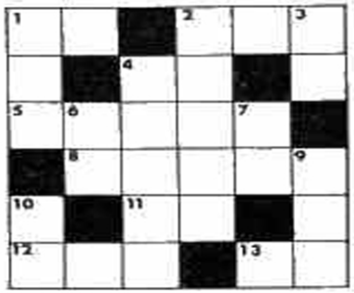
## JUNIOR CROSS FIGURE No. 33

*Prepared by the Pre-Apprentice Course, Clowne Technical College.*




CLUES ACROSS :

1. $\left[(8.485)^2-1\right]$.
2. $(2^3-2)^3$.
4. $(3.609)^3$.
5. $\sqrt{17}$ to 4 dec. places.
8. $10n^2$ when $n=83$.
11. $\sqrt{5776}$.
12. Reciprocal of 40.
13. $(4.481)^3$.

CLUES DOWN :

1. A perfect square.
2. $\sqrt{\frac{3}{4}(10)^3}$.
3. $(x^3+x)$ when $x$ is a perfect square.
4. The cube of 35.
6. The square of $x^2$ when $x$ is 2.
7. $x$ when $(x+1)^3=8{,}000$.
9. The reciprocal of 50.
10. $(n^2-1)$ given that $n$ is a perfect square.

## NOTE BENE

A traveller at a hotel handed in a five pound note for safe keeping to the landlord. Instead of putting it away in his safe, the landlord took it to his friend the baker and paid off a bill he owed there. The baker in his turn used the same note to pay a bill of five pounds which he owed to his solicitor, and finally the solicitor took the note to the hotel and used it to pay for a dinner party he had held the previous evening. The landlord then placed the note in the safe.

Next morning the traveller came and asked for his note and, being an honest man, the landlord told the guest how he had borrowed it and then replaced it later. "Of course I don't mind," said the traveller and tore up the note, "you see it is a forgery."

Who lost on the local transactions? R.H.C.

## SOLUTIONS TO PROBLEMS IN ISSUE No. 35

MORE GREEK

The English words should have been in the following order ERROR, ERASE, EERIE, ROTOR, RAISE.

SENIOR CROSS FIGURE No. 35

ACROSS : (1) 317, (4) 13, (5) 1217, (8) 204, (10) 35, (11) 756, (14) 3618.

DOWN : (1) 31, (2) 121, (3) 711, (4) 11, (6) 720, (7) 54558, (9) 172, (12) 63, (13) 16.

JUNIOR CROSS FIGURE No. 32

ACROSS : (1) 16093, (6) 405, (8) 31, (10) 64, (11) 925, (12) 668, (13) 60, (15) 04, (16) 124, (18) 28500.

DOWN : (2) 64, (3) 009, (4) 95, (5) 63957, (7) 14841, (9) 120, (10) 660, (14) 225, (16) 18, (17) 40.

The editor apologises for two slips. 18 across should have read to the nearest 100, and clues 13 across and 5 down do not agree.

POETICAL PI

The number of letters in each word gives the sequence in the decimal equivalent of $\pi$. B.A.



55377 99140 37340 43287 52628 88963 99587 94757

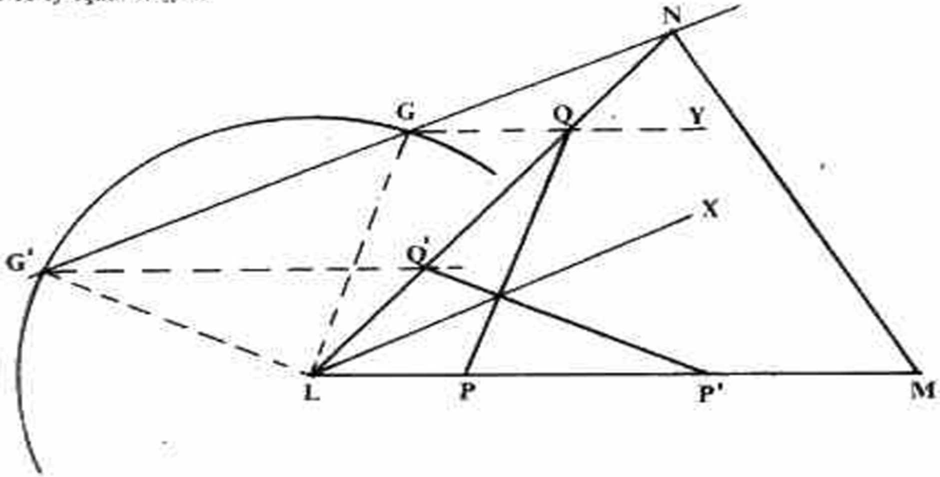



# MATHEMATICAL PIE

No. 36 — *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England — MAY, 1962


