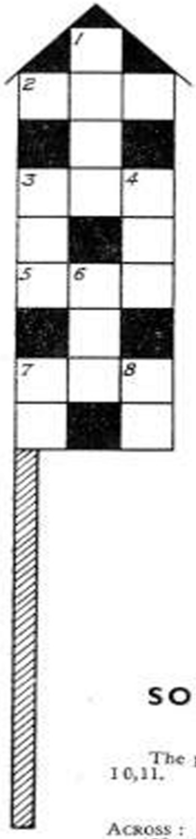
FUN WITH NUMBERS No. 4

Using each digit from 0 to 9 express zero in as many ways as possible.

R.H.C.

JUNIOR CROSS-FIGURE No. 23



ACROSS :

2. The number of 2/6d. Rockets that can be bought for " 8 down " pounds.
3. Third of " 6 down ".
5. The percentage decrease in length of a small Sparkler, 5 in. long, compared with a large Sparkler, 8 in. long.
7. Total number of Roman Candles stocked by a shopkeeper who has XL large ones, LXI medium ones and XC small ones.

DOWN :

1. Height, in inches, a Maroon bursts above the ground if its average rate of climb is 96 ft. per sec. and it takes 3 seconds.
3. Area, to nearest $\frac{1}{10}$ sq. in., of one side of a Catherine Wheel of diameter 6 in. Take $\pi = 3{\cdot}14$.
4. Weight, in lbs., of the Roman Candles in stock if they weight 4 oz., 3 oz., and $2\frac{3}{4}$ oz. (see 7 across).
6. This number follows from " 1 down ".
7. Square root of " 2 across ".
8. Cost, in pence, of a Jack-in-the-Box if six Star Shells and three Jack-in-the-Boxes or three Star Shells and five Jack-in-the-Boxes can be bought for half a guinea.

CHECK :

Total weight of the rocket is 4974·3. I.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 24

PUZZLING PAIRS

The pairs of numbers drawn were 1,3 ; 2,4 ; 6,7 ; 5,9 ; 8,12 10,11.

SENIOR CROSS-FIGURE No. 24

ACROSS : (1) 9·13 in. ; (3) 12 ; (4) 10·7 in. ; (5) 45 ; (6) 2.6 in. ; (7) 28 ; (8) 1·8 ; (9) 4·58 in. ; (11) 60° ; (12) 220 ft. per sec.

DOWN : (1) 93 ; (2) 315 ; (3) 1728 ; (5) 4840 (acre) ; (8) 1·82 sq. in. ; (10) 10.

FALLACY No. 24

Log $\frac{1}{2} = -\log 2$, therefore, when dividing by log $\frac{1}{2}$ in $\log \frac{1}{2} > 2 \log \frac{1}{2}$ the division is by a negative quantity, which changes $>$ to $<$.

MOVING IN THE BEST CIRCLES

Bill travels 3 m.p.h. faster than Chris so Bill does exactly one circuit more than Chris in 1 hour. Alan travels 9 m.p.h. faster than Chris so Alan does exactly three circuits more than Chris in 1 hour. Hence, after 1 hour, they will all three reach the same point on the track.

PEDALLING PACES

After 5 hours Johnny has travelled 40 miles and his sister has travelled 30 miles. On his return Johnny's speed relative to his sister is 14 m.p.h. and the distance between them is 10 miles, therefore, Johnny will meet his sister after a further $\frac{10}{14}$ hrs. $= \frac{5}{7}$ hr. During that time, his sister has travelled $\frac{5}{7} \times 6$ mls. $= 4\frac{2}{7}$ miles, therefore they will meet $34\frac{2}{7}$ miles from their starting point.

JUNIOR CROSS-FIGURE No. 22

ACROSS : (1) ·15 cm. ; (3) 42 ; (4) 24° ; (5) 8·32 sq. cm. ; (6) 37 ; (7) 47 ; (8) 10 in.

DOWN : (1) 1·287 ; (2) 14 ; (4) 2240 ; (6) 3·6.

I.L.C.

