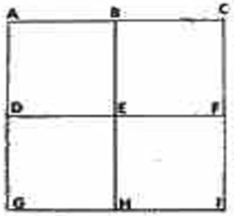
GENERALISED CUTS

If we take a line and divide it into two pieces then count up the number of different pieces or segments which could be chosen from the set of points, we find they are 01, 02, 12.

Now take a line and divide it into four pieces. Count up the number of different segments again. How many line segments can be made with the inch markings on a 12 inch ruler? Can you find a formula to give the number of different segments when there are n inches on the line?

In a similar way, a square of side two inches can have five different squares chosen from it. *ABED, BCFE, DEHG, EFIH, ACIG*. How many squares can you find in a similarly marked square of side 3 inches and how many in a foot square? Generalise the result for a square of side n inches.



Finally, the game can be extended into three dimensions. A cube of side 1 foot is divided up into inch cubes. How many different cubes can be chosen from the cubic foot and what will be the number for a cube of side n inches?

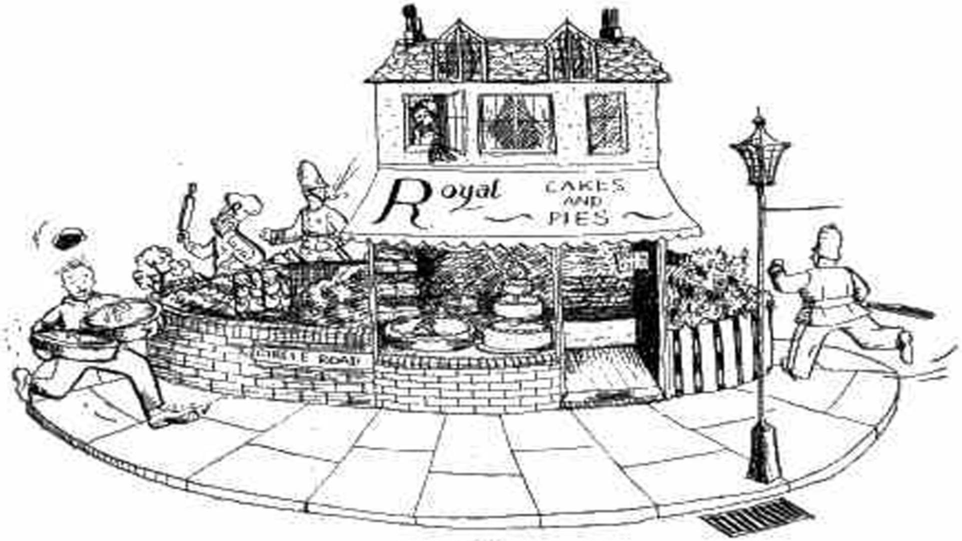
Just for fun, can you generalise your results for four dimensions? J.G.

FOOD FOR THOUGHT

$$\frac{\text{PORK}}{\text{CHOP}} = C$$

In this division sum C is greater than 2.

CIRCUMFERENCE OF A CIRCLE IS TWO PIE R



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MATHEMATICAL PIE

No. 38 Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England FEBRUARY, 1963

POINT CENTRED PATTERNS

The essential thing about a pattern is repetition of a motif. In the language of elementary geometry, a pattern is a collection of congruent figures, or more rarely similar figures, which are repeated according to some system. The particular system used determines the classification of the pattern.

The leaf pattern is an example of a point centred pattern formed by rotating the basic motif of one leaf about a fixed point through 120°. The operation of rotation through 120° which has been used to form the pattern can be applied to the pattern itself without changing its appearance.

The 6-way snowflake pattern illustrates another form of repetition. This pattern has six axes of symmetry passing through the centre point. If a mirror were placed along one of these axes, one half of the pattern would be the reflection in the mirror of the other half.

The pentacle appears at first sight to have five axes of mirror symmetry but one half of the pattern is not an exact mirror image of the other. The interlacings give the impression that the pattern has a certain depth. If we accept this and suppose that the straps really do cross, we have five axes of rotational symmetry in the plane of the paper. One half of the pattern is given by rotating the other about one of these axes as if it were a page of a book.

The fourth pattern is of a type attempted more often in Nature than in art. The pattern is formed by rotating the motif through a fixed angle and at the same time enlarging it in a fixed ratio. Such a pattern can never be completed.

