

JUNIOR CROSS-FIGURE No. 28

Submitted by Marion Mitchell, Form IV X, Bradford Girls' Grammar School.

CLUES ACROSS:

- Last year + next $\frac{1}{2}$ of this year.
- $\sqrt{289}$.
- A fifth of a furlong in yards.
- Number of acres in half a square mile.
- Number of chains in a mile.
- Half of three times $(9 - x)(6 + 2x)$ when $x = -2$.
- Area of a triangle, base 20 inches, height 5 inches.
- I spent $\frac{1}{2}$ of my money, and then $\frac{3}{4}$ of the remainder. I had 24/- left. How much had I at first?
- Third prime number squared multiplied by the sixth prime number.
- Tom has 4 times as many marbles as Dick, Jack has as many as Tom and Dick together, Sam has one less than Tom. 125 altogether, how many has Sam?
- One eighth of half of 122.
- Reverse a number and multiply the two together.



- A perfect square.
- Number of ounces in a ton, reversed.
- 103×11 .
- Number of gallons which can be held in a cistern four feet each way inside.
- Number of farthings in a third of a guinea.
- 18 cwt. 2 qr. 23 lb. in lb.
- $72 \div (62 - 22 - \sqrt{4})$.

CLUES DOWN:

- $\sqrt{729}$.
- $102 - (62 - 42 - 10)$.

SOLUTIONS TO PROBLEMS IN ISSUE No. 30

SQUARE IN THE FACE.
($\sqrt{x+1}$)² = $x + 2\sqrt{x+1}$.

ROUNDERS.

- Cotton reel.
- Inner tube.
- Outer cover.
- Flower pot.
- Gramophone record.
- Hand bell.
- Diabolo.
- Ball.
- Vase.
- Saucer.

SENIOR CROSS-FIGURE No. 30.

Across: (1) 384; (3) 173; (5) 520; (6) 26; (7) 27; (8) 65; (9) 60; (11) 128; (12) 062; (13) 484.
Down: (1) 375; (2) 4577; (3) 10; (4) 346; (6) 25; (7) 20; (8) 6284; (9) 650; (10) 464; (11) 12.

MAGIC SQUARES.

Question 1. Sequences: 1, 5, 9; 8, 5, 2; 4, 5, 6.

Question 2. Magic Total 30.

Question 3. 2, 5, 7, 8, 10, 12, 13, 15, 18. 10 is in the central cell.

Question 4. Sequences: 8, 10, 12; 2, 10, 18; 15, 10, 5; 7, 10, 13.

Question 5.

Question 6.

Question 7.

Question 8.

9	1	8	5	4	9		2			x	
5	6	7	10	6	2	6			2y-x		
4	11	3	3	8	7			4		y	

To prove the result in Question 8 put m for any number in the central cell: then using the sequence principle, the first (top left corner) cell must contain the number $2m-9$, and the magic total, found by adding up the completed diagonal, is $3m$. So all the other cells can now be filled, by making the total of each row, column and diagonal $3m$.

In the answer to Question 3 notice the symmetrical pattern formed by the numbers which do not appear

2 XX 5 X 7 8 X 10 X 12 13 X 15 XX 18

Note also that 10 is the average of all the numbers.

JUNIOR CROSS-FIGURE.

Across: (1) 13; (3) 275; (5) 4840; (6) 30; (8) 26; (9) 13; (12) 3300.
Down: (2) 340; (3) 245; (4) 70; (6) 3413; (7) 6600; (10) 33; (11) 10.

B.A.

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77917 45011 29961 48903 04639 94713 29621 07340

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If I have seen further than other men, it is by standing
on the shoulders of Giants. Sir Isaac Newton.
What names would you suggest for the giants? B.A.

237

26654 08530 61434 44318 58676 97514 56614 06800

NUMBERS IN WORDS

Suggested by E. Rowland, Esq., Huddersfield.

Any number can be expressed in words, but generally this form is used only for small numbers: it is so much quicker and neater to use numerical symbols for the large numbers. Some new and amusing results can be obtained when numbers are written in words.

1. The eight letters in FORTY SIX are different but this is neither the largest nor the smallest number that can be expressed in words in which all the letters are different. What are the largest and the smallest numbers that you can find expressed in words in which all the letters are different?

2. Find a number which contains the same number of figures when expressed in the usual way as letters when expressed in words.

3. The following simple additions are obviously correct. It is also possible to replace each dot by a letter to form addition problems in words which are still correct. The first one becomes TWO + FIVE = 7

- (i) ... + ... = 7 (v) ... + ... + ... = 11
 (ii) ... + ... = 8 (vi) ... + ... + ... = 12
 (iii) ... + ... + ... = 9 (vii) ... + ... + ... = 13
 (iv) ... + ... = 10 (viii) ... + ... + ... = 14

Paradoxes arise occasionally when numbers are written in words. The statement that there is a number which is "the least whole number which cannot be named in less than nineteen syllables" is paradoxical, because the phrase in inverted commas which has been used to define a number contains only EIGHTEEN syllables.

