

The advance and perfecting of mathematics are closely joined to the prosperity of the Nation.—NAPOLEON.

Hold nothing as certain save what can be demonstrated.—NEWTON.

If the Greeks had not cultivated conic sections, Kepler could not have superseded Ptolemy.—WHEWELL.

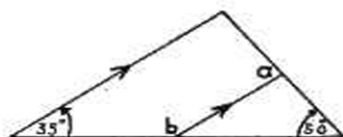
JUNIOR CROSS FIGURE No. 26

CLUES ACROSS

- Number of sides of a polygon with interior angle 150° .
- A third of 2 down plus five, minus ten.
- Simple interest in pounds on £170 invested for 4 years at 5%.
- Number of 6 inch square blocks to cover a hall floor $15\frac{1}{2}$ ft. by 11 ft.
- A prime number.
- $6.72 - 3.32$.
- Product of four consecutive prime numbers.
- a.

CLUES DOWN

- Interior angle of a regular pentagon.
 - b.
 - Add 1 across to 9 across { $2x + y = 13$
and multiply by $x + y$. { $x + 2y = 17$
 - $11xy$.
 - $(p - q)(p + q)$ when $p = 20$ and $q = 5$.
 - Area, in sq. in., of a rectangle 1 ft. 5 in. by 2 ft.
 - A prime number.
- Check clue. One digit is not used. The sum of the digits is 77.



SOLUTIONS TO PROBLEMS IN ISSUE No. 27



Write the number THE FIRST SHALL BE LAST
The other numbers will be
Adding we have
Divide this number by $(a + b + c + d)$ and the result is 1,111.

SENIOR CROSS-FIGURE No. 27
CLUES ACROSS: (1) 456; (3) 721; (5) 715; (7) 12; (9) 12.65; (12) 28; (14) 169;
(16) 864; (17) 152.
CLUES DOWN: (1) 421; (2) 67; (3) 75; (4) 192; (6) 12.36; (8) 21; (10) 52;
(11) 538; (13) 842; (14) 14; (15) 91.

FALLACY No. 26
The fallacy arises because LBDC is given as 30° . It must be 35° if the other information is true.

A PUZZLE FOR SQUARES
The chess-board cannot be covered in this way as the two squares which were removed are of the same colour.

6.5 SPECIAL
The man meets the car when it is at a distance which takes $\frac{1}{2}$ hour to travel to the station. Hence he walks for 45 minutes.

DUCKS AND DRAKES
He bought 3 ducks and the drake cost 6 shillings. The total cost was then 36 shillings.

JUNIOR CROSS-FIGURE No. 25
CLUES ACROSS: (2) 121; (5) 21; (7) 37; (8) 63; (10) 8470.
CLUES DOWN: (1) 12; (3) 23; (4) 17; (6) 1680; (9) 34; (11) 74.

B.A.

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00675 10334 67110 31412 67111 36990 86585 16398

