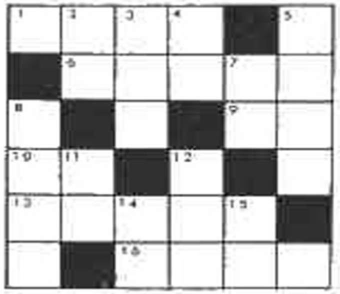
JUNIOR CROSS FIGURE No. 34



CLUES ACROSS:

1. Palindromic number.
6. Palindromic perfect square.
9. Perfect Square.
10. A prime number.
13. Product of the first eight natural numbers.
16. Sounds like the end of a count-down.

CLUES DOWN:

2. ab when $a = 2$ $b = 2$.
3. Twice the square root of 6 across.
4. Unlucky for some.
5. Palindromic anagram of 1 across.
7. A perfect number.
8. Consecutive numbers.
11. Life's span.
12. Convert 1001101110 from the binary to the decimal scale.
14. Product of two primes.
15. Reverse the base of the decimal system.

Check Clue. One digit is not used and the sum of the digits in the Cross Figure is 68.

B.A.

ANOTHER SERIES

Continue the series 60, 90, 108, 120, ... and state the rule for the series. J.G.

Time Chart—continued from page 291

Another Greek who flourished during this little renaissance of the Alexandrian School was the geometer Menelaus who wrote a treatise on the geometrical properties of spherical triangles and proved several important theorems about plane triangles, one of which appears in the chart above.

Claudius Ptolemaeus (Ptolemy) (c. 90–160 A.D.) did for astronomy what Euclid did for Geometry. He brought together in a single treatise, the "Almagest," the discoveries of his predecessors, arranging the work in such a systematic fashion as to make his work a standard reference for many centuries. R.M.S.

SOLUTIONS TO PROBLEMS IN ISSUE No. 36

PERFECT NUMBERS

The second perfect number is 28. Its factors are 1, 2, 4, 7, 14, whose sum is 28.

SENIOR CROSS FIGURE No. 36

ACROSS: (1) 482, (4) 4844, (5) 68, (6) 35, (8) 4934, (10) 126.
DOWN: (1) 48841, (2) 84, (3) 24336, (4) 46, (7) 54, (9) 92.

FUN WITH NUMBERS No. 3

$a = 2$, $b = 3$, $c = 4$, $d = 5$.

FUN WITH NUMBERS No. 4

The answers to the sums were 88, 888, 8888, 88888.

SEQUENCES

2A. $n = 5$; 3A. $a = 2$, $b = 5$, $c = 8$; 4A. $a = 4$, $b = 8$; 5A. $x = 9$, $y = 14$, $z = 15$.

THE POOR PILGRIM

The pilgrim had 5 francs 25 centimes when he entered the church.

A SIMPLE GRAPH

The result of this exercise will be announced in the next issue.

OTHER NUMBER SYSTEMS

(a) is true in the scale of 7. (b) is true in the scale of 6.

JUNIOR CROSS FIGURE No. 33

ACROSS: (1) 71, (2) 216, (4) 47, (5) 41231, (18) 68890, (11) 76, (12) 025, (13) 90.
DOWN: (1) 784, (2) 27386, (3) 68, (4) 42875, (6) 16, (7) 19, (9) 020, (10) 80.