## TRY ANGLES

*The owner of a triangular field LMN has a fence of length l with which he wants to divide the field into two parts. He requires PQ to be of length l and NQ and LP to be of equal length.*




*Construction* :—With centre $L$, and radius $l$, draw a circle. Draw in $LX$ to bisect $NLM$, and construct a line through $N$ parallel to $LX$ to meet the circle in $G$ and $G'$. $GY$ is drawn parallel to $LM$ to cut $LN$ in $Q$. Mark off $LP$ equal to $QN$. Join $PQ$. This is the position of the fence.

A second position of the fence can be found using $G'$ instead of $G$.

*Proof* :— $\angle NQY = \angle NLM = 2\angle XLN = 2\angle QNG$ (parallel lines)

$\angle NQY = \angle QNG + \angle QGN$ (exterior angle of triangle)

Hence, $\angle QNG = \angle QGN$ and $GQ = QN$.

In the quadrilateral $LPQG$, $LP = GQ$ and $LP$ is parallel to $GQ$; hence the figure is a parallelogram.

Hence $PQ = GQ = l$.

Write on a postcard, a short note to the Editor giving your views on the solution of the problem when (i) $G$ and $G'$ coincide, and (ii) the line from $N$ neither cuts nor touches the circle. A book token will be awarded to the sender of the best postcard.

The above solution uses simple geometrical methods. It is possible to find other solutions based on trigonometrical methods and the use of conic sections other than the circle. J.F.H.



51852 90928 54520 11658 39341 96562 13491 43415

## DO YOU (OR DID YOU) COLLECT CAR NUMBERS?

Whether you did or not here's an odd fact about them you can check for yourself on your bus journeys to and from School.

Choose a number from 1 to 9, say 4 for example. Now count how many registration number plates end in 4 in the next batch of 10 cars you pass. (For this purpose count DON 914 and 914 DON as both ending in 4).

Call this count the score for that batch of 10 cars and make a note of it (but *not* on the cover of your Maths. exercise book !).

Keep doing this for batches of 10 until you have 100 scores. It doesn't take very long on our crowded roads. If you've been honest with yourself and kept the scores accurately you will find that they run very close to the figures given below :

| Score (*i.e.*, number of 4's per batch) | 0 | 1 | 2 | 3 | 4 | 5 or more | Total |
|---|---|---|---|---|---|---|---|
| No. of batches with that score | 37 | 37 | 18 | 6 | 2 | 0 | 100 |

If you compare your results with those of your friends on other bus routes you will find that they are very nearly the same. In fact if you take the average of their results and yours you will find that you get even closer to the figures given above.

Isn't it rather odd that such a " chancy " business should be capable of being predicted? Predictions like this (only rather more useful) are the everyday work of statisticians. The statistician works with Insurance Firms, in Medical and Agricultural research, in the design of Nuclear Power stations and Telephone exchanges. Perhaps you'd like to be one—ask your Mathematics teacher what it involves and how you set about it. R.M.S.

## PERFECT NUMBERS

A perfect number is a number such that it is equal to the sum of all its factors, including unity but excluding itself. Thus, leaving out unity, 6 is the smallest perfect number with factors 1, 2 and 3 which, of course, add up to 6.

There are not many of these numbers, in fact, so far only twelve have been found. The next but one perfect number after 6 is 496 with factors 1, 2, 4, 8, 16, 31, 62, 124, 248.

Somewhere in between the numbers 6 and 496 there is another perfect number. It is much nearer to 6 than 496. Can you find it? J.F.H.



## SEQUENCES

| D \ A | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 6 | 24 | |
| 2 | 5 | 9 | 11 | | 60 |
| 3 | 10 | 28 | | 8 | 20 |
| 4 | 16 | | 27 | 3 | 5 |
| 5 | | 126 | 38 | 0 | 1 |

Fill in the blank squares. 1D, 2D, 3D, 4D, 5D and 1A are regular sequences of numbers whose methods of formation you have to discover.

Rows 2A, 3A, 4A, 5A are obtained from the following formulae :—

2A : $n$, $2n - 1$, $2n + 1$, $3n$, $12n$.

3A : $ab$, $4b - c$, $2b + c$, $4a$, $\frac{1}{2}bc$.

