JUNIOR CROSS FIGURE No. 31

Submitted by R. C. Bulpin, The Grammar School, Minehead.

1	2	■	3	4
■	5		■	
6		■	7	
	■	8		■
9		■	10	

CLUES ACROSS :

1. " Top of the house."
3. Perfect square.
5. Simple interest on £600 for 3 years at $3\frac{1}{2}$% p.a.
6. $2x+y=18$ Find $3(x+1)$. $x-y=3$
7. Express 36 as a percentage of 90.
8. $\frac{1}{2}$ (6 across + 4) × 4.
9. CDVIII ÷ VI.
10. $x^2 - 2y$ (See 6 across).

CLUES DOWN :

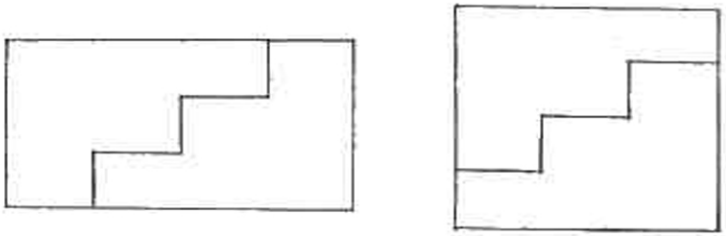
2. Number of half crowns in £120 10s. 0d.
4. Internal angle of a regular nonagon.
6. $8(\frac{4}{7} \times \frac{21}{5} \div \frac{1}{15})$.
7. Average speed of a plane which travelled 1,624 miles in $3\frac{1}{2}$ hours.

SOLUTIONS TO PROBLEMS IN ISSUE No. 33

3 SECOND QUIZ

(*a*) 1/- ; (*b*) 6d. ; (*c*) 8d. not 9d. which is the usual quick answer.

IT'S A TWIST



SENIOR CROSS FIGURE No. 33

ACROSS : (1) 21 ; (3) 315 ; (7) 441 ; (8) 648 ; (9) 1400 ; (11) 68 ; (12) 159 ; (14) 39 ; (16) 220 ; (18) 101; (20) 84 ; (21) 15, (22) 24.
DOWN : (1) 24 ; (2) 14 ; (3) 310 ; (4) 160 ; (5) 546 ; (6) 8888 ; (9) 11 ; (10) 458 ; (13) 901 ; (14) 315 ; (15) 90 ; (16) 28 ; (17) 24 ; (19) 14.

AN ODD QUESTION

The smallest number is 103.

USING THE HEAD AS WELL AS THE FEET

Collecting tickets in the order *OACBGFDEO* requires covering seven hundred yards. Return to the centre line after visiting *A* and *D*. Continue down past *E* and return to *O* along the bottom line.

IT'S ALL GREEK

The correct order was Least, Class, Amiss, Tacit, and Sails.

DO YOU LIKE IT

'A Midsummer Night's Dream.' Puck circled the earth in 40 min. His average speed must have been approximately 37,700 miles per hour.

JUNIOR CROSS FIGURE No. 30

ACROSS : (1) 264 ; (4) 30 ; (5) 84 ; (6) 40 ; (7) 81 ; (9) 144 ; (10) 106 ; (11) 22 ; (13) 112 ; (16) 156.
DOWN : (1) 2816 ; (2) 64 ; (3) 444 ; (4) 38 ; (8) 169 ; (10) 121 ; (12) 33 ; (14) 10 ; (15) 28.

O, O, ANTON I O

You sigh for a cipher, but I sigh for thee,
O sigh for no cipher, but O sigh for me ;
Then will your sigh for no cipher be,
But give sigh for sigh, for I sigh for thee.

31666 36528 61932 66863 36062 73567 63035 44776

MATHEMATICAL PIE

No. 34 — *Temporary Editorial Address :* 135 Peartree Avenue, Southampton — OCTOBER, 1961

CLOCK ARITHMETIC No. 1

Addition and Subtraction

You will all know that $4 + 5 = 9$ and will readily agree that the process can be illustrated by using a ruled scale as shown in Fig. 1.

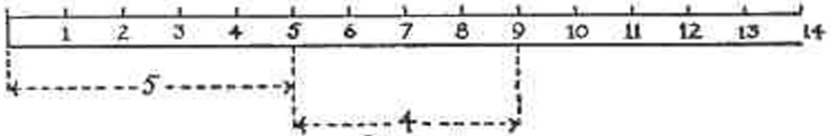


Figure 1

Suppose we now try to do this addition on a circular scale instead of a linear one. Such a circular scale will look like a clock face and let us take first one which has only six numbers on it, as in Fig. 2.