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MATHEMATICAL PIE

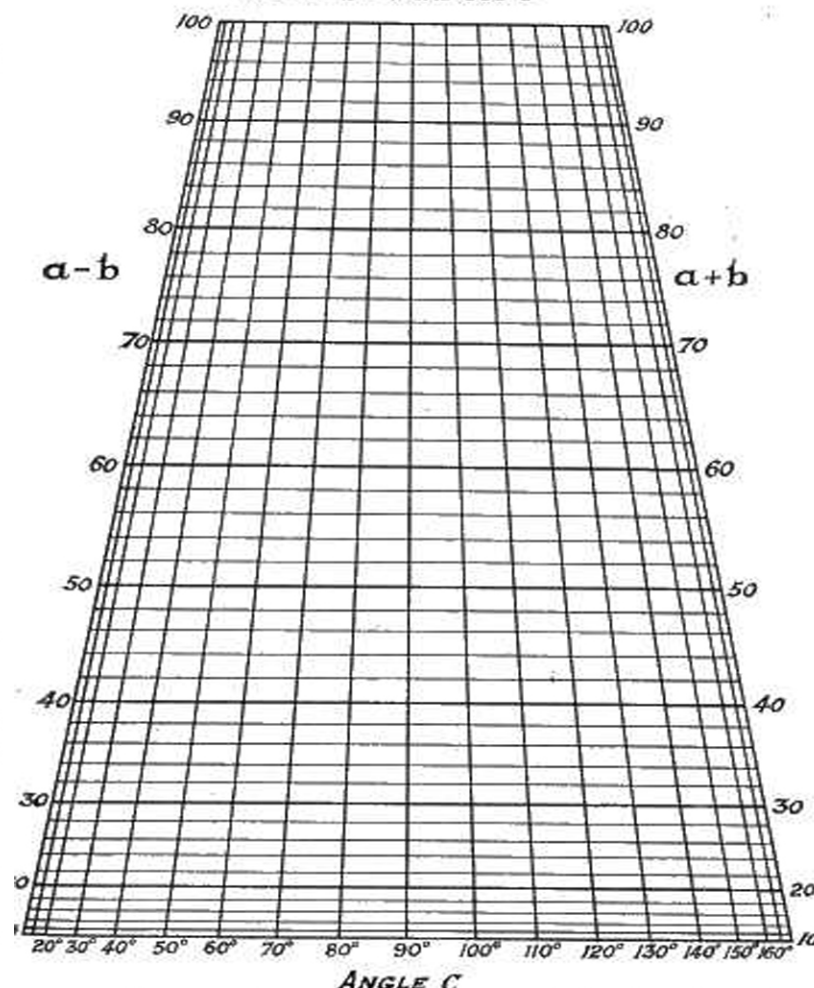
No. 28

Editorial Offices:
97 Chequer Road, Doncaster

OCTOBER 1959

A NOMOGRAM FOR THE COSINE FORMULA

$$c^2 = a^2 + b^2 - 2ab \cos C$$



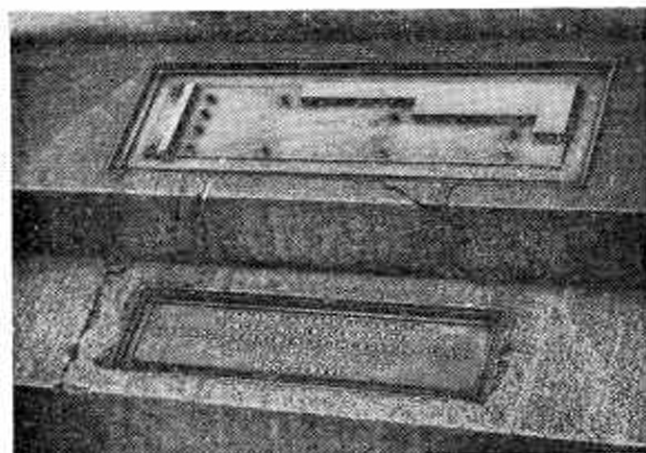
This nomogram can be used to find the angles of a triangle when the three sides are given, and to find the third side when two sides and the included angle are given. The horizontal lines represent values of c , the vertical lines values of C , and the two side scales represent $a + b$ and $a - b$.

Example: In $\triangle ABC$ $A = 60^\circ$, $a = 5$ in., $b = 3$ in. Find c .

Lay a straight-edge from the point $a + b = 8$ to the point $a - b = 2$. The straight-edge cuts the line $C = 60^\circ$ at $c = 4.35$. C.V.G.

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10055 08106 65879 69981 63574 73638 40525 71459



One of the many public semi-standards for one foot, two feet, and one yard.

American feet are to be a little smaller. As greater accuracy of measurement has been achieved the old standards of length have become unsatisfactory. The English standard yard has varied by about one-twenty-fifth of an inch since the first legal standard was made over 700 years ago. In 1922 it was discovered that the current standard (made in 1845) was slowly shrinking. The inch was therefore defined as 25.39996 millimetres. In 1951 it was redefined as 25.4 millimetres exactly.

