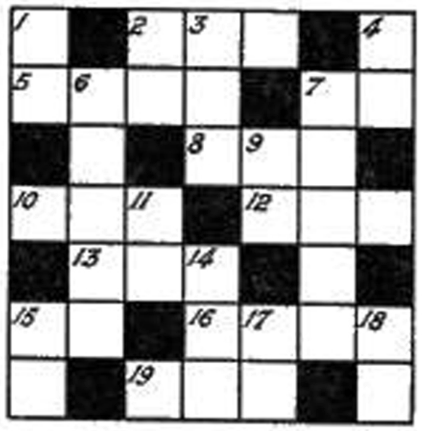
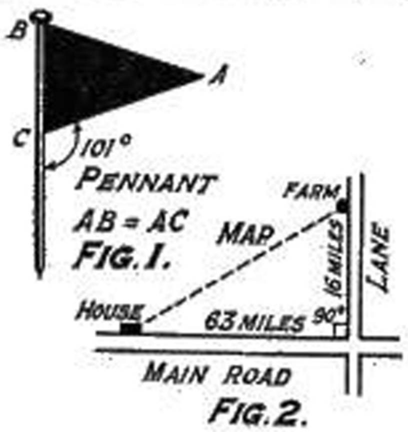
JUNIOR CROSS-FIGURE No. 21

Suggested by C. Ovendon, 5th Form, Grammar School, Dartford, Kent.



ACROSS

2. Number of centimetres in an inch.
5. 2s. 3d. as a decimal of £1.
7. The square root of 1156.
8. DXLI
10. ab^2 if $2a - 11b = 25$ and $3a - 25b = 12$.
12. A multiple of "2 down".
13. A perfect square.
15. How long will it take a class of 18 girls to bake 18 cakes if 25 girls can bake 25 cakes in 25 minutes?
16. The reciprocal of the square root of "13 across"
19. Supplement of 35°.



DOWN

1. Average of "7 across" and "15 down."
2. Angle A of pennant (see fig. 1).
3. L.C.M. of 111 and 185.
4. A boy goes by road for a holiday on a farm, later his father flies direct. How much farther, in miles, does the boy travel? (see fig. 2).
6. The cube of "15 across".
7. Thousands of seconds in a year.
9. c if $5(c - 20) - 2(c + 11) = 1$.
11. $5(\frac{2}{3} - \frac{4}{15})i \div \frac{1}{14}(2\frac{2}{15} + \frac{1}{3})$.
14. The area, n square miles, of the ground between the main road, lane and direct route. (see fig. 2).
15. Pounds in a quarter.
17. The shortest distance, in miles, between the house and the farm (see fig. 2).
18. Angle B of pennant (see fig. 1).

Check clue: The sum of all the digits inserted is twice the difference between "4 down" and "18 down."

I.H.

SOLUTIONS TO PROBLEMS IN ISSUE No. 22

HANDY PROBLEM

The time of departure was $36\frac{12}{13}$ minutes past 4.

Let the time be x minutes past 4, then the hour hand will be at $\frac{x}{12}$ minutes after 4. This means that the time of return will be at $20 + \frac{x}{12}$ minutes past the hour, so the hour hand in this case will be at $\left(20 + \frac{x}{12}\right) \times \frac{1}{12}$ minutes after 7. The equation in x will therefore be $x = \left(20 + \frac{x}{12}\right)\frac{1}{12} + 35$.

SENIOR CROSS-FIGURE No. 22.

Across: (1) 4·7; (3) 35°; (4) 73·5; (6) 36°; (7) ·162; (9) 70·7; (11) ·04; (12) 21·9 cm.; (13) 4·9; (14) 72.

Down: (2) 7760; (3) 351; (5) 324; (6) 3·75; (8) 6097; (10) 729.

FALLACY No. 22

60 oranges at 5 for 2d. is the same as 36 at 3 for 1d. and 24 at 2 for a 1d. This is 6 more at 3 for 1d. and 6 fewer at 2 for 1d. than in the first arrangement; giving 1d. less on the sale.

COPS AND ROBBERS

35 policemen must be dispatched from A in order to catch the robber.

BON-FIRE NIGHT BARGAIN

Let x be the number of sparklers in the bundle. Since these cost 1s. the price was $\frac{12}{x}$d. each, i.e. $\frac{144}{x}$d. per dozen. When 2 more were added this became $\frac{144}{x+2}$ d. per dozen, the equation in x is therefore $\frac{144}{x} - \frac{144}{x+2} = 1$, which gives $x = 16$.