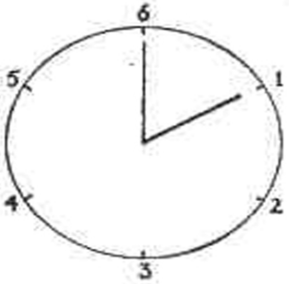


Figure 2

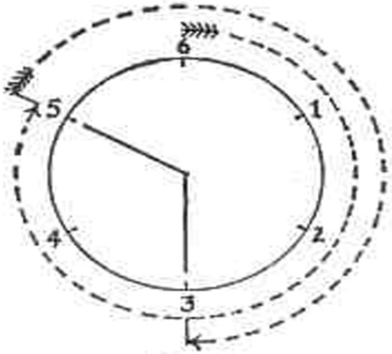


Figure 3

Then $4 + 5$ on this kind of scale would be done as shown in Fig. 3. This clock face arithmetic therefore gives $4 + 5 = 3$. Similarly, $3 + 4 = 1$. Can you give the clock face values of $5 + 2$, $3 + 2$, $2 + 2$, $4 + 4$, $6 + 3$? There is no 0 in this set of numbers. What number performs the function of zero in this arithmetic?

+	1	2	3	4	5	6
1						
2			5^b			
3						
4						
5				3^a		
6						

Figure 4

Now complete the addition table as shown in Fig. 4.

$5 + 4 = 3$ (see *a*)
$2 + 3 = 5$ (see *b*)

Notice that the first number in the addition sum is always indicated in the extreme left hand column and the second number in the top line, the answer being found in the body of the table.

09243 23789 69070 69779 42236 25082 21688 95738

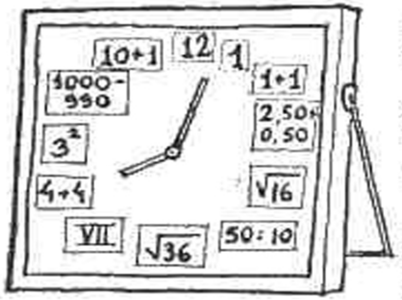
This game is known by mathematicians as addition in Modulo 6 arithmetic (No number larger than 6 appears in the table). Can you now make up an addition table for Modulo 12 arithmetic, *i.e.*, using the normal clock face numbers 1 to 12? This time some of your sums may include 7, 8, 9, 10, 11 and 12, which of course were not allowed in your first game.

Another piece of research is to make up similar tables for games of Modulo arithmetic for all Modulo arithmetics up to 12.

Problem.

Can you now use these tables to do subtraction and then make up subtraction tables for each Modulo arithmetic.

Example.



Reproduced by courtesy of *Punch*, 2nd. December 1959.

Modulo 6 subtraction $5 - 2 = 3$ and $1 - 3 = 4$. Do you notice anything queer about the answers you get?

But what use can be found for Modulo arithmetic? It is strange that you should have been using it but have not noticed it.

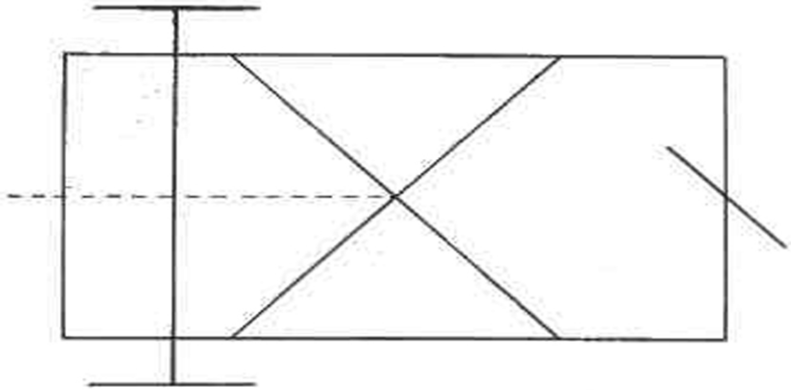
Example in Modulo 7.

In counting the days of the week, if 1 indicates Sunday, 2 indicates Monday, 3 Tuesday, etc., then five days after Tuesday is $3+5=1$, *i.e.*, Sunday.

Can you find an example in Modulo 12 arithmetic?

In one of our future issues we will deal with the problem of multiplication in Modulo arithmetic.

RUNNING IN CIRCLES



The diagram shows the skeleton chassis of a tricar. The front wheel is turned through 30° from the normal position. What are the radii of the circles described by the three wheels when the car is set in motion, given the distance between the axles is 12 feet, the distance between the wheels on the axle is 5 feet, and the diameter of the wheel is 2 feet? J.G.