09402 52288 79710 89314 56691 36867 22874 89405

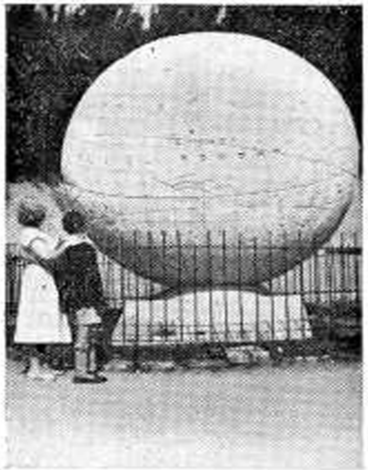
MATHEMATICAL PIE

No. 25 — *Editorial Offices :* 97 Chequer Road, Doncaster — OCTOBER, 1958

40 TONS OF SOLID GEOMETRY

The Great Globe at Swanage

Have you noticed how much help you receive from a good diagram when you are trying to solve a geometrical problem? You should make sure it is a good diagram, however, or you may find yourself led up the garden path by a slipshod construction.



Some of the geometrical fallacies that you meet depend on a free-hand diagram ; once you use a ruler and compasses the fallacy vanishes. It is easy enough for you to take a sheet of paper and drawing instruments and, with them, to provide a satisfactory diagram for a plane geometrical problem, because only two dimensions are involved. Suppose you were working in three dimensions, however, and had to prove some proposition concerning a sphere. You have no difficulty in folding paper to make a cube, but constructing a sphere is a different matter. Fortunately, we can sometimes use objects that may be handy, say tennis balls or even apples, to help form an idea of a spherical solid.

Mr. Hope-Jones, who sometimes contributes to MATHEMATICAL PIE, is well remembered for his lectures in which he used an apple to illustrate features of a sphere. The lucky boy or girl who helped in the demonstration usually got the apple as a reward when the lecture was over !

Our Earth is very nearly spherical in shape and there are many important features that we should know about—latitude, longitude, great circles, the tropics—and these can best be appreciated if we see them marked on an actual globe.

The globes that we have at home or at school are usually of modest size and some of the details are difficult to distinguish : in any case, the paint seems to wear off after a time and the details are lost. Neither of these disadvantages applies to the globe that Mr. George Burt caused to be made and erected at Durlston near Swanage, Dorset, in 1887. This globe is ten feet in diameter, and, as our photograph shows, the details are deeply chiselled so that they stand out clearly seventy years later. It was made in sixteen sections, the actual carving from Portland stone being done at the works of John Mowlem and Co. in Greenwich, London.

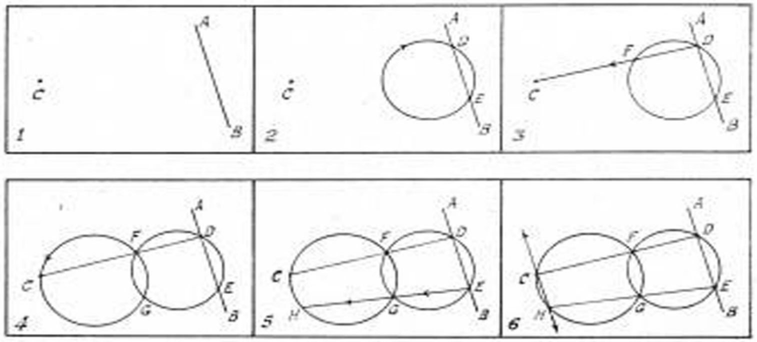
99725 24680 84598 72736 44695 84865 38367 36222

If you happen to be in or near to Dorset during your holidays, you will find it well worth while to call and have a look at the Great Globe of Swanage ; both your Mathematics and Geography teachers will be very much in favour of your doing so ! Even if you are not so good at these subjects, it is worth going to see just what 40 tons looks like in one solid lump. J.F.H.

PIED PIPER

A tobacconist bought a quantity of pipes at 2s. 1d. each and others at 4s. 1d. each. He spent £8 6s. 8d. on the pipes. How many of each did he buy? R.H.C.

CURIOUS METHOD OF DRAWING A PARALLEL, SHOWN IN STEPS



1. Given a line AB and a point C.
2. Draw any circle cutting the line at D and E.
3. Join the given point C to either of the intersections, say D. F is the intersection with the circle.
4. Draw any circle through C and F, meeting the first circle again at G.
5. Join EG and produce to meet the second circle at H.
6. Join CH, producing both ways. This is the required parallel. J.G.

POSTMAN'S KNOCK

How many different digits are needed by a builder in order to number all the 288 houses in a street? (Front doors only).