4A : $a^2$, $b^2 - 1$, $a + 3b - 1$, $\frac{1}{4}(a + b)$, $a + 1$

5A : $x + y$, $xy$, $x - y + z$, $y - z + 1$, $z - y$

What are the numerical values of the symbols used? J.G.

## THE POOR PILGRIM

### (adapted from Le Facteur X)

A poor pilgrim, very pious and with very little money, entered a church where he found the three great saints ; St. Peter, St. Paul and St. John and addressed to each in turn a prayer as follows :

" Greetings, great saint, I pray that you will be pleased to double the money that I have in my purse ; I promise that to mark my gratitude, I shall give 6 francs to this Church."

His wish was granted each time, and each time he kept his promise with perfect honesty. At the end, he left the church without a sou. How much had he when he entered? J.F.H.

## A SIMPLE GRAPH

Plot the graph of $y = x^3(5 - x^2)$ for values of x between $-2$ and $+2$.

Submit solutions to the Editor stating your age. J.G.

## OTHER NUMBER SYSTEMS

(a) In what number system is the identity $202 = 13 \times 13$ correct?

(b) " 524 is the first perfect square larger than 441." In what number system is this statement correct? B.A.

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**Time Chart**—Continued from page 281

this he calculated the diameter of the Earth to be 7,850 of our miles—only about 50 miles out ! He also estimated the sun's distance to be 100 million miles, which is reasonably correct. His work on prime numbers is also well known—the " Sieve " of Eratosthenes being a device for sifting out the composite numbers and leaving only the primes.

We have been able to deal with only a few of the many mathematicians of this very fruitful period—a period of creative thought which was not to be matched till Newton's day. We must postpone one of the greatest of them, Archimedes, until our next chart. He has been described as " one of the greatest mathematical physicists of all time." R.M.S.

## MAGIC SQUARES No. 2

*Contributed by Canon D. B. Eperson, Bishop Otter College, Chichester.*

|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

Fig. 1

In a magic square with 9 cells the sum of the numbers in each row, column, and diagonal is the same. In addition, the numbers in the central column, the middle row, and each diagonal, form sequences called Arithmetic progressions. In Fig. 1 these progressions are 1, 5, 9 : 3, 5, 7 : 2, 5, 8 : 4, 5, 6 : Their common differences are 4, 2, 3 and 1 respectively.

*Question* 1. Can you find the four progressions in Fig. 2, and their common differences ?

|   |   |   |
|---|---|---|
| 15 | 2 | 13 |
| 8 | 10 | 12 |
| 7 | 18 | 5 |

Fig. 2

Such sequences in magic squares can be used to form patterns. For instance, in Fig. 3 we have a dot marked at the centre of each cell, with lines through the four sets which correspond to the sequences mentioned above. These form a pattern similar to a Union Jack.

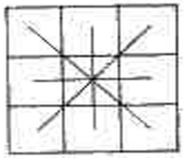


Fig. 3

*Question* 2. What patterns do you make by joining the dots representing the following sequences in Fig. 1?

[a] 1, 2, 3 ; 4, 5, 6 ; 7, 8, 9 ; [common difference 1]
[b] 1, 3, 5 ; 3, 5, 7 ; 5, 7, 9 ; [common difference 2]
[c] 1, 4, 7 ; 2, 5, 8 ; 3, 6, 9 ; [common difference 3]
[d] 2, 4, 6, 8. [common difference 2]

*Question* 3. What progressions in Figure 2, will give the same patterns as those in Questions 2[a] and [c]. What are their common differences ?

*Question* 4. Form patterns by joining up the dots corresponding to the numbers taken in increasing order of magnitude in Figs. 1 and 2.

|   |   |   |
|---|---|---|
| 7 + 1 |   | 7 + 4 |
|   | 7 |   |
| 7 − 4 |   | 7 − 1 |

Fig. 4

The two patterns are not alike, but all magic squares of 9 cells belong to one or the other of these two patterns. Test this by making your own magic squares in this way :— Choose any two numbers which are not in the ratio 1 : 2 or 2 : 1 ; then choose a third number not less than 5, which is greater than the sum of the first pair. [*e.g.*, the numbers could be 1, 4 and 7]. Put the third number in the central cell and fill the two diagonals with numbers which form A.P's whose common differences are the original pair of numbers. Then fill in the remaining cells, remembering that the rows and columns must all have the same total as the diagonals [see Fig. 4]. The join up the dots corresponding to the numbers in the cells taken in ascending order of magnitude, and you will get one of the two patterns occurring in Question 4.

|   |   |   |
|---|---|---|
|   |   | 7 |
| 5 |   |   |
|   | x |   |

Fig. 5

*Question* 5. [for those who like algebra] Can you find the value of $x$ such that Fig. 5 can be made into a Magic square whatever number is put into the central cell, and the others filled with numbers that obey the sequence rule shown in Fig. 3 ?