In the United States each state maintained its own standards until 1832 when a Senate Committee found considerable disagreement between these standards and ordered the distribution of new standards based on the British standard yard. In 1893 the Senate redefined the standard yard as $\frac{3600}{25}$ metres. This makes the inch equal to 25.4000508 millimetres. Since 1951, however, the American Bureau of Standards has used the British inch and this is now to be made the legal United States standard.

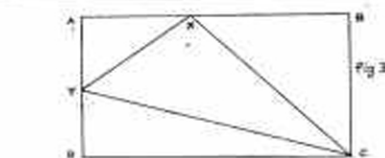
The metre used for the definition is not the French standard. It is the length of 1,553,164.13 wave lengths of Red Cadmium light. This figure was obtained as the average of a number of determinations which differed only in the last figure, and has been adopted as the international standard. It will probably soon be superseded by a definition in terms of Mercury light.

C.V.G.

Continued from page 215

CLUES DOWN

- Value of $(a+b)^2$ when $2a+b=31$ and $2b+a=32$.
- Difference between $n(n+h)^2$ and $n^2(n+b)$ when $n=13$ and $b=1$.
- ABCD are four successive points on a line. AD=24 in., AC=18.84 in., BD=18.61 in. What is BC?
- Radius of a circle drawn through the vertices of a rectangle 30 in. by 16 in.
- An even number of three figures which may be described as x to the power x .
- Number of cu. in. in two equal cubes of 1 ft. side.



- Four consecutive integers.
- How many m.p.h. is 105.6 ft p.s.?
- Hypotenuse of a right-angled triangle; base 60 in., area 750 sq. in.
- Fig. 3. Area of triangle XYC. AB=20 ft., BC=12 ft., AX=8 ft., and AY=6 ft. J.G.

Mrs. Cook was busy cooking the dinner when the heat supply to the stove suddenly failed. She found that the slot-meter required a shilling to be inserted to restore the supply. Unfortunately, she had no change so, handing Charlie a one-pound note, she said: "Run round to the grocer's and get me twenty shillings' worth of silver". On his return, Charlie handed his mother the change and ran out to play before she counted it. She found that there were twenty coins and that these certainly added up to twenty shillings; while there were half-crowns, florins and sixpences, however, there was not a single shilling! What coins did Charlie bring back? J.F.H.

STAMP COLLECTORS' CORNER No. 7



France 1937, 90c Copper Red.

RENE DESCARTES (1596-1650), was the man who developed the work on graphs. After education in a Jesuit college he became dissatisfied with accepted philosophy and turned to mathematics, as Pythagoras had done, as a means of understanding the universe. After a few years divided between mathematics and gambling he became a soldier. During this time he invented the idea of using the methods of algebra in the solution of geometrical problems. In 1621 he gave up soldiering and began a series of investigations into problems of physics and astronomy. He wrote a treatise which was ready for publication when Galileo was brought before the Inquisition. Publication was suspended, but at last, in 1637, with the encouragement of Cardinal Richelieu, "A Discourse on Method" was printed.

C.V.G.

FALLACY No. 27

The following problem with its solution was found by the Editor whilst looking through an old magazine, "The Universal Magazine of Knowledge and Pleasure" Vol. II, April 1748—

"A gentleman hath an oblong garden, whose length is 620 feet and breadth 400 feet, round which he would make a trench 15 feet deep, so as the earth flung up should raise the garden one foot higher. Qu. The breadth of the trench?"

In the May edition, the following solution is given—

620 feet long	620 feet long
400 feet broad	400 feet broad
248000 sq. feet in the G.	1020 half the compass
	2
	2040 feet in the compass
30600)248000 solid feet of earth	15 feet in depth
	30600 square in compass and depth.
	306) 2480
	220
	14
Feet 8, 1 wide	The trench must be 8 feet 1 wide.
And the answer is wrong. Can you follow the method, and show why it is wrong?	

It is, therefore, all the more surprising that the Babylonians and Egyptians could calculate areas and volumes and make astronomical calculations with considerable accuracy.