JUNIOR CROSS-FIGURE No. 20

Across: (1) 819; (3) 17; (5) 3·645; (7) 66; (9) 21; (13) 6080; (14) 117.

Down: (1) 816; (2) 93; (3) 14; (4) £750; (6) 65°; (8) 62; (10) 167 sq. in.; (11) 180; (12) 31.

I.L.C.

19652 85022 21066 11863 06744 27862 20391 94945

MATHEMATICAL PIE

No. 23 — Editorial Offices: 97 Chequer Road, Doncaster — FEBRUARY, 1958

SOME IMPORTANT CURVES IN MATHEMATICS

3—The Conchoid

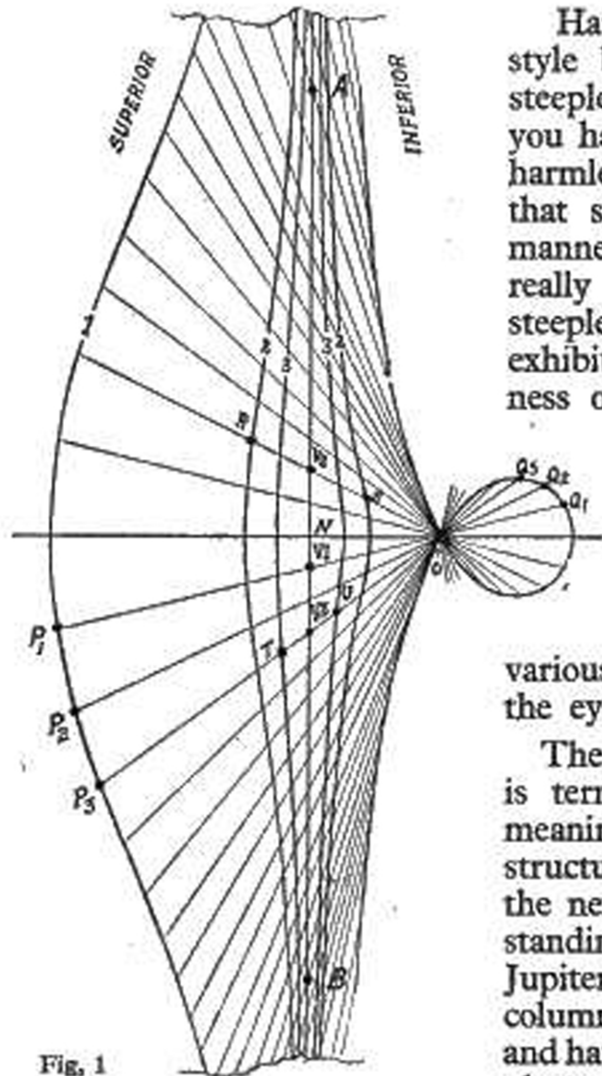


Fig. 1

Have you ever looked at a classical-style building or a church with a tall steeple? If you have there is a chance that you have been the victim of a mild, but harmless, deception. Those tall columns that seem to taper in such a uniform manner from the base to the capital are really slightly convex in outline. The steeple, apparently a perfect pyramid, may exhibit a similar departure from straightness on its flanks.

The architect deliberately incorporates this slight convex curve in what would otherwise be long straight runs to correct for the appearance of concavity that the outline of the various parts would normally convey to the eye.

The convexity of the shaft of a column is termed *entasis*, from a Greek word meaning stretching; it has been applied to structures since ancient times. Fig. 2, on the next page, shows some columns still standing in the remains of the Temple of Jupiter at Baalbec in the Lebanon. These columns are 63 feet from base to capital and have a diameter of about $7\frac{1}{2}$ feet. The photograph has been taken at such an angle that the swelling is plainly seen in the gap between the third and fourth columns.

The Temple of Jupiter was built in A.D.190, but entasis was used much earlier. The sides of the 6 foot diameter columns of the Parthenon in Athens, which was completed in 438 B.C., deviate from a straight line by about an inch in their length of 34 feet; the greatest swelling is at about one third of their height.

Why does entasis make these columns and other details "look right"? One explanation would be that our eyes cannot take in all the bulk of a large building even when viewed from a distance. In consequence, we tend to scan the view by slight movements of our eyes or head and may thereby

81647 06001 61452 49192 17321 72147 72350 14144

introduce an effect like that of a mildly distorting mirror, an effect that is corrected by the entasis.