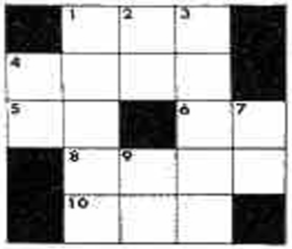


## POETICAL PI

Now I, even I, would celebrate
In rhymes inapt, the great
Immortal Syracusan, rivaled nevermore,
Who in his wondrous lore,
Passed on before,
Left men his guidance how to circles mensurate.

A. C. ORR

## SENIOR CROSS FIGURE No. 36




CLUES ACROSS :

1. Value of $a^2 - b^2 + 2$ when $a^3 + b^3 = 8100$ and $a + b = 30$.
4. The digits of this number are $x, 2x, x, x$ ; but if written $x, x, 2x, x$ it would decrease by 360.
5. Value of $s(s-a) + (s-b)(s-c)$ when $2s = a + b + c, b = 4$, and $c = 17$.
6. A clock is 7 minutes slow at 4 o'clock and 5 minutes fast at 5 o'clock. In how many minutes after 4 o'clock will it be correct ?
8. Value of $b^2 - ac - 14$ when $b + 2a = 122$, $b - c = 52$ and $a - c = 2$.
10. How many rectangles (including squares) in a rectangle 6" by 3" ruled into square inches ?

CLUES DOWN :

1. L.C.M. of 2873 and 3757.
2. Angle between the hands of a clock at 6.48 a.m.
3. Area of a trapezium, sides 1021, 25, 1007, 25.
4. Hypotenuse of a right angled triangle, perimeter 10", area 2 sq. in. (to 1 dec. place).
7. Number of diagonals in a duodecagon.
9. Numerical value of $x^2 - 6xy + 9y^2 + 23x - 69y + 42$ when $x - 3y = 2$.

J.G.

## FUN WITH NUMBERS No. 3

$$1+2+1 = a^2$$
$$1+2+3+2+1 = b^2$$
$$1+2+3+4+3+2+1 = c^2$$
$$1+2+3+4+5+4+3+2+1 = d^2$$

What are the values of a, b, c, d and can you write down quickly some similar problems and answers ?

## FUN WITH NUMBERS No. 4

$9 \times 9 + 7$
$98 \times 9 + 6$
$987 \times 9 + 5$
$9876 \times 9 + 4$

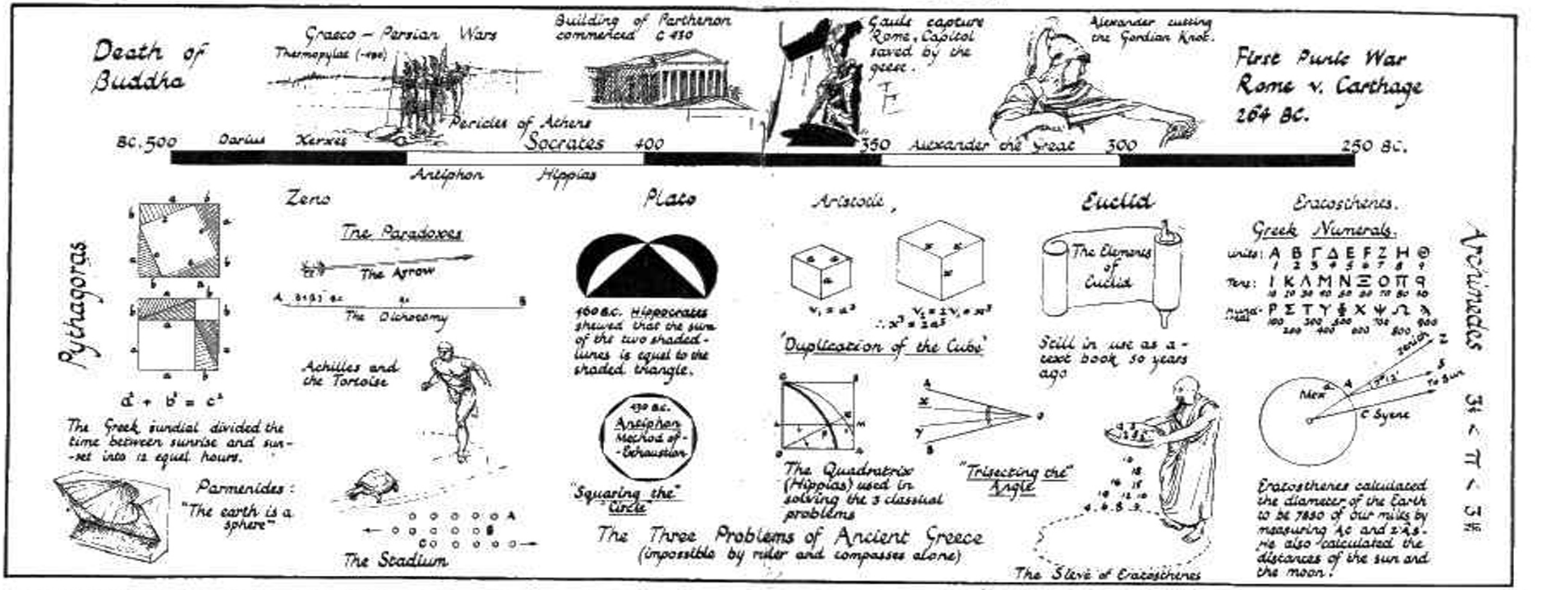
Work out these sums.