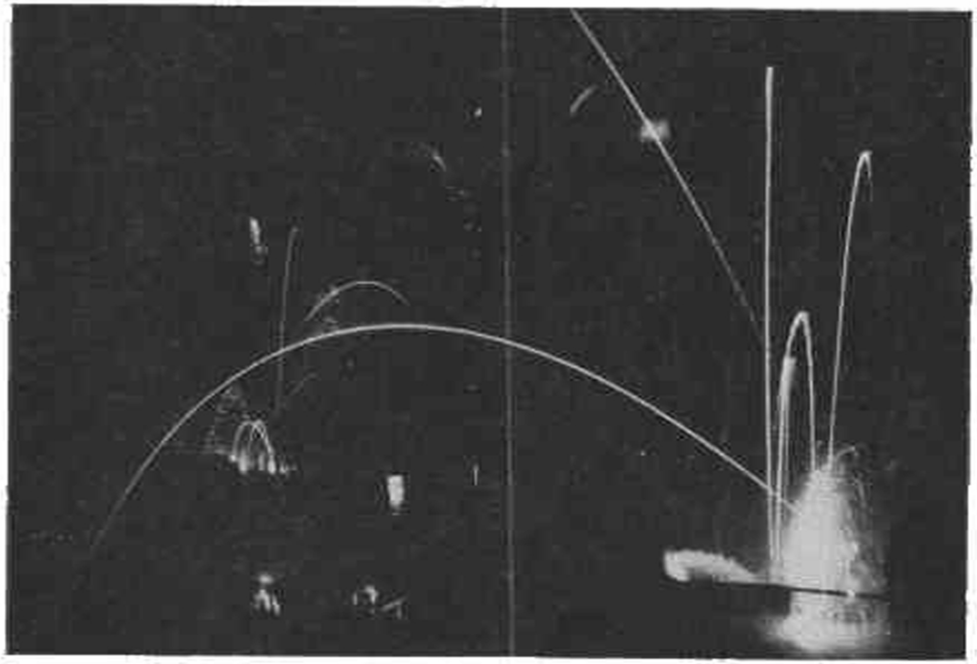
B.A.

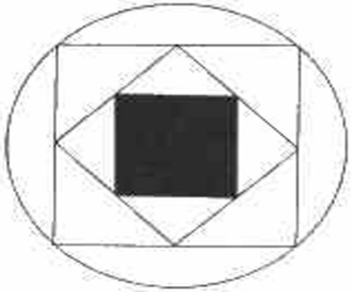
26327 50529 83491 87407 86680 88183 38510 22833

MATHEMATICAL PIE

No. 37 — Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England — OCTOBER, 1962

FIRED!





Three squares are inscribed in a circle 34 inches in diameter as shown in the diagram. The black square represents a window which has to be filled in. Will a board 12 inches square do the trick?

DO YOU FIND MATHEMATICS DIFFICULT?

This is said for those who are dismayed at the outset of their studies, and then set out to gain the mastery over themselves, and in patience to apply themselves continuously to those studies.

From these one sees that from these studies there result things marvellous to relate.

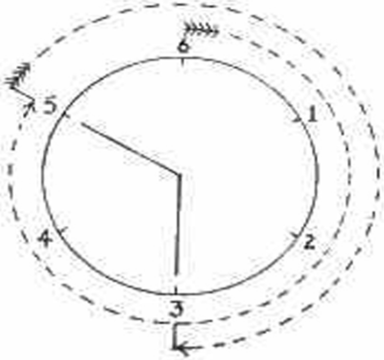
Leonardo da Vinci, 1452–1519. J.G.

29174 64263 57455 25407 90914 51357 11136 94109

CLOCK ARITHMETIC No. 3

Multiplication and Division

In the first article (Issue No. 34) you were shown how to make out an addition table for a circular face of scale 6 instead of a linear one in which $2+3=5$ and $5+4=3$ and from these results we constructed the addition table shown.



+	1	2	3	4	5	6
1	2	3	4	5	6	1
2	3	4	5	6	1	2
3	4	5	6	1	2	3
4	5	6	1	2	3	4
5	6	1	2	3	4	5
6	1	2	3	4	5	6

In the second article (Issue No. 35) you were shown the subtraction table and it was pointed out that the addition table above could be used to get subtraction results by working it backwards. This is similar to the use of logarithm tables to obtain anti-logarithms.

But what can we mean by multiplying numbers in modulo 6 arithmetic?

Now 3×4 means ' three fours ' *i.e.*, $4+4+4$

and 2×5 means ' two fives ' *i.e.*, $5+5$

so that every multiplication sum can always be translated into a process of repeated addition.

Thus, using the addition table above

$3\times4=(4+4)+4$ $\quad 2\times5=5+5$ $\quad 4\times5=(5+5)+(5+5)$

$\quad =2+4 \quad = 4 \quad = 4+4$

$\quad =6$ (see *a*) $\quad$ (see *b*) $\quad =2$ (see *c*)

Now complete the multiplication table. Look carefully at your table and see if you can suggest a number which behaves as *O* does in ordinary arithmetic.