37986 23001 59377 64716 51228 93578 60158 81617

JOHNNY IS NO SQUARE

John wrote down the cube of his age in years. He then subtracted his age from this figure and obtained 4080. How old is he?

FUN WITH NUMBERS—2

$2 \times 2 - 1 \times 3$
$3 \times 3 - 2 \times 4$
$4 \times 4 - 3 \times 5$
$5 \times 5 - 4 \times 6$
$6 \times 6 - 5 \times 7$

What do you notice about these expressions? Deduce the *n*th. line.

£ s. d.

What like fractions of a pound, a shilling and a penny when added together will make exactly one pound? R.H.C.

GOALS, MORE OR LESS

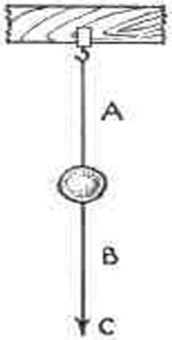
If the star performer in the school eleven had scored two more goals than he actually did, his average would have been 3 goals per match. With a score of two goals less, the average would have been 2. What was his total score? J.F.H.

FLOOR SPACE

Show how you can cover half a square two feet on each side and still have a square which is two feet from top to bottom and two feet across.

IT'S NOT WHAT YOU DO (IT'S THE WAY THAT YOU DO IT)

(Adapted from Le Facteur X)



The diagram represents a heavy ball of metal suspended by means of a wire *A* of uniform thickness. From a point diametrically opposite to the point of suspension, a piece of similar wire *B* hangs underneath the ball. What will happen when

(*a*) a gradually increasing downward pull is applied at *C*?

(*b*) a very sudden and powerful downward jerk is applied at *C*?

J.F.H.

SHAKESPEARE AS MATHEMATICIAN, RE SIGNS

Bolingbroke : Are you contented to resign the crown?

Richard II : Ay, no ; no, ay ; for I must nothing be ;
Therefore no no, for I resign to thee.

Shakespeare seems to have known the rule of signs, *viz.* : $2 - 2 = 0$; $-2 + 2 = 0$; and two minus signs operating on each other give a plus.

12019 05166 09628 04909 26360 19759 88281 61332

The reason is quite simple. It is a question of cost. If a printer is to be able to print all the numbers from 1 to 999 he needs three of each of the figures from 1 to 9 (He could leave spaces for the zeros). In the scale of two we write one thousand and twenty-three as 111,111,111. To print numbers up to this needs nine figures, but since the only figure used is 1 we need only nine altogether. The same comparison holds for all large numbers. Using the scale of ten the printer needs a stock of type three times as great as the stock needed for the scale of two.

In a computor some piece of apparatus is needed to represent each of the digits to be used in its calculations. Therefore a computer using scale of ten is three times as big as a computer using scale of two. As large computers cost hundreds of thousands of pounds this represents an enormous difference in cost even though a few hundred pounds has to be spent on an input machine to change numbers from scale of ten to scale of two, and an output machine to change numbers from scale of two to scale of ten.

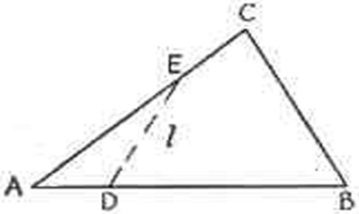
C.V.G.

Claudius said he could take one from four and have five left. Thereupon Julius showed how he could take eleven from twenty nine and have twenty left.

DIGITAL COMPUTERS

TRY ANGLES

(adapted from Le Facteur X)



The owner of a triangular field has a fence of length l with which he wants to divide the field into two parts as shown. He further requires that the portions AD and EC shall be of equal length. How is he to determine point D?

J.F.H.

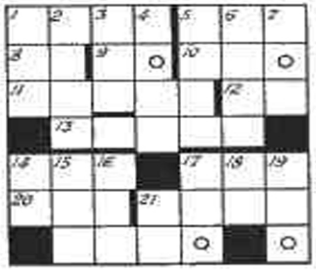
NOT VERY OBVIOUS

Two right angled triangles have equal hypotenuses and equal perimeters. Are they congruent?

27777 10386 38773 43177 20754 56545 32207 77092

SENIOR CROSS FIGURE No. 34

A small circle indicates zero. Fractions should be decimalised. The number of significant figures is indicated by the double lines.



