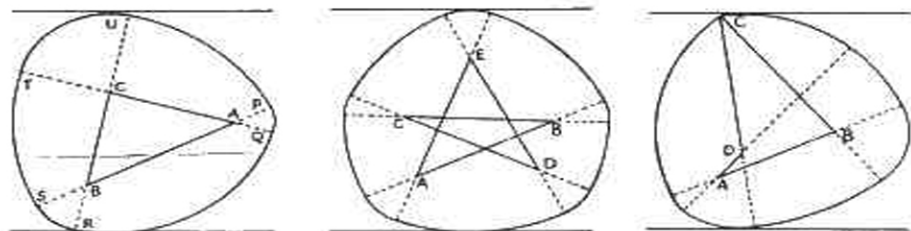


When the attempt was made to manufacture ball bearings by grinding between parallel plates some very queerly shaped objects were produced. There had been a confusion between a theorem and its converse. The proposition that "pairs of parallel tangents drawn to a circle of diameter d are distance d apart" is true, the converse:—"if all pairs of parallel tangents to a curve are the same distance d apart then the curve is a circle of diameter d " is not true.



A simple closed curve is formed by a continuous curve which returns to its starting point without coming to any other point on itself. Thus a circle, a convex polygon, and an ellipse are simple closed curves but a figure eight is not. The breadth of a simple closed curve is found in the following way. Draw a line in the direction in which its breadth is required, drop perpendiculars from each point on the curve to the line, and the segment of the line which contains the feet of all the perpendiculars is defined as the breadth of the curve in this direction. Most closed curves have different breadths in different directions but there are some which have the same breadth in every direction, such curves are called curves of constant breadth.

The simplest type of non-circular curve of constant breadth is formed by drawing on an equilateral triangle, three arcs whose radii are equal to the length of the side of the triangle and whose centres are the three vertices, each arc lying between two vertices. Can any regular figure be treated in this way to give a curve of constant breadth?

The next simplest type of non-circular curve of constant breadth is formed by six circular arcs whose centres are the vertices of a triangle. ABC is a triangle (see first figure). With centre A (it is advisable to start at the vertex opposite the shortest side) and with any radius describe an arc cutting CA at Q and BA produced in P . Then with centre C and radius CQ describe an arc cutting AC produced at T , with centre A and radius AT describe an arc to cut AB produced in S , with centre B and radius BS describe an arc to cut CB in R . It is now easy to prove that the arc with centre C and radius CR passes through Q , so closing the curve, and also that $AP + AS = BR + BU = CQ + CT$. Two parallel tangents to the curve must touch arcs with the same centre, therefore the perpendicular distance between tangents is always equal to PS .

The next two figures are based on a pentacle and a re-entrant quadrilateral respectively. The only limitation on the polygon used is that the sum of the acute and obtuse angles plus the supplements of any reflex angles must be two right angles. (to be concluded). C.V.G. B.A.

308

44604 77464 91599 50549 73742 56269 01049 03778

MATHEMATICAL PIE

No. 39

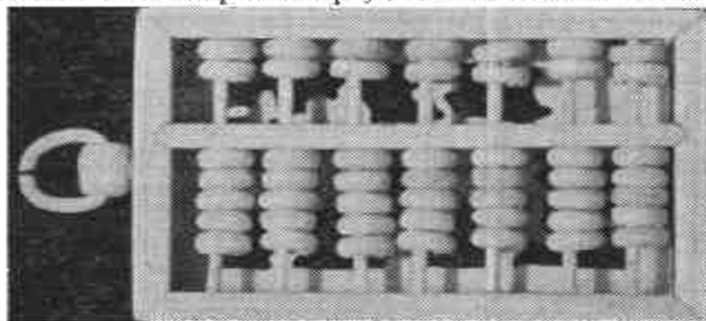
Editorial Address: 100, Burman Rd.,
Shirley, Solihull, Warwicks, England

MAY, 1963

MATHEMATICAL INSTRUMENTS No. 8

The Abacus

The little ivory Chinese abacus, reproduced full size, must have needed dainty fingers. Those in common use in China are rather larger, about the size of a foolscap envelope, and made of wood and wire. Some kind of



abacus, or counting frame, has been used in almost every part of the world. The Romans used marbles on a grooved board. Our word calculation is derived from the Latin word *calculus* which means a small pebble. In the Middle Ages in

Europe, an abacus was a board ruled in squares and the counters were like the "men" used in the game of draughts. The Chinese abacus, the Japanese abacus and the Bulgarian abacus, used in Balkan countries until quite recently, have beads on wires. The five beads on the right hand wire stand for units. The two beads above are fives. On the next wire the five beads represent tens and the two beads above represent fifties. The beads count when they are pushed towards the centre bar. Each wire of a Japanese abacus has only four beads below the bar and one above.