Can you find any more paradoxes?

MATHS

I like doing Maths.
 Working sums and making graphs.
 Finding the area of a square,
 Giving it the utmost care.
 Ruling a line, learning a table.
 Doing it the best as I am able.
 Algebra, geometry, fractions too
 In maths there's certainly a lot to do.

Marilyn Boydle, North Manchester High
 School For Girls, Church Lane, Moston,
 Manchester, 9.



By courtesy of the "Daily Mirror."
 "Have you finished my homework yet, Dad?"

ROUND AND ROUND

P is a point on a sphere of radius 6". A pair of compasses is opened to a radius of 4" and a circle centre P is drawn with the compasses on the sphere. What is the radius of this circle? What is the radius of the largest circle that can be drawn on the sphere?

FUN WITH NUMBERS—1

$$1 = 1$$

$$2 + 3 + 4 = 1 + 8$$

$$5 + 6 + 7 + 8 + 9 = 8 + 27$$

$$10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64$$

Deduce the next line and check the arithmetic. Then try to guess a general rule for the n th. line and prove it.

INTELLIGENCE TEST

A general knowledge quiz was set to five of the class, John, Brenda, Robin, Harold and Joyce. These were the questions

- (1) What is the name of the nearest cinema?
- (2) What is the name of the mayor.
- (3) How many miles is it to London?
- (4) What are the colours of the Rovers.
- (5) What is the Christian name of a contrary young lady.
- (6) How many centimetres in an inch?

These were the answers the master received.

John :	Astra	Painter	150	Red	Kitty	3
Brenda :	Ritz	Green	170	Blue	Mary	2½
Robin :	Grand	Watkins	140	Blue	Mary	3
Harold :	Gaumont	Painter	140	Yellow	Lucy	2½
Joyce :	Gaumont	Watkins	140	Yellow	Lucy	3

Each of the five has two correct answers and four incorrect and each question has been answered correctly by someone.

What are the correct answers.

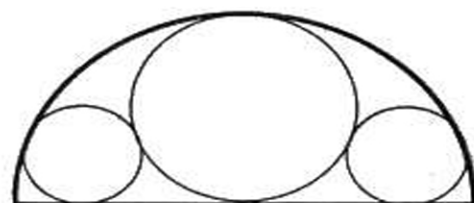
R.H.C.

PROBLEMS

1. Two sides of a parallelogram are 7" and 9". If one of the diagonals is 8", how many inches are there in the other?
2. Find the smallest integral value of X that will make $1260X$ a perfect square.
3. An integer between 20 and 30 is added to its cube and the sum is 13848. Find the integer.

ANOTHER TIGHT FIT

Three circles, two of them equal are drawn in contact with a semicircle as shown. If the diameter of the semicircle is 12", what are the diameters of the smaller circles?



THE TWO BUCKETS

Two exactly similar buckets are as full of water as they can be, but one has a large piece of ice floating in it. Which weighs heavier?

THE POWERS THAT BE

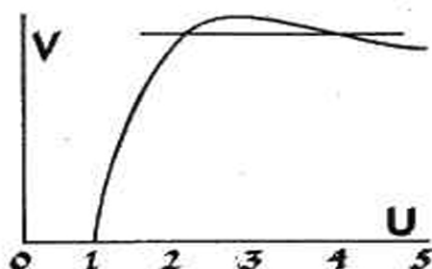


Figure 1. $v = \log u$.

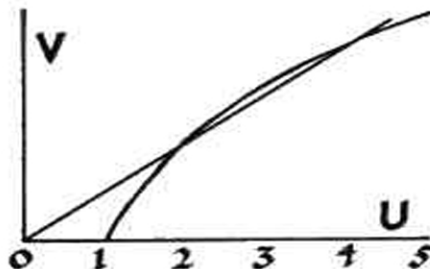


Figure 2. $v = \log u$.

Over a hundred readers sent solutions to the problem in Mathematical Pic No. 29 on finding pairs of numbers to satisfy $x^y = y^x$.

Those who tried a graphical solution changed $x^y = y^x$ to $(\log x)/x = (\log y)/y$. If the graph of $v = (\log u)/u$ is plotted, pairs values for x and y are given by the intersections of horizontal lines with the curve (Figure 1). Alternatively, pairs of solutions are given by drawing lines through the origin to intersect the curve $v = \log u$ (Figure 2). It can be seen from the graphs, or proved in other ways, that the smaller of the two numbers must lie between 1 and e (2.718...), so that there can be no integral solution other than 2 and 4.