LINEAR PATTERNS

To make a linear pattern, a motif is repeated by translation through a fixed distance in a fixed direction. The pattern below has repetition by translation only.

The motif repeated by translation may itself be a point centred pattern formed by the reflection or rotation of an element, so that the linear pattern may have transverse or longitudinal axes of mirror symmetry,

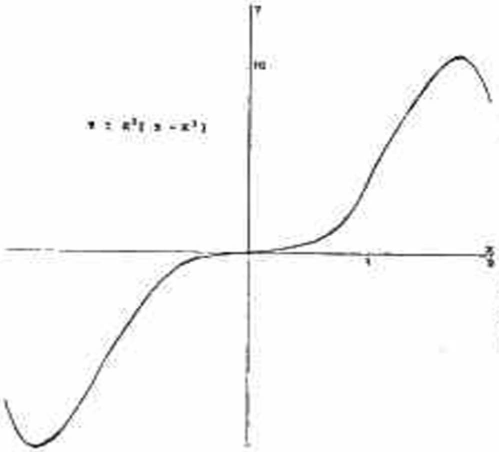
45085 04860 82503 93021 33219 71551 84306 35455

rotation through 180°, with or without symmetry.

and another type of symmetry called glide reflection, which is produced by moving the mirror reflection in an axis sideways by half the translation distance.

C.V.G.

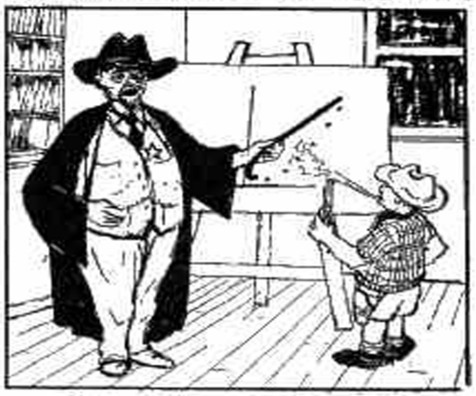
A SIMPLE GRAPH



There were several good attempts at plotting the graph of $y=x^3(5-x^2)$ submitted in response to the invitation in Issue No. 36. The winner of the competition was P. W. Vincent, of Dover, aged 16 years.

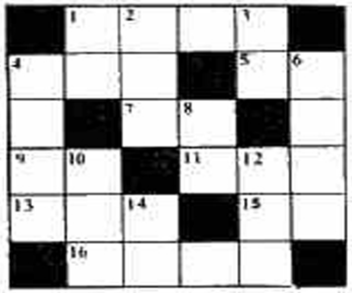
The main feature of the graph is that if only integral values of x are considered, the points lie on a straight line. Some of our younger readers discovered this and wrote to ask for guidance, as they recognised that the graph should be a curve. By plotting intermediate values, the shape of the graph becomes clearer and by using differential calculus the true shape of the graph can be confirmed. The greatest value in the range occurs when $x=\sqrt{3}$ and the least value of y occurs when $x=-\sqrt{3}$.

The graph shows the danger of generalising results from a limited number of observations B.A.



Now plot the point where $x=7$

JUNIOR CROSS-FIGURE No. 35



Given $a=6$, $b=7$, $c=4$, and $d=5$. Ignore decimal points.

CLUES ACROSS:

1. $2(4x+3)=19$. Find x.
4. $b(b-c)^4$.
5. $b(a+b)$.
7. $y+z-c$ (See 2 and 4 down).
9. $c(a+1)-b$.
11. $(a+d)^2$.
13. $2c^4+c$.
15. $a(b-d)$.
16. $10(a^3+b^3+c^3+d^3)$.

CLUES DOWN:

1. $a+\frac{1}{2}cd$.
2. $3y-7=20-y$. Find y.
3. $(b+2)(c+2)+d$.
4. $2z-100=101-2z$. Find z.
6. a^3b.
8. $bd-c$.
10. $(a+b)(c+d)$.
12. abd.
14. c^3.

B.A.

TREAD SOFTLY!

The famous leaning tower of Pisa has been slowly falling out of true ever since it was built 800 years ago because the foundations on one side are slowly sinking into the subsoil faster than those on the other. If we suppose that it is 180 feet high, 36 feet in diameter and has become 16 feet out of true since it was built, when is it likely to topple over? (Supposing nothing is done about it in the meantime). J.F.H.