Addition, subtraction, and multiplication or division by two are easy. To divide a number by two start at the right hand side. To double a number start on the left hand side. For multiplication the method of duplication or "Russian multiplication" is used. Only very small multiplications could be performed on the little ivory abacus.

Using figures instead of beads this is how multiplication of 23 by 19 is done

19	23	23
9	46	69
4	92	69
2	184	69
1	368	437

The three columns represent different parts of the working on the abacus. The answer is found in the last column.

The numbers 19 and 23 are entered on the abacus. 19 is odd therefore enter 23 in the last column.

301

56803 44903 98205 95510 02263 53536 19204 19947

Line 2. Halve 19 and double 23, omitting remainders. 9 is odd therefore add 46 to the last column.

Line 3. Halve 9 and double 45

Line 4. Halve 4 and double 92

Line 5. Halve 2 and double 184. 1 is odd therefore add 368 to the last number in the last column.

The number in the last column is now 23/19. Try it using match sticks.

Division is a different matter. When Pope Sylvester published a treatise explaining how it could be done many people thought that he must be in league with the Devil. C.V.G.

RULER AND COMPASSES ONLY

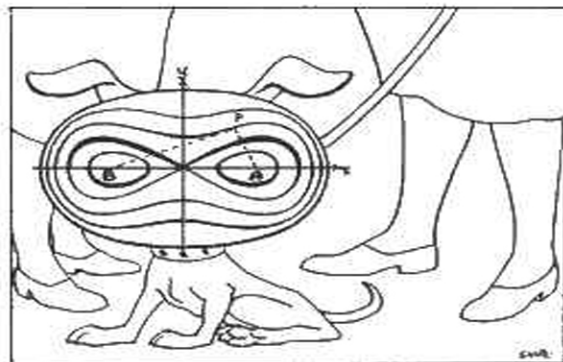
Regular polygons with six and eight sides are easy to construct by dividing up a circle. With a prime number such as 5, 7, 11 or 13 representing the number of sides, the task of drawing a regular polygon is not easy. The following method, however, gives a very close approximation to a truly regular polygon.

With centre O, describe a circle. Draw two diameters XOX' and YOY' at right angles to each other. From X' draw a line making an angle of 30° with $X'OX$; let this cut the circle at A. Draw OB perpendicular to $X'A$. Join YB; with compasses set to this distance, the circle may be divided into ten. Joining every other point on such a division gives a pentagon. In the same way use $X'B$ for a 7-sided polygon (heptagon). You can try

some others as follows: DE for 13 sides; DF for 19 sides; X'G for 17 sides; X'J for 23 sides; and X'K for 26 sides. J.F.H.

AN ODD RESULT

Add 1,000 to a certain whole number and the result is actually more than if the original number were multiplied by 1,000. What is the number?



'He's looked like that ever since I fed him on Cassini's Ovals.'

JUNIOR CROSS-FIGURE No. 36

Submitted by R. Annie Horton, Girls Grammar School, Maidstone.

Correct each answer to the appropriate number of significant figures.



CLUES ACROSS:

- Last year, John was half Anne's age. Now he is 13, how old is Anne?
- Prime number, large as possible.
- The water is just about to boil.
- Earth's mean radius in miles.
- Reciprocal of 1.1.
- Log 2.048.

CLUES DOWN:

- (LXIX)2.

- Express the decimal number 25 in the binary scale.
- $\tan 9^\circ 36'$.
- $x - y = 15, 2y + 3x = 9$. Find y.
- $x + 114$ (See 6 down).