For many centuries numbers were used solely for the purposes of calculation. It was not until the Golden Age of the Greeks that the properties of numbers were considered and arithmetic was divided into calculation on the one hand and the theory of numbers on the other. I.L.C.

KNOWLEDGE INCREASES

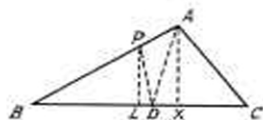


Fig. 1.

From a piece of fairly stiff paper—drawing paper will do very well—cut out a number of fairly large triangles such as ABC (Fig. 1). Fold it so that AC lies on AB, and then crease it plainly at the fold. The crease, AD, will show the bisector of the angle A. Flatten out the triangle again and form, in the same way, the crease bisectors of the angles B and C. Examine your handiwork and

you will see that the three bisecting creases appear to meet at one point (the incentre).

Now take another triangle; fold and crease it so that C lies on B. You will obtain the perpendicular bisector LP of BC. If you crease similarly to show the perpendicular bisectors of the other two sides, you will see again that the three creases appear to meet at one point (the circumcentre).

Using another triangle, fold and crease so that C lies at any point on BC. The crease will be at right angles to BC. If before creasing you adjust the fold so that the crease passes through A, you will obtain AX, the perpendicular height (or altitude). By refolding, you may obtain the perpendiculars from B and C to the opposite sides. Again the three creases appear to meet at a point (the orthocentre).

Finally, with a fourth triangle, make three creases from the corners to the middle points of the opposite sides. You will thus obtain the centroid, or centre of gravity, of the triangle.

The four points you obtain will all be different unless the triangle is isosceles or equilateral. A large triangle folded to show all the points may suggest to you that three of them lie in one straight line. Which of the four is the stranger? J.G.



A three-day cricket match by Sandra Noreliffe, Holme Valley Grammar School, Honley.

PENNY PLAIN, TWOPENCE COLOURED!

This is a game of "just suppose". Imagine that there are two kinds of numbers, very much like ordinary ones, one kind being red (R) and the other kind deep blue or indigo (I). The colour letters R or I can be put in front of figures so that R10 means "red ten" and I7 means "indigo seven". For the purpose of our game suppose that the numbers are in pairs, e.g., R10+I7, where the two sorts are kept separate just as 10x+7y would be written in algebra. In the rules of the game we can have addition and subtraction, just as in algebra, so that (R10+I7) - (R5 - I2) equals R5+I9.

The real fun of the game starts when we multiply or divide, because the colours, kept separate in addition and subtraction, now get mixed; suppose they sort themselves out, however, according to these simple rules: $R \times R = R$; $R \times I = I$; $I \times I = -R$; (take note of that minus sign!).

Let us try using these rules to multiply the two numbers mentioned above, that is, to find the meaning of (R10+I7) \times (R5 - I2). Taking the terms in order we get R10.R5 - R10.I2 + I7.R5 - I7.I2 which makes, by the rules, R50 - I20 + I35 + R14 or R64 + I15.

You can now try your hand at the game with the following:

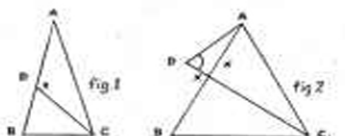
- (R4+I3) \times (R3+I2)
- (R3+I4) \times (R2+I3)
- (R4+I3) \times (R4 - I3)
- For division, first practice by dividing the products in a, b and c by their factors, then try (R9+I38) \div (R4+I3).

The principle of this simple game is of great service in many mathematical problems ranging from the solution of difficult equations to the practical one of designing a motor car silencer! J.F.H.

SENIOR CROSS FIGURE No. 28

CLUES ACROSS

- Value of $\frac{x^3-x}{x^2-x}$ when x is 40.
- Fig. 1. The angle x when B is 78°, AB=AC and AD=DC.
- The next number in the sequence 3, 8, 15, 24, 35, ...
- If 0.055 of a leap year is past, how many complete days remain?
- Value of $\frac{11ab+312}{5}$ when a=123 and b=456.
- Volume of rectangular block. Base is 19" \times 6". Total external surface area is 428 sq. in.
- Value of (4x+1) (x+6) when the first bracket is 6 less than three times the second.
- Fig. 2. The angle x. B is 63° AB=AC=CD and AD=AX.
- Value of x² when (x-10) (x+4) = (x-6) (x-4).
- How many lb. per sq. in. is 5.4 tons per sq. ft.?