By defining division as the converse of multiplication, so that $3\times4=6$ means $6\div3=4$ and $4\times5=2$ means $2\div4=5$, we can use the complete multiplication table to do modulo 6 division. What do you make of the following divisions?

$4\div2 \qquad 3\div3 \qquad 2\div5$

The first two questions have more than one answer and the last answer need not be a fraction.

R.H.C.

x	1	2	3	4	5	6
1						
2					4^{b}	
3				6^{a}		
4					2^{c}	
5						
6						

GERONIMO'S GEOMETRY

Submitted by Marian House, Lister County Technical School.

The squaw on the hippopotamus is equal to the sons of the squaws on the other two hides.

Time Chart—continued from page 289

by which an angle could be trisected, and Diocles who invented the cissoid, which may be used in the duplication of the cube.

At about this time (150 B.C.) we get the beginnings of Trigonometry, the division of the circle into 360° and the development of a sexagesimal system of fractions (" minutes " and " seconds ") which had been suggested by the Babylonians in the first place. Hipparchos of Rhodes (c. 150 B.C.) worked out what was virtually a table of sines and developed a kind of spherical trigonometry (used in navigation). He was an excellent astronomer and left a catalogue of 850 fixed stars, which was increased to about 1,000 by Ptolemy about 300 years later, but was not materially added to until comparatively recent times.

We do not usually think of Julius Caesar as a Mathematician but he was well versed in Astronomy and planned extensive surveys of the Roman Empire. His chief claim to our interest is probably the reform of the calendar. His system of leap years was accurate enough to be only 11 days out 1600 years later and with Pope Gregory's modification about century years will need no further adjustment for the next 3,000 years.

Most of you will know Heron's formula for the area of a triangle $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$. He was an Egyptian living in Alexandria about 50 A.D. and wrote a treatise summarising the Egyptian methods of land surveying. He invented various machines, including a simple steam turbine nearly 2,000 years before the days of the steam turbine battleship and ocean liner.

continued on page 292

THE WORLD ENCOMPASSED : No. 3

The French Mathematicians

In company with many other problems that were studied during the early years of mathematics, investigation of the size and shape of the earth was neglected for many centuries following the promising start made by the Greeks. It was not until such questions as " What is the earth made of ? " and " Is the earth a perfect sphere ? " were once more examined by mathematicians that further serious attempts were made to measure the earth and assess its shape.

In the 17th century, Newton suggested that on account of the earth's rotation it would tend to differ in shape from a true sphere and be slightly flattened at the poles. If such flattening existed, then the pole-to-pole diameter should be less than an equatorial diameter. Newton suggested that the shortening would be of the order of 1/230th part of the longer diameter.

One way of checking this possibility was by travelling along a meridian and measuring the " length of a degree " at various intervals along the meridian. Thus, supposing two places to be at sea level, one at latitude 50° and the other due north at latitude 51°, the distance between them (supposed free from mountains or hollows) would be accurately measured. Similarly, measurements further north and further south would be made, the latitudes being checked by corrected astronomical observations (Note that if the earth were a true sphere and the Pole Star were at the true astronomical north pole, the elevation of the star in degrees would be the same as the latitude at any observation point in the northern hemisphere). In regions of flattening, it would be necessary to travel further along the meridian in order to make a change of 1° in the elevation of the Pole Star than in regions of " bulge."

The French Mathematicians of the time were anxious to investigate Newton's suggestion although he warned them that the differences in the length of a degree at, say, Bayonne and Dunkirk would be so slight as to be undetectable with the instruments available. Despite the warning, Jacques Cassini made some measurements in 1718 and concluded that the length of a degree was less at a certain part of a meridian than it was at a part some distance further south. This result implied that the shape of the earth tended more to that of a lemon than to that of an orange, *i.e.*, the poles would be elongated rather than flattened.

In 1737, a team that included Pierre Maupertuis and A. C. Clairaut went to Lapland and at latitude 66° 22′ N determined the length of a degree to be 69.403 miles. Another team that went to Peru included Charles M. La Condamine, Pierre Bouguer and Godin. Their measurements, in 1744, showed that at latitude 0°, the length of a degree lay between 68.713 and 68.732 miles. Newton's conjecture was thus vindicated.