SOLUTIONS TO PROBLEMS IN ISSUE No. 38

A MONUMENTAL PROBLEM

AB was seven yards,

FUN WITH NUMBERS—No. 6

$$1. \frac{1}{1+1}, 2. \frac{1}{1-1}, 3. \frac{1}{1-1+1}, 4. \frac{1}{1+1+1+1}, 5. \frac{1}{1+1+1+1+1},$$

$$6. \frac{1}{1+1+1+1+1+1}, 7. \frac{1}{1-1-1-1-1}, 8. \frac{1}{1-1-1-1-1}, 9. \frac{1}{1-1-1-1-1},$$

$$10. \frac{1}{1-1-1-1-1-1}, 11. \frac{1}{1-1-1-1-1-1}, 12. \frac{1}{1-1-1-1-1-1}$$

BEARING UP

Event (2) causes the ball-bearings to move further.

SENIOR CROSS FIGURE No. 38

ACROSS: (1) 131, (4) 113, (7) 1027026, (9) 940, (10) 1061208, (13) 12, (14) 18, (15) 176, (17) 120, (18) 15, (19) 13.
DOWN: (1) 110111, (2) 30, (3) 1296, (4) 1002, (5) 12, (6) 362880, (8) 741, (11) 0271, (12) 0121, (16) 65, (17) 11.

SITTING ON THE FENCE

The cost per mile is £125. Each property must be a mile square as this is the only rectangle with the same perimeter as the first property.

JIMMIE'S HOMEWORK BOOK

Problems are, tens and units, shillings and pence, pounds and ounces, stones and pounds, Pounds and shillings.

JUNIOR CROSS FIGURE No. 35

ACROSS: (1) 1625, (4) 567, (5) 91, (7) 53, (9) 21, (11) 121, (13) 516, (15) 12, (16) 7480.
DOWN: (1) 16, (2) 675, (3) 59, (4) 5025, (6) 1512, (8) 31, (10) 117, (12) 210, (14) 64.

TREAD SOFTLY

The top has moved 1 foot every 50 years out of true. Hence the next 20 feet will take another 1,000 years.

ALL ROUND

The farmer has not moved around the squirrel.

AM I ALL RIGHT JACK?

The bridge would collapse as the force required to maintain the two bows in the air is greater than the weight of one wood.

FOOD FOR THOUGHT

$$9867 \div 3289 = 3.$$

CIRCUMFERENCE OF A CIRCLE IS TWO PIE R

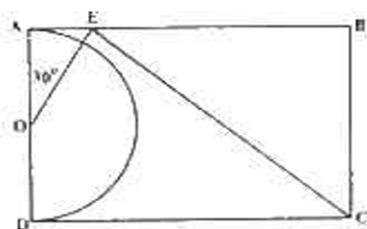
Will the girl who submitted this cartoon please write to the editor as we have lost her address? R.A.

Continued from page 305

Crystallographers are concerned with patterns in three dimensions in which the motif is the group of atoms forming a molecule of a crystalline substance. There are exactly 230 possible patterns in three dimensions.

The analysis of point patterns, such as might be used for a medallion, of linear patterns, and of plane patterns was carried out by da Vinci in the 15th century. The analysis of three dimensional patterns was made by a number of crystallographers in the 19th century. C.V.G.

APPROXIMATE CIRCLE SQUARING



Draw a rectangle $ABCD$ where the ratio AB to BC is as $3:2$. Describe a semicircle with AD as diameter. From O , the centre of AD , draw a radius making an angle of 30° with OA . Let this radius meet AB at E ; join EC . Show that EC is very nearly equal to the length of the semicircle AED .

J.F.H.

AN ODD FARMER'S PROBLEM

Is it possible to put nine pigs in four pens so that there is an odd number of pigs inside each of the four pens?

THOSE CAR NUMBERS AGAIN!

In Issue 36, we asked you to carry out an experiment by counting the number of cars which had 4 (say) as the last digit on their number plate in each batch of 10 cars which passed you.

Now two members of your Editorial Board have been indulging in a friendly wrangle about the figures we published in that issue. The point at stake is that there are two ways of calculating the figures according to which of two distributions we assume to fit the case. (For those of you who may have heard of them they are the Poisson and Binomial Distributions). Our two friends agree to differ about which is correct.

Here are the two sets of figures.

Score (i.e., No. of 4's in batch of 10)	0	1	2	3	4 or more
According to Poisson .. No. of times	37	37	18	6	2
According to Binomial .. in 100 batches	35	39	19	6	1

Before our two friends resort to loaded Slide Rules at ten paces we want you to carry out an experiment to help them. Will you carry out a test on at least 100 batches of 10 cars (the more the merrier) and send the results of your test on a post-card to the Editor? Mark your P/C clearly CAR NOS. in the bottom left hand corner.