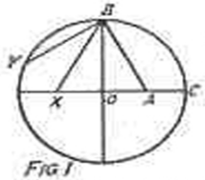


FIG 1

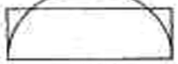


FIG 2

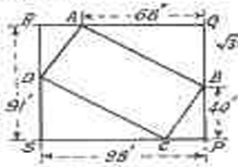


FIG 3

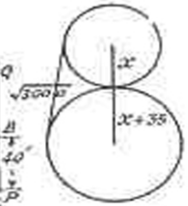


FIG 4

FIG 5

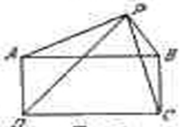


FIG 6

ACROSS:

1. A number which is a perfect square, whose four digits in order may be written $x+1$, $x-1$, $x-1$, $x+2$.
5. Stable hands in 44 ft.
8. The angle BOY (Fig. 1). Diameters perpendicular. $OA = \frac{1}{2}OC$, $AX = AB$, $BY = BX$. (Calculation or scale drawing).
9. A lady kept 84 pets—cats and goldfish. One day each cat ate 2 goldfish and then there were only 12 left. How many goldfish originally?
10. A price in shillings which, when increased by a quarter of itself, further increased by 15%, then reduced by 3s. in the £, is £39 2s.
11. Recurring figures in the decimalised $\frac{10}{21}$. (Exclude zero).
12. How many cats? (See 9 across).
13. $(a+b)^2-(a-b)^2$ if $a=222$, $b=111$.
14. $(x^2+y^2)\ (1+k^2)$ if $x - ky = 14$, $kx + y = 13$.
17. A floor is tiled with 6 in. square tiles. A circle of 4 ft. radius is drawn with its centre in the middle of the floor at the junction of 4 tiles. How many complete tiles are enclosed?
20. $\dfrac{a^3-b^3}{a-b}$ if $a+b=30$, $ab=48$.
21. (Fig. 2). Rectangle and semicircle of equal areas. Radius of semicircle is 10 in., find the height of the rectangle. (Take $\pi = 3{\cdot}1416$).
22. (Fig. 3). Area of rectangle ABCD inscribed in rectangle PQRS.

DOWN:

1. Becomes a perfect square if you add 10 or subtract 33.
2. Less than 53×53, but leaves no remainder when divided by 53, and leaves a remainder 53 when divided by 7×53.
3. Sum of the areas of all the different rectangles (including squares), whose adjacent sides may be any of the numbers from 1 to 6.
4. Two numbers, the larger first. They are the radii of the circles in Fig. 4. One radius is 35 in. longer than the other and the common tangent is $\sqrt{3000}$.
5. Freezes when divided by 53. Boils when divided by 8.
6. (Fig. 5). Diameter of a semicircle touching the shorter sides of a 3", 4", 5" triangle. (First 3 dec. pl.).
7. Sum of the 3 altitudes of an isosceles triangle whose sides are 75, 75, 90.
14. $a - b$ if $a^2 - ab = ab - b^2 = 228$.
15. Three consecutive digits in descending order. Reverse and add to the original and the result is 1110.
16. Two numbers. The values of a and b if the roots of

$$x^2 - bx + a = 0 \quad \text{are} \quad \frac{22 \pm \sqrt{464}}{2}.$$

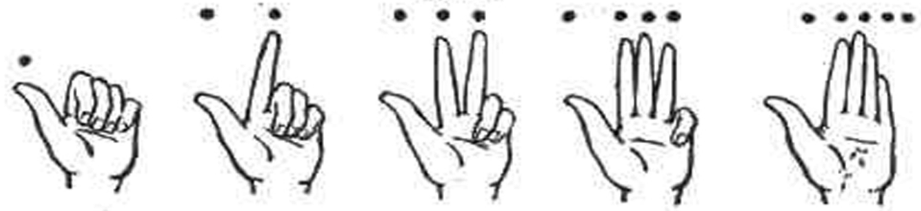
17. pv. When p is reduced by 3 and v is increased by 3, pv is unaltered. If p is increased by 3 and v is reduced by 2, pv is again unaltered.
18. (Fig. 6). Sum of the areas of the triangles APD and BPC. The rectangle is 15 in. by 10 in.
19. This number increased by 25% three times is 859⅜.

J.G.