To find rational solutions an algebraic approach is necessary. Let x be the smaller of the two numbers, and put $y = kx$. Then $x^{kx} = (kx)^x$. Therefore $y = x^k$ and $kx = x^k$. Rearrangement gives $x = k^{1/(k-1)}$ and $y = k^{k/(k-1)}$. Pairs of values for x and y are given by substituting any value for k . To obtain rational values of x and y put $1/(k-1) = n$.

$$\text{Then } x = \left(\frac{n+1}{n}\right)^n \text{ and } y = \left(\frac{n+1}{n}\right)^{n+1}$$

Substituting $n=1$ gives $x=2$, $y=4$, $n=2$ gives $x=\frac{9}{4}$, $y=\frac{27}{8}$ and so on.

Book tokens have been sent to:—

A. Balfour (Edinburgh), C. H. Biggs (Tonbridge), J. Bonnici (Malta G.C.), B. K. Booty (Whitchurch), R. Bourke (Newcastle), H. L. Kotkin (Enfield), A. Lavington (Whitgift School), P. R. Mogridge (Exeter School), E. Violet (Wigton).



R. BOURKE

STAMP COLLECTOR'S CORNER No. 9

Sir William Rowan Hamilton, 1805—1865, was Ireland's greatest Mathematician. His first work was in the theory of optics. Later he investigated the fundamentals of algebra and created a new algebra which he called quaternions. His quaternions and his work in optics began the developments which lead to Einstein's theory of relativity and to modern quantum mechanics. In his old age, Hamilton became solitary and eccentric. One of his unusual habits was that of using mutton chops as book marks.

C.V.G.

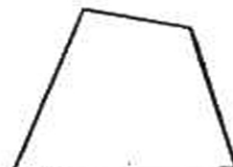


Ireland 1943 2d Brown and Green

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61581 40025 01262 28594 13021 64715 50979 25923

MODERN GEOMETRY



The sum of the inside angles of this quadrilateral is 360° . Without adding any more lines prove that the sum of the interior angles of a pentagon is 540° .

TIM-Ber!

Which is the greatest and which is the least of

(a) $\log(3+5)$, (b) $\log 5 + \log 3$, (c) $\log(8-4)$, (d) $\log 8 - \log 4$.

SENIOR CROSS-FIGURE No. 31

CLUES ACROSS:

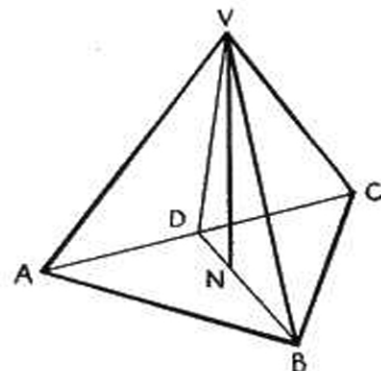
1. Δ VAC.
4. Four BD².
5. Sum of the lengths of the edges.
7. Twice DV.
8. Area Δ BVD to nearest square unit.
9. Angle BVD to nearest degree.
11. 4DN reversed.
14. BN².
15. Angle BVN plus three.
17. Volume reversed.
19. Total surface area.
20. BN reversed.



CLUES DOWN:

1. Angle VDN.
2. Three VB to nearest unit.
3. Area VAD to nearest sq. unit.
4. Sum of sloping edges to nearest unit.
5. Area Δ ABC.
6. $\frac{1}{2}(VB^2 - BN^2)$.
10. Rearrange 5 down.
12. VB.
13. Angle VBN.
16. Perimeter of VDB to nearest unit.
18. Angle AVC to nearest degree.
19. Perimeter of VDN plus one.

B.A.