R.N.S.

STAMP COLLECTORS' CORNER No. 22



SIMON STEVIN (1548-1620), of Bruges, gave the first systematic treatment of decimal fractions and campaigned unsuccessfully for the introduction of a decimal system of weights and measures. In applied mathematics he studied problems of equilibrium and solved problems on bodies resting on inclined planes.

C.V.G.

Belgium 1942,
50c—10c fawn.

HAPPY BIRTHDAY

A man was born in the nineteenth century. He was x years old in the year x^2 . Find his age in 1875.

PAINTER'S PUZZLE

A paint tin weighs 5 lb. when half full and 4 lb. when it is one third full of paint. Find the weight of a full tin of paint.

DESERT VICTORY

An explorer wishes to cross a barren stretch of land which will take 6 days to cross. However he can only carry 4 days' supplies. He hires native bearers who can each carry 4 days' supplies. What is the smallest number of bearers that can make the trip possible?

SENIOR CROSS-FIGURE No. 39

Submitted by J. G. B. Byatt-Smith, King Edward VI G.S., Totnes.

CLUES ACROSS:

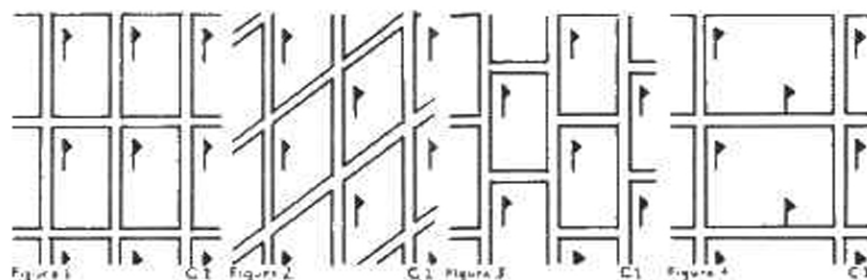
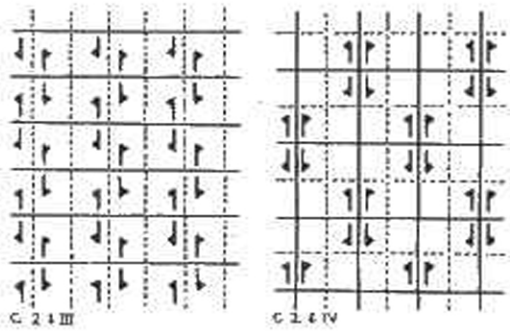
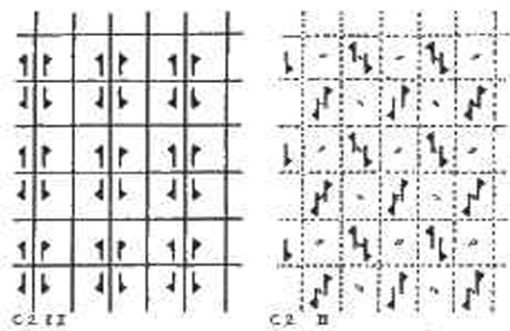
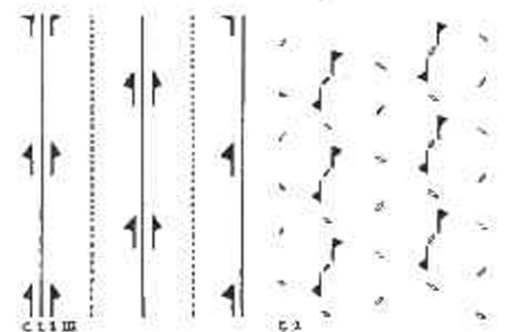
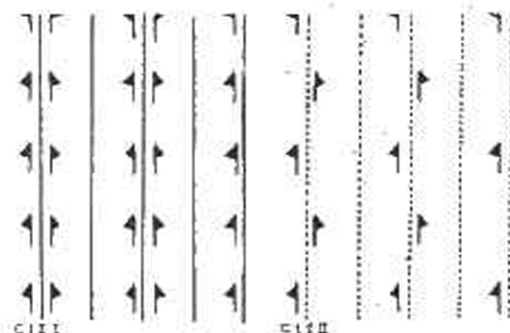
- Minimum value of $11\left(x + \frac{1}{16x}\right)$
- $\sin 2\theta$, see figure.
- Sum of the first 21 terms in the progression 25, 60, 95,
- $3, 6, 1 + 6$.
- Product of two perfect numbers minus ten times the smaller.
- Number of feet in a mile.
- The square of the sum of the digits gives the number reversed.
- A prime number whose digits are each perfect squares.
- $25\frac{1}{2}$ miles per hour in knots.
- $355\left[1 - \frac{1}{1} + \frac{1}{3} - \frac{2}{3} + \frac{1}{3} - \frac{2}{3}\right]$ continued fraction
- An obtuse angle whose sine is one half.
- Sum of the squares of the roots of $x^2 + 4x - 621 = 0$.