55782 97352 33446 04281 51262 72037 34314 65319

People who count on their fingers are usually very young or not very bright. Whatever the other cavemen may have thought of him, the first man to count on his fingers was probably not very bright by our standards, but he has had a tremendous influence on the mathematics that have come after him. He decided that when he thought of one thing he would stick up his thumb, and that when he thought of two things he would hold up a finger as well, and that when he thought of three things he would hold up another finger. Like this :—

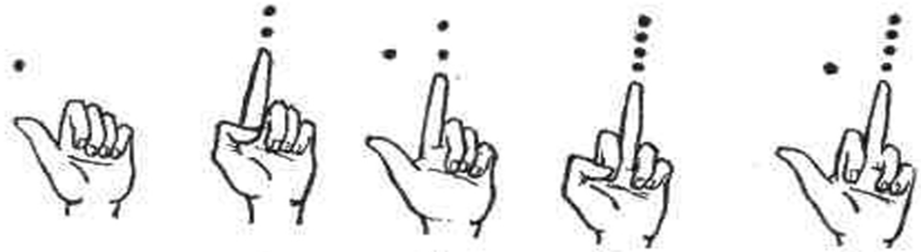
Figure 1



one two three four five

Now if he had been a little brighter he might have decided to stick up his thumb when he thought of one thing, but to stick up his finger instead when he thought of two things. Then when he thought of three things he would stick up his finger and thumb. Like this :—

Figure 2



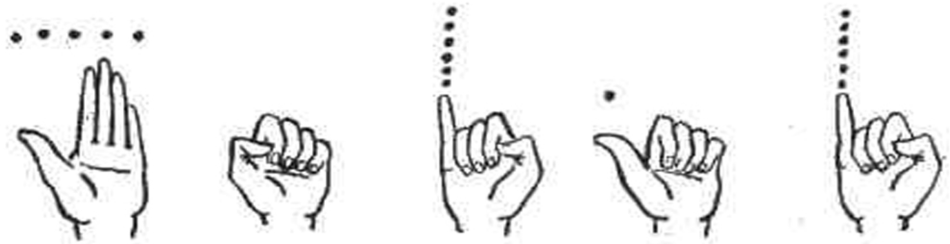
one two three four five

Then instead of being able to count up to five on the fingers of one hand, up to ten on both hands, and up to a score if he used his toes as well, he could have counted up to thirty-one on one hand, up to one thousand and twenty-three using both hands, and to over a million if he used his toes.

Of course primitive men never had that many of anything so it did not matter at first. If the first man who had to count above five had thought much about large numbers he might have speeded up the progress of arithmetic by thousands of years. If, when he had used up the fingers of one hand, he had stuck up the thumb of his second hand and put down the fingers of the first hand, and then started again on the first hand, as in *figure 3*, he would have been able to count up to thirty-five on his fingers and would have made the important discovery that one sign can stand for a group of things.

77774 16031 99066 55418 76397 92933 44195 21541

Figure 3



five six seven

When men started doing calculations with pebbles they represented thirty-two like this

0000000000
0000000000 0
0000000000 0

It took many centuries before any one thought of this

0 0
0 0
0

where one large pebble represents ten small ones. The first man to do this probably said that the pebbles represented three men and two fingers.

Less than a thousand years ago people in Europe still wrote XXX11, where X stands for two hands and 1 stands for one finger.

However someone in India realised that a pebble in the second column could represent ten units without having to be a different size of pebble. So he represented thirty-two like this :—

0
0 0
0 0.

Still later more easily written symbols were invented for and a symbol for an empty column, and the scale of ten was perfected.

0
0 0
o **0** 0
0 **0** 0

and so on,

Our first cave man might have invented the scale of two in which the number up to twelve would be written like this :—

1 10 11 100 101 110 111 1000 1001 1010 1011 1100.

Our second cave man might have invented the scale of six with the numbers written like this :—

1 2 3 4 5 10 11 12 13 14 15 20.

In the scale of ten 111 means One (ten $\times$ ten) and one (ten) and one. In the scale of six 111 means one (six $\times$ six) and one (six) and one. In the scale of two it means one (two $\times$ two) and one (two) and one.

The readers of MATHEMATICAL PIE have become used to the scale of ten, but some of them may have found it hard at first. Because our first mathematician was not very bright every child learning arithmetic has to learn 81 facts like this :—$3\times4=12$ and 81 facts like this $3+4=7$. With the scale of two the only figures would be 0 and 1, and the only fact to learn would be $1+1=10$.

Perhaps in the long run it was a good thing that the first mathematician missed his chance. Arithmetic would have been so easy that it would not have challenged the intelligence. Certainly there would have seemed no need for logarithms, slide rules and calculating machines, so that it is perhaps rather odd that large electronic calculating machines work in the scale of two.

34189 94854 44734 56738 31624 99341 91318 14809