J.F.H.

AN ARTICLE OF CLOTHING ?

Using the same scale on both axes take values of x from 0 to 11 and values of y from 0 to 11, then draw the following as accurately as possible :—

(*a*) straight lines from (1,6) to (1,2) ; from (2,2)to (2,6) ; from (3,6) to (3,2) ; from (4,2) to (4,6) ; from (5,6) to (5,2) ; from (6,2) to (6,6) ; from (7,6) to (7,2) ; from (8,2) to (8,6) ; from (9,6) to (9,2) ; and from (10,2) to (10,6).

(*b*) straight lines from (1,5.6) to (10,5.6); from (1,5.4) to (10,5.4); from (1,2.6) to (10,2.6) ; and from (1,2.4) to (10,2.4).

(*c*) semicircles upwards from (10,6) to (8,6) centre (9,6) radius 1 unit ;from (8,6) to (6,6) centre (7,6) radius 1 unit ; from (6,6) to (4,6) centre (5,6) radius 1 unit ; and from (4,6) to (2,6) centre (3,6) radius 1 unit.

(*d*) straight lines from (9,2) to (10,2) ; from (1,6) to (1.5, 7.5) ; and from (2,6) to (1.5, 7.5).

(*e*) straight lines from (1,0) to (0,4) ; from (10,0) to (11,4) ; from (2,8) to (0,11) ; from (4,7) to (3,11) ; from (4,7) to (3,11) ; from (6,7 to (7,11) ; and from (9,8) to (11,11).

(*f*) semicircles downward from (1,2) to (3,2) centre (2,2) radius 1 unit ; from (3,2) to (5,2) centre (4,2) radius 1 unit ; from (5,2) to (7,2) centre (6,2) radius 1 unit ; and from (7,2) to (9,2) centre (8,2) radius 1 unit.

For solution to " An Article of Clothing ? " see page 194. I.H.

62609 91246 08051 24388 43904 51244 13654 97627

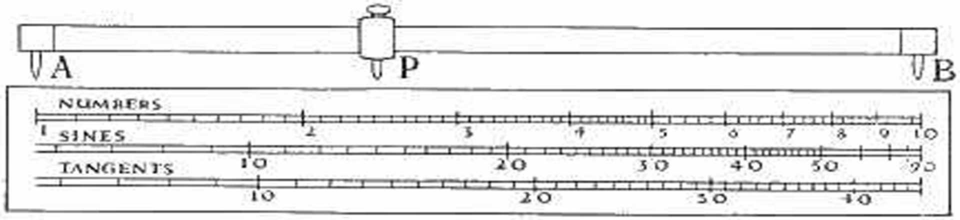
MATHEMATICAL INSTRUMENTS No. 5

Gunter's Scales

In 1614 Lord Napier of Merchiston published his invention of logarithms. Six years later Edmund Gunter, Professor of Astronomy at Gresham College, London, designed what he called " logarithmic lines of numbers ". This invention consisted of a boxwood scale, usually about two feet long, on which was marked logarithmic scales of numbers from 1 to 100, sines, tangents, versines and other functions used in navigation. A logarithmic scale of numbers means that the distance from the mark "1" at the end of the scale to the mark for any other number is proportional to the logarithm of that number.

To multiply two numbers using logarithms tables we look up the logs of the two numbers, add them together and then look up the antilog of the sum. Using Gunter's Scale there is nothing to write down. For example to multiply 5·6 by 2·7 a pair of dividers is opened so that the points span the distance from 1 to 2·7 on the scale. One point is then placed on the mark for 5·6 and the other point shows the product. For numbers not between 1 and 10 the decimal point is ignored when using the dividers and then inserted in the result by inspection. As the scales were large, some were six feet long, beam dividers were used.

A later improvement was a special type of beam dividers with two fixed points and one movable point. These enabled scales to be used with graduations from 1 to 10 only. In the illustration P is the movable point and A and B are the fixed points. The distance between these points is equal to the distance between 1 and 10 on the scale. To multiply 3·5 by 1·8, the point A is placed on the mark 1 and P moved to the mark 3·5, then the beam is moved along and A placed on the mark 1·8. P now shows the product, 6·3. If we try to multiply 3·5 by 4·8 in the same way we find that P comes beyond the end of the scale, but by placing B, not A, on the mark for 4·8, P is brought a scale's length to the left and points to 1·68. Therefore the required product is 16·8. You can try this on the scale illustrated, using marks on the edge of a sheet of paper to represent the pointers. C.V.G.