ALL ROUND

A farmer sees a squirrel on the trunk of a tree and raises his rifle to shoot, but just then the squirrel ran around to the opposite side of the tree. The farmer kept his rifle aimed at the tree and slowly circled round it. However the squirrel kept moving ahead of him and the farmer went all round the tree without seeing the squirrel. He had gone all round the tree but had he gone all the way around the squirrel?

AM I ALL RIGHT JACK?

A bowls player carrying 3 woods came to a bridge that would only carry his weight and one wood at a time, so he decided to juggle them as he crossed so that 2 are always in the air. Smart?

SOLUTIONS TO PROBLEMS IN ISSUE No. 37

THREE SQUARES

The side of the black square is $8.5\sqrt{2}$ inches, which is just over 12 inches

HAPPY BIRTHDAY

The grandfather was 66 years and the grandson 6 years.

QUICK QUIZ

The first column of figures has the larger sum.

SENIOR CROSS FIGURE No. 37

ACROSS: (1) 74188, (5) 74, (6) 48, (7) 9699690, (10) 37, (11) 02, (12) 1362880, (15) 66, (16) 46, (17) 30031.

DOWN: (1) 779, (2) 17976, (3) 849, (4) 180, (6) 4928, (8) 6336, (9) 60860, (12) 169, (13) 240, (14) 021.

JUNIOR CROSS FIGURE No. 34

ACROSS: (1) 1221, (6) 12321, (9) 81, (10) 37, (13) 40320, (16) 3210.

DOWN: (2) 21, (3) 222, (4) 13, (5) 2112, (7) 28, (8) 2345, (11) 70, (12) 622, (14) 33, (15) 01.

ANOTHER SERIES

The series is 60, 90, 108, 120, 128$\frac{4}{7}$, 135, ... The terms give the internal angles of regular polygons.

B.A.

Time has shewn that they tackled the problem from such different standpoints and used such different notations that each may be accorded a full share of the honour of the discovery.

Leibniz' interest in Mathematics really stems from his meeting with Huygens, the inventor of the pendulum clock and of the wave theory of light. Leibniz and Huygens studied the properties of the Catenary which is the curve taken up by a chain hanging freely between two points. Leibniz was also the inventor of a calculating machine of much greater potential than the simple adding machine of Pascal—indeed many modern calculating machines use a version of the Leibniz Wheel.

Newton really requires an article to himself. Like Archimedes he tried his hand at many things and illuminated everything that he touched. In two years which he spent at home because the University of Cambridge was closed during the Great Plague he laid the foundations of much of his later work. He invented the Calculus, which he called the Method of Fluxions, stated the three fundamental laws of mechanics, discovered the Universal Law of Gravitation and applied it to the Solar System, proved the general Binomial Theorem, invented numerical methods of solving equations, studied Friction, resolved white light into its component colours, and invented a telescope . . . In any Mathematical or Scientific work which stems from that period we can be sure to find the print of Newton's hand somewhere. R.M.S.

TRY ANGLES AGAIN

Dear Editor, Exmouth.

There is a misprint in the last line of the proof. It should read :— $PQ=GL=1$.

When G and G' coincide the line from N is a tangent to the circle, $\angle NGL=90°$, $\angle QGL=\angle QLG=90°-\angle LNG$, and therefore $NQ=QL$ and $LP=\frac{1}{2}NL$.

When 1 is such that the line from N neither cuts nor touches the circle, the fence will not be long enough to divide the field whilst satisfying the given requirements.

Yours faithfully,
KATHRYN SAMPSON (15 years).

Kathryn's was the best of those received after the article in Issue No. 36, and she has been sent a book token. B.A.

SITTING ON THE FENCE

Smith and Jones, two bargain hunters, were seeking estimates for the fencing in of their estates. " My property is exactly a square mile " said Smith. " Mine is exactly a mile square," said Jones.

" Then as far as I am concerned," said the contractor, " I will fence either property for £500." What did the contractor charge per mile of fencing? R.H.C.

JIMMIE'S HOMEWORK BOOK

28	28	28	28	28
+39	+39	+39	+39	+39
67 ✓	65 ✓	61 ✓	63 ✓	517 ✓

Teacher gave him full marks. How was this? R.M.S.