The question as to what is the correct amount of convexity is met by using the curve known as the *Conchoid* (Greek *konchoeidēs* from *konchē*, shell and *eidos*, form). This curve (Fig. 1) was invented by Nicomedes between 240 and 210 B.C. and is very easy to construct. Its essentials are a straight line AB and a point or pole O not on the line. A whole series of radiating lines may be drawn through O cutting AB in V_1, V_2, V_3, etc., including ON perpendicular to AB. If, on the rays, equal lengths are marked off from the points V where the rays cut AB so that $V_1P_1 = V_1Q_1$ a series of points $P_2, P_3, \ldots$ and $Q_2, Q_3, \ldots$ will be found sufficient to describe the curve No. 1. The points $P_1, P_2, P_3, \ldots$ are on the *superior*, and $Q_1, Q_2, Q_3, \ldots$ are on the *inferior* parts of the curve. In curve No. 1 V_1Q_1 is greater than ON the inferior part therefore contains a loop. If smaller distances are taken on either side of the points V, say $RV_2 = V_2S$ or $TV_3 = V_3U$, both less than ON, then curves such as No. 2 and No. 3 are obtained with no loop. For the outline of a column, a very slender conchoid, more slender even than No. 3 would be used.

Fig. 2

The conchoid is a useful curve in other respects; the ancients used it for inserting two mean proportionals between two given numbers and for the related problems of the duplication of the cube, the trisection of an angle and the quadrature of the circle. Some of these solutions may be illustrated in future issues of Mathematical Pie together with simple instructions for making a machine that will draw a conchoid.

J.F.H.

FOOD FOR THOUGHT

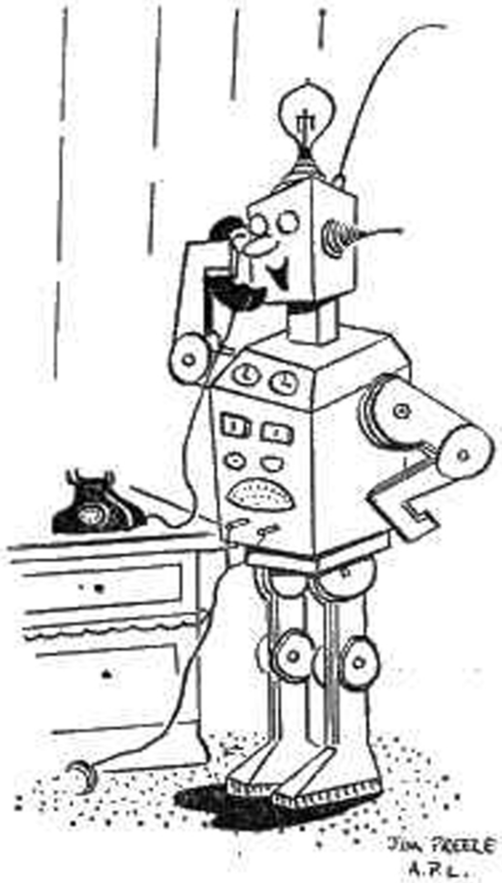
The day the power failed in the school kitchens, John, Roger and Peter went to the little café down the road for their lunch. They looked at the menu which read:

Fried Egg and Ham	1/6
Fried Egg and Chips	1/7
Ham and Chips	1/8
Fried Egg, Ham and Chips	2/2

John said, "I'm not very hungry, a Fried Egg would suit me, but it doesn't say how much." Roger, remembering that morning's algebra lesson, remarked: "That's easy, it's just a lot of simultaneous equations. Take No. 1 from No. 3 and you get Chips minus Fried Egg equals 2d. Substitute in No. 2 and you get Fried Egg costs $8\frac{1}{2}$d." "That can't be right," said John. "Take No. 3 from No. 4 and you get Fried Egg costs 6d." Peter, who was reflectively munching a bread roll taken from the tray in the centre of the table, said: "You are both wrong; a Fried Egg alone will cost you 11d."

His was the correct price; why?

J.F.H.



"Central Press Features"

"If you forget my number—just think of the square root of 955815164964"

MUTE ROOTS

Any number which can be expressed exactly as a ratio of two integers is called a 'rational' number, and, if this is not possible, the number is then called 'irrational.' Hence, $\sqrt{2}$ is an irrational number.

Pythagoras, Plato and many other Greek philosophers turned their minds to the study of such numbers as $\sqrt{2}$, considering it as the length of the diagonal of a square of side 1 in.

Greek mathematicians of the Alexandrian school used the word 'logos' (a word) as a technical expression for a rational number. Another Greek word 'alogos', meaning 'without a word', was used to describe a number which was irrational.