MATHEMATICAL LOVER

£ s. d.

What like fractions of a pound, a shilling and a penny when added together will make exactly one pound? R.H.C.

55936 34568 17432 41125 15076 06947 94510 96596

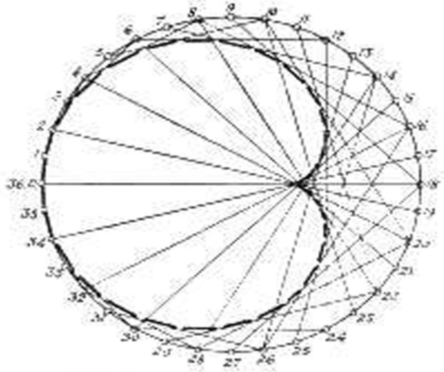


Fig. 9
Embroidered Cardioid.

number exceeds 36, we subtract 36 from it; thus, hole 24 is joined to hole 12 (*i.e.*, 48—36).

Any formula for hole spacing will yield a design, and there is here much room for research, using straight lines and circles only.

Do not be content with mere copying of designs: try to invent some for yourself. Finally, if you cannot understand the proofs of your theorems, try stitching the diagrams in coloured string. J.G.

SEVENTH HEAVEN?

The three sons of a deceased wine merchant were left 21 equal casks of which 7 were full of wine, 7 were half full and 7 were empty. These they had to divide among themselves, without pouring any wine from one cask to another, but in such a way that each brother received the same amount of wine and the same number of whole casks. How should they do it?

J.F.H.

SEE HOW THEY RUN

Three fierce dogs, Alpha, Beta and Gamma, stand at the vertices of a large equilateral triangle. At a given signal, Alpha chases Beta, Beta chases Gamma and Gamma chases Alpha, all with the same speed. Sketch the paths run by each of the dogs. J.G.

ATENSION!

An engine coupled to six equally loaded trucks is pulling them at a steady speed. In which of the six couplings is the tension greatest?

If the tension in the first coupling is 100 lb., what are the (theoretical) tensions in the other five? J.G.

STAMP COLLECTORS' CORNER No. 5

GERBERT (?—1003) was born in Auvergne. He studied mathematics in Spain and later taught in Rheims. He became Bishop of Rheims and of Ravenna. He was appointed Pope by Emperor Otto III and took the name Sylvester II. He published books on geometry, on division in arithmetic, and on the use of the abacus. His activities stimulated interest in mathematics and encouraged western scholars to search for and translate Greek and Arabic manuscripts.

Hungary 1938,
1 f. deep violet,
10 f. red orange.

C.V.G.

Solution to "An Article of Clothing": A jumper.

66727 82398 64565 96116 35488 62305 77456 49803

QUOTATION

"*Without geometry and arithmetic, nobody can hope to become a painter*".
(Giovanni Paolo Lomazzo, A.D. 1584).

Would Picasso agree?

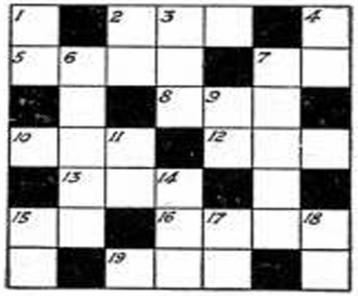
HISTORY OF A DEGREE

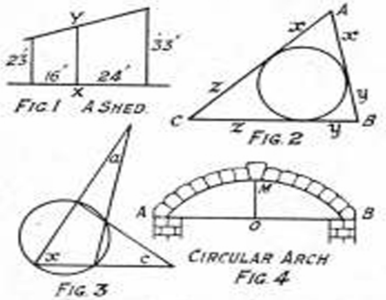
In early Babylonian reckoning the year consisted of 360 days and it is believed that this gave rise to the division of the circle into 360 equal parts. One reason for this belief is that the Babylonians were aware that the radius of a circle could be stepped round the circumference exactly six times. Each of these parts contained 60 divisions, which was a mystic number to the Babylonians.