CLUES DOWN:

- $2BC^2 (AB+AC)$, see figure.
- Sum of roots times the product of the roots of $x^2 - 13x + 450 = 0$.
- Half the sum of the 4th, 6th, and 10th terms of 2, 4, 8, 16, —
- Square of a prime number. The first two, and also the last two reversed, are perfect squares.
- One twelfth of the coefficient of x^7 in $(1+x)^{20}$.
- Area of the ellipse $x^2 + 100y^2 = 49$.
- $a^3 (3a+5)$ when a is the first prime number.
- 17 down written to the base 4.
- $x(x-1)(x+1) - 4$ when $x=6$.



- One tenth of the product of the partial fractions of $\frac{-40}{21x^2 - 26x - 11}$ when $x=3$.
- xyz when $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 64$ and $x^2 = \frac{4900 - y^2 z^2}{y^2 + z^2}$
- $\frac{1}{2}\pi$ radians in degrees.
- Area between the graphs $y = 12x^2 - 12x + 1$, $y = 0$, $x = 1$, and $x = 3$.



Every pattern has a motif, which may be a group of flowers or figures, a scene or just a geometrical shape, and it has a system for the repetition of the motif. In the two patterns above the motif is a simple shape which is repeated by translation in two directions. Joining corresponding points of four adjacent repeats gives a parallelogram (or rectangle or square), called a unit cell, which contains the motif. These parallelograms together form the lattice of the pattern. Geometrically, rectangles and squares are included in parallelograms, therefore simple one-way patterns which are produced by translation alone are classed together as type C1. C is the symbol for a plane pattern, the suffix 1 indicates a one way pattern without axes of symmetry. Designers of fabrics and wallpaper usually prefer to deal with rectangles. Instead of the oblique grid of figure 2, they would generally consider this pattern as based on half drop rectangles. Metallurgists and crystallographers, who are interested in patterns of atoms and molecules, also prefer to deal with rectangular lattices whenever possible and might divide the pattern of figure 2 into double cells.

Every pattern must repeat by translation. A pattern can also repeat in three other ways:—(1) by reflection or reversal, (2) by a combination of reflection and translation called glide reflection, (3) by rotation through 180°, 120°, 90°, 60°. The symbols for an axis of reflection or symmetry, for a glide axis and for the four types of rotation are shown below.

If we add axes of symmetry to the basic one-way pattern we form three new types. (To show that there are axes of symmetry we add a suffix / to the type name). C1/I with two sets of axes of symmetry, C1/II with two sets of glide axes and C1/III with axes of symmetry and glide axes. By making a tracing of the simple pattern of figure 1, reversing the tracing and placing it over the figure patterns of these three types are produced.

Rotating a simple one-way pattern through 180° produces a simple two way pattern of type C2. In the figure the centres of rotation are marked by the symbol for 180° rotation. It is interesting to note that there are four centres of rotation in each unit cell. Adding reflection to the 180° rotation produces four more types:—C2/I with two sets of axes of symmetry at right angles, C2/II with two sets of glide axes at right angles, C2/III with axes of symmetry in one direction and perpendicular glide axes and type C2/IV with axes of symmetry and glide axes in each direction.

Axes of symmetry at 120°, 90°, and 60° produce patterns based on 60° rhombi and square lattices. The names of the various types are C3, C3/I, C3/II; C4, C4/I, C4/II; C6, C6/I. This brings the number of types of plane patterns to seventeen and no more are possible.

Continued on page 307