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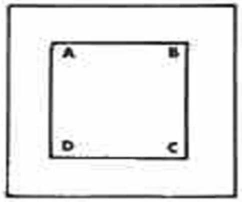
A MONUMENTAL PROBLEM

(adapted from Le Facteur X)



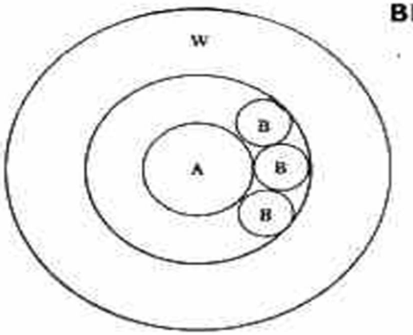
The base of a monument is a square—say $ABCD$. It is surrounded by a plinth also with a square outline. The outer edges of the plinth are distant 3 yards from the base of the monument. The area of the plinth is 120 square yards. Find the length of the side AB.

J.F.H.

FUN WITH NUMBERS No. 6

Using four 1's can you express the numbers 1 to 12? *e.g.*, $13=11+1+1$. R.H.C.

BEARING UP



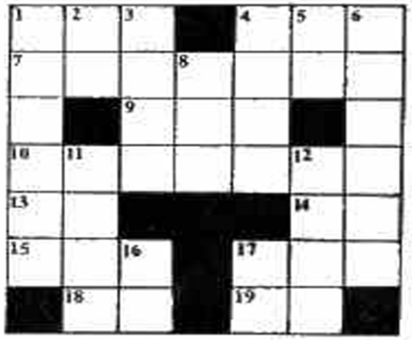
W is a wheel and A is its axle ; in between wheel and axle are ball-bearings BBB . . . . Consider two possibilities :

(1) The axle is held and the wheel is turned for one complete revolution.

(2) The wheel is held while the axle is turned for one complete revolution.

In which of these two events do the ball-bearings travel further? J.F.H.

SENIOR CROSS-FIGURE No. 38



CLUES ACROSS :

1. 55 in the scale of 6.
4. Change 1,110,001 in the scale of 2 to the scale of 10.
7. $\frac{1}{10}$ the product of the first eight primes plus $3\times7\times11\times13\times19$.
9. Change (800 — 26) from scale of 11 to scale of 10.
10. Perfect cube.
13. It has 1, 2, 3 and 4 as factors.
14. See 5 down. Shillings.
15. $13\frac{1}{2}^2 - 2\frac{1}{2}^2$.
17. Five !
18. Product of two prime numbers.
19. The sixth prime number.

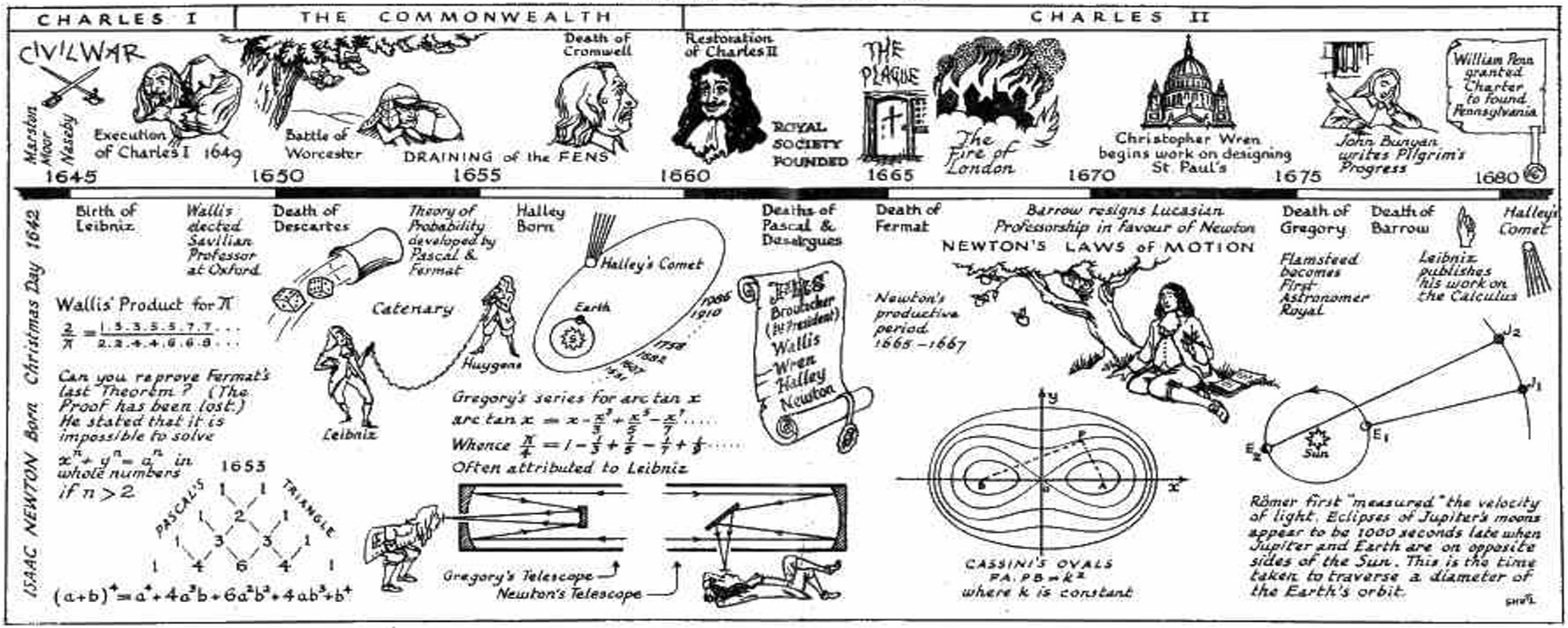
CLUES DOWN :