Since those early days, progress in the design, construction and operation of instruments has greatly improved the accuracy of observations. The latest determinations of the earth's shape give the polar flattening as one part in297. This is roughly equivalent to the thickness of the tissue paper compared to the diameter of orange wrapped in it, so is not as great as some folk imagine.

Let us admire the courage and pioneering spirit of these 18th century French Mathematicians and learn from their mistakes—just as they did !

J.F.H. I.L.C.

FUN WITH NUMBERS No. 5

Express each of the fractions 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8 and 1/9 by arranging the nine digits 1,2,3,4,5,6,7,8,9 using each digit once and only once.

HAPPY BIRTHDAY

A man and his grandson have the same birthday. For six consecutive birthdays the man is an integral number of times as old as his grandson. How old is each at the sixth of these birthdays ?

(*From Maths Student Journal*)

QUICK QUIZ

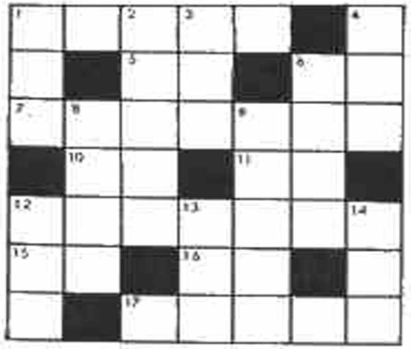
123456789	1
12345678	12
1234567	123
123456	1234
12345	12345
1234	123456
123	1234567
12	12345678
1	123456789

You are allowed ten seconds to decide which column of figures when added will give the larger result. Check your result after you have made your guess.

R.H.C.

SENIOR CROSS-FIGURE No. 37

$a+3b=23$, $a^2-ab+6b^2=386$, $3c \div d = cd = 12$.



CLUES ACROSS :

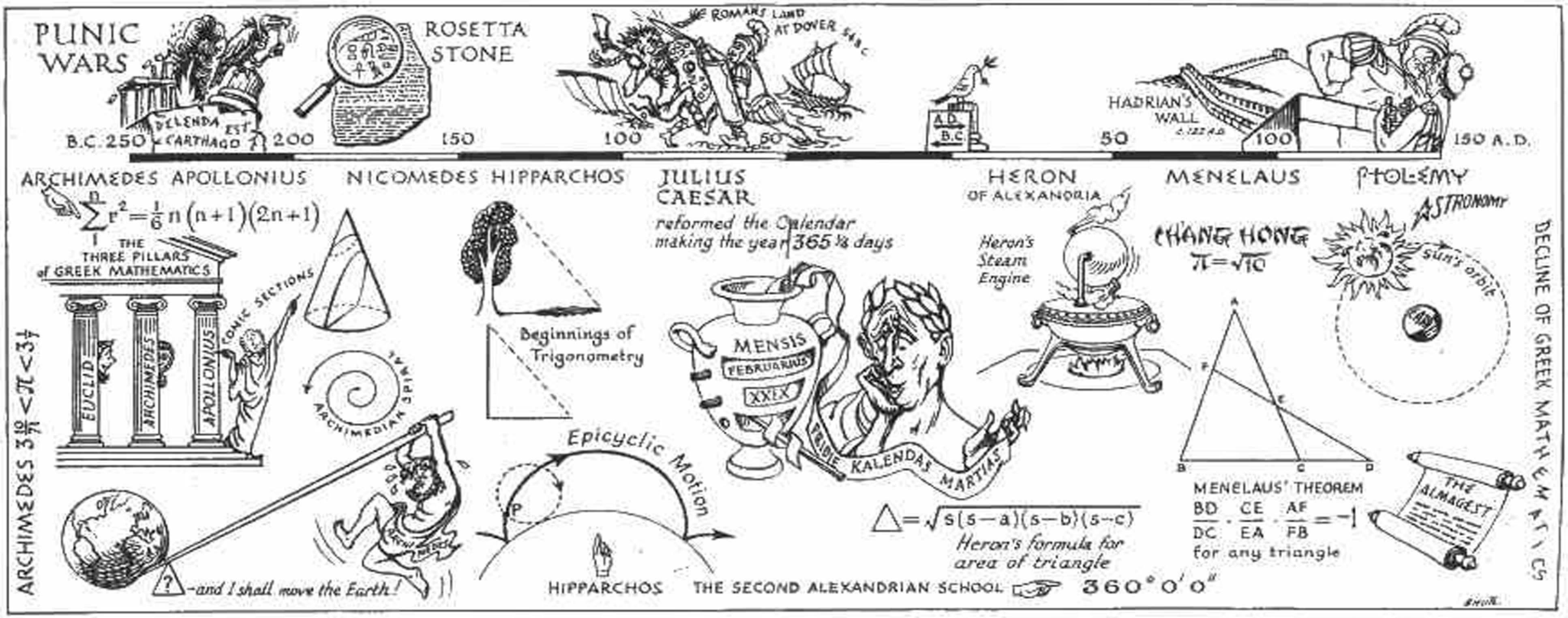
1. $(2a+c)^3 \div 5a$.
5. $c(d^2-b^2)$.
6. Sum of the digits of 7 across.
7. Product of the first eight prime numbers.
10. Half of 5 across.
11. Reverse a.
12. A million and nine !
15. $3(a+c)$.
16. $c(a+b+c)$.
17. Leaves a remainder of 1 when divided by each of the first six prime numbers.

