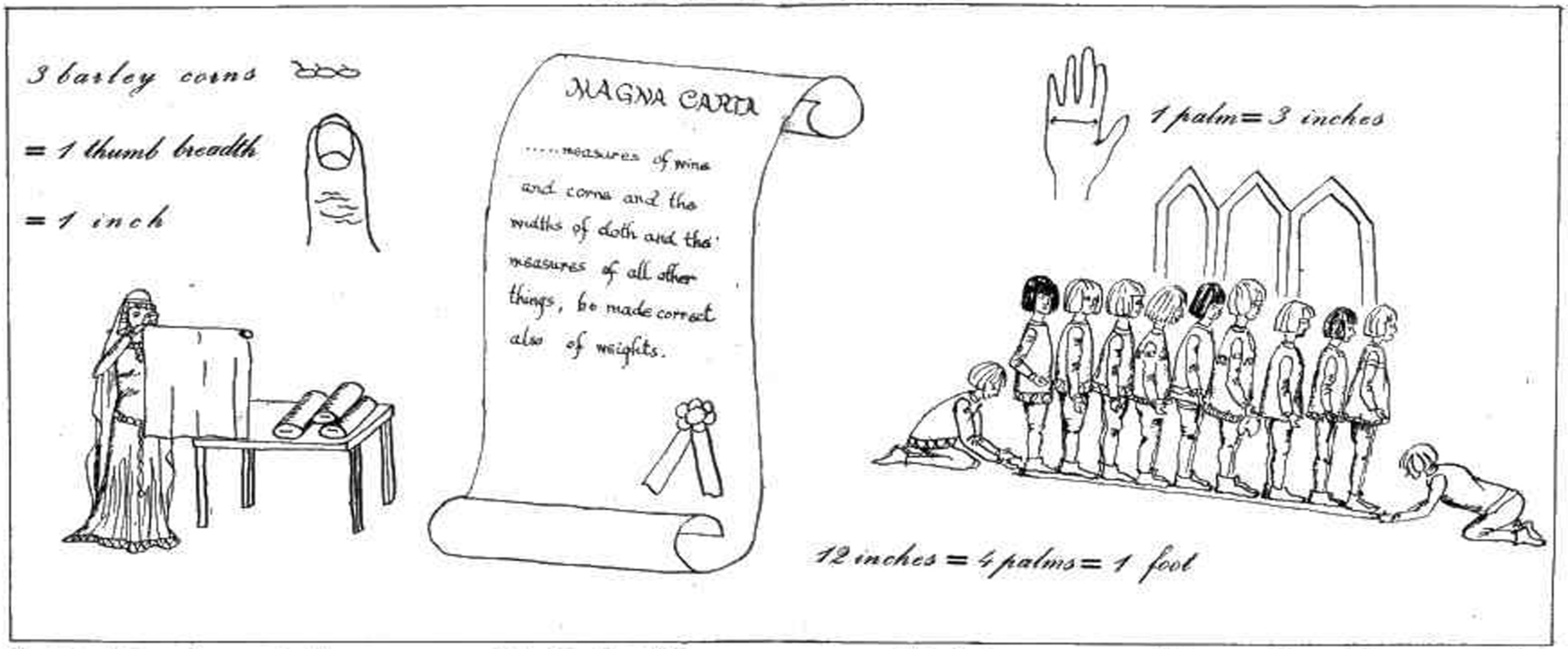
1. $\dfrac{(ax+b)^2}{a} - \dfrac{b^2 - ac}{a}$
4. $(ax - 3c)(a - c) + ac(x+3) + a(a^2+b)$.
6. What must be taken from $4c^3+2b^3$ to give $b^3+c^3-a^2$.
8. $\dfrac{c(c^4-1)}{(c-1)}$
9. The square root of $b^4 - 8acb^2 + 16a^2c^2$.

CLUES DOWN:

1. Divide $ax^3 - a^3x$ by $x - a$.
2. The numerator of $\dfrac{x}{x-b} - \dfrac{x+a}{x+a+b}$ when written as a single fraction.
3. Expand $c(1+c)^3$.
5. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) + 3abc$.
7. $ac\left(\dfrac{a}{c} + \dfrac{c}{a}\right)$
8. x when $\dfrac{x}{c} + 4a = 1$.

Ignore signs and place one term of each answer in each box. J.G.

66795 70611 08615 33150 44521 27473 92454 49454



The first of this series on units of measurement, published in May 1958, described how the early civilisations in the Mediterranean countries introduced units of length for the measures that were required and how some of these units were eventually standardised. Such units as the cubit, span, etc. continued in use for many hundreds of years and were introduced to new countries as culture spread throughout Europe.

However, although the Egyptians had realised the need for standard units of measure as early as the time of the building of the pyramids, the measures used in Europe in, for example, Anglo-Saxon times were as unreliable as any of the early cubits had been. As new units were introduced so was the pattern of history repeated. The yards, feet and inches we use today came from different body measurements just as did their predecessors the cubit, span, palm, etc. and consequently depended on the size of the person making the measurement.

The inch was measured in either of two ways. It could be the length of three full grown barley corns placed end to end or the breadth of the thumb nail measured at the base of the nail. If you make this last measurement on yourselves and compare with your friends you will see how much the different "inches" can vary. Another measure you can check is the palm, which was given as three inches. This is simply the breadth of your hand measured across the palm.

65450 55518 24188 34895 38980 25342 98695 35193

The foot as a measure probably spread through Europe via the Romans who had originally adopted it from the Greeks. This again obviously varied very much from person to person and was eventually linked with the existing measures in England by Henry I. The story is that a number of men were lined up so that each man's toes touched the heels of the man in front and from the total length was calculated the average foot length. As this came to be twelve inches they either had large feet or small inches !

The yard was also in use in the time of Henry I and was most used for the measurement of cloth. You have probably seen cloth measured approximately in just the way it was measured by the merchants of the 11th century by using the distance between the tip of the nose and the end of the outstretched arm. Naturally such a rough method of measurement led to a certain amount of cheating on the part of the merchants and this was one of the reasons for the clause in Magna Carta stating that measures must be made correct.

However, it was not until the reign of Edward III that standard measures were created by law, which stated ". . . . we will, and ordain, that one weight, one measure and one yard be used throughout all the land."

Today one can still see, in the Standards Office, the standard yard instituted by Henry VII. This is an eight sided brass bar, one side of which is divided into three feet with one of them marked in inches ; and another side marked to show $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ of a yard. I.L.C

71115 00340 64216 64280 55720 60291 77236 28041