1. 55 in the binary scale.
2. Two more than a perfect number.
3. Perfect square.
4. Smallest number over one thousand whose digits add up to three.
5. Take a sum of money less than £12. Interchange pence and pounds and find the difference. Interchange the pence and pounds in the answer and add to the answer. Number of pounds. See also 14 Across, 17 Down.
6. 9 !
8. An arithmetic progression.
11. 8 short of the number of cubic inches in a cubic foot reversed.
12. Consecutive numbers.
16. Product of two primes.
17. See 5 down. Pence.

B.A.

84683 39363 83047 46119 96653 85815 38420 56853

TIME CHART No 10



This period was one of the most productive in the whole of mathematical history. The improvements in algebraic notation which took place in the first half of the 17th century allowed ideas, which for long had been only half perceived, to be expressed with crystal clarity.

The period abounds with great names. Pascal was still at the height of his powers. In 1653 he wrote a treatise on the remarkable properties of the triangular array of numbers which goes by his name although some of its properties had been known to Omar Khayyam, the Persian Poet (c.1100) and to the Chinese two centuries later. Two of its many consequences are the Binomial theorem for expanding $(1+x)^n$, and the Theory of Probability which Pascal and Fermat (1608—1665) worked out in a now famous correspondence.

Fermat was a first rate mathematician but remained an amateur and we learn of his work mainly through his correspondence with Pascal, Descartes and others. His work on the Theory of Numbers was outstanding. This is a field of Mathematics in which the results are easy to state and to verify but surprisingly difficult to prove. *e.g.*, if p is a prime number and not a factor of a then $(a^{p-1}-1)$ is divisible by p. (Test it yourself with $a=5$ and $p=7$ and other values). Fermat's famous Last Theorem (see chart) also belongs to the field of Theory of Numbers.

Wallis (1616–1703), chaplain to Charles II and one of the founders of the Royal Society, extended the use of indices to include negative and fractional indices as well as positive integers. He obtained the equivalent of

$\dfrac{ds}{dx} = \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ giving the length of an element of a curve and was able to 'integrate' such curves as $y=x^n$ and $y=x^{1/n}$.

Mathematics has always been closely linked with Astronomy and we have a brilliant galaxy here including the first Astronomers Royal of Britain and France, Flamsteed and Cassini, Halley, of Comet fame and Gregory, inventor of the Gregorian telescope who also developed the series

$$\text{arc tan } x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 \ldots\ldots$$

which is used in almost all modern calculations of π. He was one of the first to distinguish between convergent and divergent series and yet he was only 37 when he died. Another famous astronomer mathematician who later turned to Architecture was Christopher Wren, who rebuilt London after the Fire of 1666.

Head and shoulders above their contemporaries stand Newton (1642–1727) and Liebniz (1646–1716). There was great bitterness between them at the time, each claiming to have been the first to have invented the Calculus.

38621 86721 86725 23340 28308 71123 28278 92125 07712 62946 32295 63989 89893 58211 67456 27010