CLUES DOWN :

1. $(a-b)\ (2a+b)$.
2. $(a+d)^3+a^2b$.
3. $b(a+b+c-d)^2+c^3$.
4. $a(b+c-d)$.
6. LCM of 224, 616, 704.
8. $4c^2d^2\ (a-b-c-d)$.
9. $\dfrac{9!+3.6!-5!}{3!}$
12. A perfect square.
13. $abcd$.
14. ad reversed.

B.A.

TIME CHART No. 3: (250 B.C.—150 A.D.)



For a time we have a continuation of the golden age of Greek Mathematics. Archimedes of Syracuse (287—212 B.C.) was a mathematical physicist of the first order. We can get some idea of the greatness of the man if we look at a few of his contributions to Mathematics and to Science.

He calculated a value of π by considering the area of a circle to lie between the areas of the polygons inscribed and circumscribed to it. By increasing the number of sides again and again, he got closer and closer approximations to π. By taking polygons of 96 sides he found that π lies between $3\frac{1335}{9347}$ and $3\frac{1335}{9345}$. This was a remarkable achievement when one considers the clumsiness of Greek Arithmetic.

He invented a system of reckoning in octads or eighth powers of 10. By this means he extended the Greek number system as far as 10^{63} to calculate the number of grains of sand in the solar system—this involves knowledge of the laws of indices $a^m \times a^n = a^{m+n}$. He also calculated the volume of the sphere, studied the properties of several types of spirals and found that $1^2+2^2+3^2+4^2 \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$. Besides all this he put the science of Hydrostatics on a firm footing and studied the properties of pulleys and levers. He is reputed to have said " Give me a fulcrum and a place whereon to stand and I will move the Earth."

It was a great loss to mankind when an infuriated Roman soldier slew him at the capture of Syracuse because he was too absorbed in a geometrical problem to notice that the soldier had asked him a question.

The third of this great trio of Greek Mathematicians was Apollonius of Perga (c. 260—c. 170 B.C.). Like Euclid he is known chiefly for his Geometry, although he made valuable contributions to Arithmetic, notably improving the system of Archimedes by basing it on the smaller and more convenient base of 10^4, a number which under the name of *myriad* had long been in use in the East. His most important contribution to mathematics was the study of the section of a cone made by a plane. These conic sections, principally the parabola, ellipse, and hyperbola (names given to them by Appollonius and still in use today) are of great practical importance. The parabola is the path followed by a projectile if we neglect air resistance and the curvature of the earth, the approximate shape of a suspension bridge chain, and the section of a car headlamp reflector or a radio telescope bowl. Most planets and comets follow elliptical orbits round the sun and the ellipse has been used for bridge arches. The hyperbola is used extensively in the LORAN system of radar navigation. All three curves will be increasingly important in the near future in space navigation.

With the death of Apollonius we really come to the end of the Golden Age of Greek Mathematics. There were a number of minor geometers such as Nicomedes, who invented a curve known as the *conchoid* (shell-shaped)

Continued on page 291