The division of the circle into 360 parts was certainly practised by Ptolemy (c.150 A.D.) and other Greek astronomers, and $\frac{1}{360}$ of a circle was taken as a unit of circular measure, known by the name *μοιρα*, meaning step. The Arab scholars translated the name as *daraja*, meaning ladder, scale or step; and this is believed to be the original form of the word "degree".

I.L.C.

SENIOR CROSS-FIGURE No. 25





ACROSS

2. Value of a^2+b^2 if $10a+b=222$ and $10b+a=42$.
5. Log log of a million.
7. Fig. 3: The angle x when $a=35°$ and $c=47°$.
8. The square root of the first integer greater than 200000 which is a perfect square.
10. The angle ACD when ABCD are consecutive corners of a threepenny piece.
12. Fig. 2: AB = 11, BC = 13, CA = 12. Lengths of the tangents in descending order.
13. Increase in the value of $\frac{n(n+1)(2n+1)}{6}$ when n increases from 20 to 21.
15. Value of v if $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$; and $u=60$, $f=240$.
16. Subtract 78, this becomes a perfect square. Add 79 to the original number and we obtain the next highest square.
19. The area of a rectangle, perimeter 60, diagonal three times the square root of 58.

DOWN

1. The length XY in fig. 1.
2. Value of a^2+b^2 if $(a+b)^2 - (a^2 - b^2) = 40$ and $(a^2 - b^2) + (a - b)^2 = 56$.
3. Increase this number by a quarter of itself, then increase the result by 10% and you will obtain 1133.
4. Fig. 4: The radius of the inner arc. The inner span AB is 40 ft. and OM, the "rise" at the middle, is 8 ft.
6. Square yards in 15[illegible] acres.
7. Value of $(a+b)^2 - (a-b)^2$ when $a=193$ and $b=63$.
9. The following results are obtained from the formula $y=mx+c$: What number should occupy the blank space?

x	2	4	9
y	12	22	

11. Fig. 2: If 0 is the centre of the circle and the angles AOB and AOC are 120° and 123°, what is the angle A?
14. Number of dots on a set of dominoes, the highest of which is double six.
15. Value of $\frac{a^3+b^3}{a+b}$ if $a-b=8$ and $ab=20$.
17. The cube root of $x^3 - 3x^2 + 3x - 1$ when $x=20$.
18. Value of x: $(a-x)^2+(b-x)^2=(c-x)^2$ when $a=20$, $b=21$ and $c=29$.

J.G.

80797 71569 14359 97700 12961 60894 41694 86855

Mathematical Embroidery

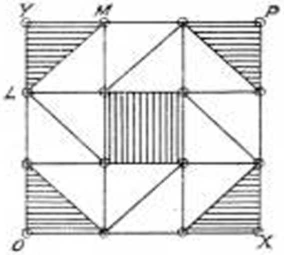


Fig. 1
An 18th Century design based on squares.

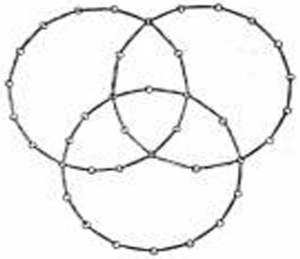


Fig. 3
Three intersecting circles.

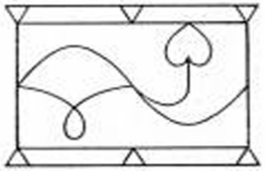


Fig. 5.
Part of a border composed of Sine curve, Cardioid and Folium.

The object of this article is not to enable the reader to obtain a knowledge of the art of embroidery. It is to present some interesting results that may be obtained by a simple method of producing designs by stitching, or, more correctly, by passing string through holes pierced in a background of cardboard or thin plywood. For the best results the string (or wool, or thread) should be thin and coloured.

It has often been said that without a basis of mathematics—geometry and arithmetic in particular, no really satisfactory design in painting, music or architecture is possible.

The holes are usually evenly spaced on straight lines or circles. They may be made by a stout needle or a sharp bradawl. Another needle is used to take the thread. You think of a design, draw it if necessary on the cardboard, plan the position of the holes, knot the end of the thread to anchor it and sew from hole to hole with a single thread.

The chief rule to follow when actually executing the work is that when you have stitched through a hole, then, when on the reverse side, stitch through the adjacent hole. You may be assured that, with care, the results, especially when colours are skilfully used, will be very interesting.