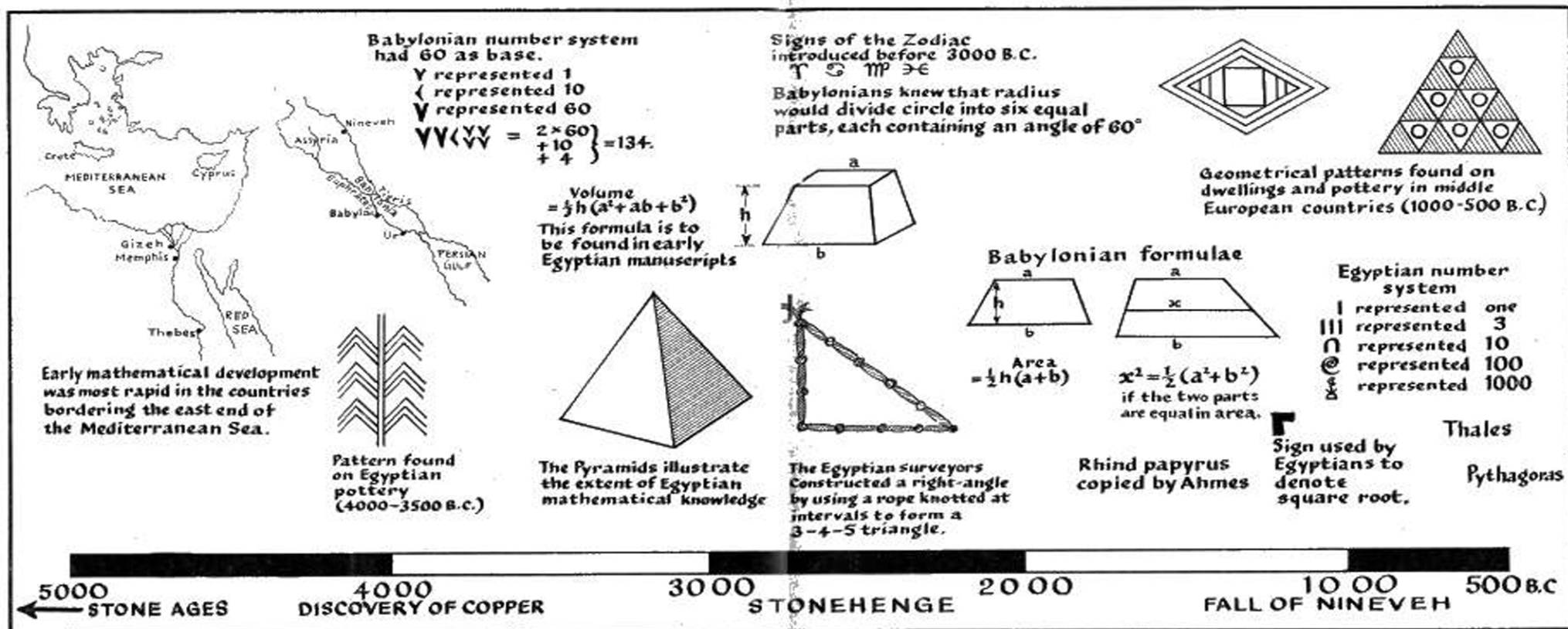
VABC is a right tetrahedron. ABC is an equilateral triangle of side 12 units. $VA = VB = VC$. D is the mid-point of AC. VN is perpendicular to ABC. $VN = 12$ units. B.A.

AFTER THE 11 PLUS

Three boys in a secondary school and the headmaster all had their birthdays on the same day. The product of the ages of the boys and the headmaster was 2,652. The sum of the ages of the boys was exactly the age of the headmaster. How old was he?

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94561 31407 11270 00407 85473 32699 39081 45466



The beginning of mathematics came not with some sudden and startling discovery, but with a gradual conception of number and form. Looking back through the last 7000 years of history we find that wherever there was culture, even of the most primitive kind, there was also mathematics. The cave paintings in France and Spain show a remarkable understanding of form, and the pyramids in Egypt, dating back to about 3000 B.C., illustrate an understanding of geometrical design and construction.

In these early times the Egyptians and Babylonians were perhaps the most advanced people in the realm of mathematics. While many tribes were still at the stage of simple counting, these two great civilizations had well-developed number systems and were able to write and calculate with fairly high numbers.

Most of our knowledge concerning Egyptian mathematics has come from the Rhind papyrus. This is believed to have been written before 3000 B.C. and is known to have been copied by the scribe Ahmes in about 1700 B.C. The papyrus is one of the earliest mathematical texts and contains problems in calculation which include work with fractions. By this time the Egyptians also had a system of units of measurement which included measurement of area as well as length. After many years of observation of the sky, the priests of Egypt were able to compile star catalogues and astron-

omy was being studied seriously before 1000 B.C.

During these early centuries similar progress was also being made by the Babylonians. Their number system, though differing in form from that of the Egyptians, was equally well developed. As far back as 1800 B.C. the Babylonians had compiled tables for multiplication and division, and primitive ideas of geometry and algebra followed very quickly. Their astronomy seems to have developed from the study of astrological omens and, by the 8th century B.C., the court astronomers had compiled a calendar of eclipses of the moon.

Throughout these years of mathematical development there was very little exchange of knowledge between the Egyptians, Babylonians, Hindus and Chinese, each pursuing their own line of cultural progress. The credit for drawing together this diversity of knowledge belongs to the Greeks. Whereas the Egyptians and Babylonians studied mainly for utilitarian purposes, the Greeks were rapidly developing into a nation of philosophers. One of the first Greek philosophers was Thales of Miletus, whose pupil, Pythagoras, spent a number of years travelling in the Mediterranean countries learning from the priests and scribes. After his travels Pythagoras, like Thales, founded his own school and his work added much to the existing fund of mathematical knowledge.

I.L.C.