(Continued on page 214)

Babylonian Number System

$\Upsilon = 1, 60, 60^2$ etc.

$< = 10$

$\Upsilon > = 100$

$\ast = \frac{1}{2}$

$\text{XX} = \frac{1}{3}$

$\text{XX} = \frac{1}{3}$

$\zeta = 0$

$\Upsilon = \text{minus}$

$\langle \Upsilon \rangle = 20 - 1 = 19$

$\Upsilon \zeta \Upsilon \Upsilon = 60^2 + 4 = 3604$

$\Upsilon \Upsilon \zeta \Upsilon \ast = 2 \times 60 +$

$5 \times 10 + 1 + \frac{1}{2} = 171\frac{1}{2}$

YYY

YYY = 9

YYY

later, 9 was written :-

YYY



Table of 3's as it would be written on a clay tablet

Egyptian Number System (hieroglyphics)

1 = one

10 = 10

100 = 100

$\frac{1}{10} = 1000$

1 = 10,000

$\frac{1}{2} = \frac{1}{2}$

11 = 12

$\frac{1}{12} = \frac{1}{12}$

All fractions except $\frac{1}{2}$ were unit fractions, i.e. fractions with numerator 1

11 100 $\frac{1}{2}$ =

$4 + 30 + 200 + 1000 = 1234$



Hieroglyphic for 6000
The meaning is "The Falcon King led captive 6000 men"

Egyptian multiplication 32×12

11 100	1	32×1
11 1000	11	64×2
111 100	111 /	$128 \times 4 /$
111 1000	111 /	$256 \times 8 /$
11 10000	1111 /	Sum 384

Egyptian multiplication was effected by a process of doubling and adding. In this ex. doubling twice gives 4×32 , three times gives 8×32 , and the sum of these gives 12×32 .

The early civilizations had systems of counting long before they wrote numerals and it was from the operation of counting that simple arithmetic evolved. The need to count was as natural to man as his need to communicate his thoughts in words. Our knowledge of early mathematics has been derived from documents which take us back about 5,000 years; and from these we find evidence of the existence of well developed number systems in Babylon, Egypt and the Orient.

The principles on which these number systems were based are similar in many ways, though there is a wide variation in the type of symbols used. The similarity of these systems lies mainly in the use, in varying degrees, of the additive principle. This principle is the one used in Roman numerals, where a number symbol is written as many times as is necessary to add up to the number required. For example, a single stroke (1) denoting the number one would be written three times (111) to denote the number three.

You will see from the illustration that most of these early systems had a symbol for one which was used to compile numbers up to nine and then a new symbol for ten, so that numbers could be expressed as so many ones and

so many tens. This division into tens was most probably a natural consequence of man's use of his fingers for counting.

In the early Babylonian system we find a positional notation used to a certain extent. Our modern number system uses this principle; for example, in the number 23 the position of the figure 2 tells us that it means two tens; in the number 234, the position of the figure 2 tells us to read two hundreds. The Babylonians used this idea beyond the number 59, for the symbol for 60 was the same as the symbol for one. The position of this symbol could make it mean 1, 60, 60^2 , etc., or $\frac{1}{60}$, $\frac{1}{60^2}$, etc.

This use by the Babylonians of the sexagesimal system, that is a number system based on powers of 60, may have arisen from their early belief that the year contained 360 days. Consequently, they divided a circle into 360 degrees and the fact that the radius can be stepped six times round the circumference gave the number 60 ($\frac{1}{6}$ of 360) a magic property. There is, however, no conclusive evidence for this belief.

With each of these early number systems the cumbersome notation made calculation, especially with fractions, a long and complicated process.