Figures 1, 2 and 3 show simple examples. Small circles represent holes made when the diagram is completed. Notice the simple plan used in figure 4. The next, figure 5, shows the use of well-known mathematical curves as a basis for design. Many others might well be used. Figure 6 shows how to stitch the " curve of pursuit ". It is the curve (*i.e.*, locus) formed when a dog at the bottom left hand corner pursues a man who runs along the top edge of the rectangle, their speeds being constant. The dog naturally always points towards the man, wherever he may be. The lines join the dog to the man throughout the motion. The segments of the curved line are all equal, so that though the holes for the various positions of the man may be pierced before the curve is obtained, the holes for the dog, though equally spaced, must be made during the construction of the path.

Figure 7 explains itself ; the four curves are parabolas. Figure 8 shows the result of joining equally spaced holes in straight lines and circles. It is a most satisfying design when stitched. Figure 9 is based on the simple mathematical formula y – 2x. By this is meant that hole 1 is joined to hole 2 ; hole 2 to 4 ; hole 3 to 6, etc., hole 17 to hole 34. When, however, the double

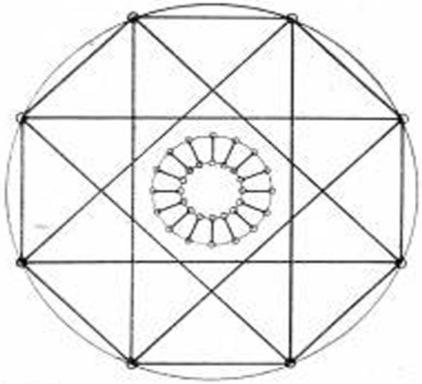


Fig. 2
Medieval design using octagons, squares, triangles.

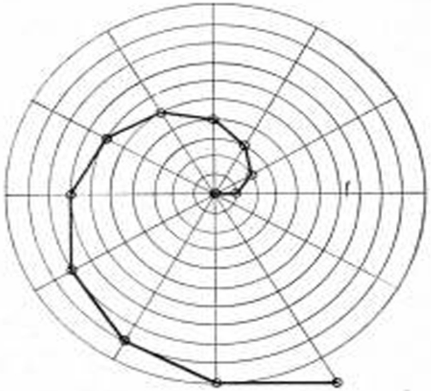


Fig. 4
The Spiral of Archimedes.

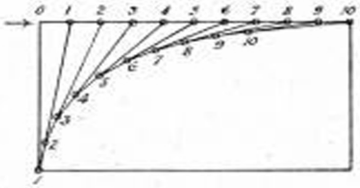
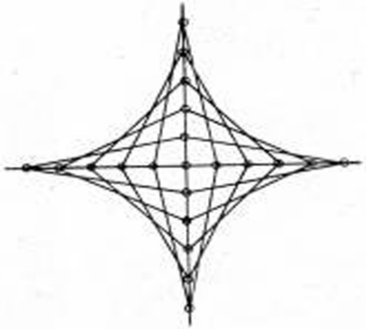


Fig. 6
Curve of Pursuit.



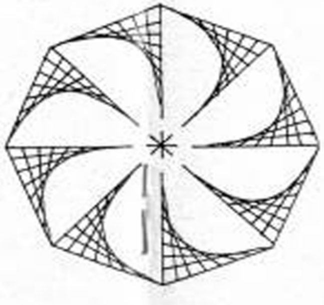
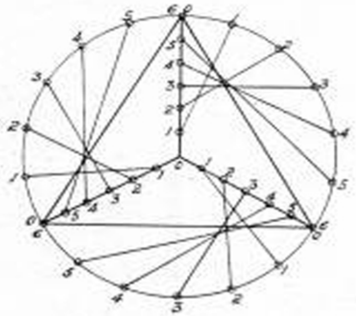


Fig. 7 *Curves formed by lines. These are effective if different colours are used.*



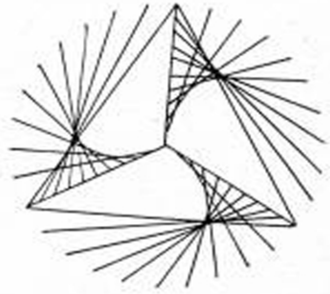


Fig. 8 *A striking example of balanced rhythm.*

58484 06353 42207 22258 28488 64815 84560 28506 01684 27394 52267 46767 88952 52138 52254 99546