Can you now extend the sequence of questions and answers ?



20856 61190 62545 43372 13153 59584 72460 29016 26482 93972 85847 83163 05777 75606 88876 44624

# TIME CHART No. 2 : (500 B.C. — 250 B.C.)




At the end of the 6th Century B.C. we come to the death of Pythagoras and the birth of the philosopher Zeno, who is remembered chiefly for his four paradoxes, questions which challenged the ideas of some of his contemporaries who believed that space and time could be divided into an infinite number of very small parts. For example, the one about Achilles and the Tortoise :—

Achilles, who can run 10 times as fast as the tortoise, gives him 100 yards start. When he reaches the place where the tortoise was, it has gone 10 yards further on. By the time Achilles has covered this 10 yards the tortoise has gone an extra yard. When he has covered this yard the tortoise is still $\frac{1}{10}$ yard ahead and so on. How does Achilles catch the tortoise ? (Ask your teacher about the other 3 paradoxes).

The importance of this problem of infinite division is bound up with the three great problems of antiquity—Duplication of the Cube, Squaring the Circle and Trisecting the Angle. Greek arithmetic was hampered by the lack of a suitable simple number system. Various systems were in use, the simplest being the alphabetical one shewn in the chart. No wonder they avoided Arithmetic and tried to express all their ideas geometrically !

Originally the 3 problems were tackled using straight-edge and compasses only, but when this was found to be impossible, Hippocrates (c.460 B.C.) and Antiphon (c.430 B.C.) solved the Squaring of the Circle (*i.e.*, finding its area) by methods which were crude forms of our integral calculus.



82468 57926 03953 52773 48030 48029 00587 60758

Antiphon inscribed a polygon in the circle and found its area. By doubling the number of sides again and again he " exhausted " the shaded area which is the difference between the areas of polygon and circle.

The Three Problems were also tackled by the use of special curves. Deinostratus (c.350 B.C.) used the *Quadratrix* discovered by Hippias (c.425 B.C.). Neater solutions were made by using the Conchoid of Nicomedes, but this belongs to our next chart.

Two Greek Philosophers who, though not primarily mathematicians themselves, did much to put geometry on a sound basis, were Socrates and Plato. They insisted on accurate definitions, clear assumptions and logical proof. The Platonic school at Athens produced many brilliant men such as Aristotle (who became a tutor of Alexander the Great). Aristotle's chief interest in mathematics lay in its application to physics.

Alexander founded the city of Alexandria in Egypt, whose most famous inhabitant was Euclid, who performed the monumental task of collecting together all the mathematical knowledge of his time and arranging it in a logical sequence. The resulting " book " was still in use as recently as the beginning of our own century !

Another scholar of the University of Alexandria was Eratosthenes. He knew that the sun was at its zenith (*i.e.*, overhead) at Syrene on the Nile and measured its deviation from the zenith at Alexandria 625 miles away. From

Continued on page 283



25104 74709 16439 61362 67604 49256 27420 42083