The Arab scholar Al-Khowârizmî translated alogos literally using an Arabic word meaning deaf or mute. Hence he would classify $\sqrt{2}$ as deaf or mute. Later, European mathematicians rediscovered these works, along with many other Arab documents, and they translated the Arabic word for deaf or mute by the Latin 'surdus', meaning 'deaf'. From this we have come to use the name 'surd' for an irrational number.

I.L.C.

GOING FOR A SAIL

Every day at noon a ship leaves New York for Southampton and at the same instant a ship leaves Southampton for New York. Each trip lasts exactly 144 hours, or six days. How many ships from Southampton will each ship from New York meet?

J.F.H.

"SEASON TICKET" PROBLEM

This problem produced a very large entry, with 149 correct solutions. Most of the rejected solutions were disqualified for the same reason, namely, not conforming to the restriction that no digit be repeated.

The ordinary level competitors on the whole were very even in quality, although the following (awarded book tokens) were above average—

J. Thewlis, Roysse's School, Abingdon, Berks.
Stella Sterman, Bournemouth Grammar School for Girls.
Angela Greenwood, Cheltenham Ladies' College.
Ann Jones, Hereford High School for Girls.

In addition, I could scarcely resist the plea of A. Barham, Bablake School, Coventry, who said—

"Nine thousand, eight hundred and one
Is the square of ninety nine.
If this is so, and it is I know,
A book-token will maybe be mine."

The more advanced investigation resulted in some twenty excellent solutions, combining ingenuity with thoroughness, whilst many others, although correct, were very tortuous. Book tokens go to—

K. G. Whitehead, The Judd School, Tonbridge, Kent (also an excellent late solution to the "Square Numbers" problem).
J. Dawes, St. Paul' School, London, W.14.
E. C. Lance, Dulwich College, S.E.21.
P. J. Morley, Solihull School, Warwickshire.

We welcome also solutions from Malta and Gibraltar, and once again thank many adult solvers for their interest; it would be invidious to mention these, although they may be assured that their camaraderie is highly appreciated.

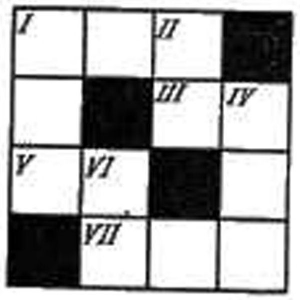
T.M.

CHARLIE COOK

Harry Hook, having been appointed Fourth Form monitor, was given a tray of inkwells and told that $\frac{1}{3}$ of them were for Form IV.A, $\frac{1}{4}$ for IV.B and $\frac{1}{5}$ for IV.C. Having counted the inkwells and found 47, he was at a loss how to divide and share them out until he met Charlie Cook at lunch break. "Leave it to me," said Charlie, "I know where the inkwells are kept." So, while no one was looking, he borrowed 13 inkwells and added them to Harry's. They then took $\frac{1}{3}$ of the total = 20 for IV.A, $\frac{1}{4}$ = 15 for IV.B and $\frac{1}{5}$ = 12 for IV.C. Since these totalled 47, Charlie was able to sneak back and replace the borrowed 13. Charlie, of course, had no conscience, but Harry is still worried. Can you see why? J.F.H.

ROMAN CROSS-FIGURE—II

Place Roman Numerals in the squares



ACROSS:

- I. Square the reverse of III across.
- III. Reverse the sum of V across, IV down and VI down.
- V. Square root of the sum of IV down and the reverse of VI down.
- VII. Difference between IV down and the reverse of VI down.

DOWN:

- I. Divisible by the reverse of V across.
- II. The product of V across and IV down.
- IV. A perfect square.
- VI. V less than the remainder on dividing I down by one quarter of III across.

I.H.

STAMP COLLECTORS' CORNER No. 4

Gaspard Monge (1746-1818), when a student at a military academy, invented descriptive geometry, now usually called geometrical or machine drawing. He was made a professor of the academy and developed many geometrical methods and constructions, but for 15 years geometrical drawing remained a closely guarded military secret. He profoundly affected the course of history by his improvements in the design and manufacture of cannon which nearly made Napoleon master of the world.

C.V.G.

France 1953
18 fr. + 5fr. Dark blue

THOUGHT READING

Talking to David about thought reading, John said he would give a demonstration. He asked David to think of a number; silently David chose 5,238. John then requested him to add a zero at the end, subtract the original number, and add 54.

David was then told to omit any one of the digits in the resulting number, and to call out the remaining ones. When he said these were 4, 1, 9, 6, John promptly informed him that the missing digit was 7.

John could have told David to add any multiple of 9 instead of 54, and still have been able to discover the missing digit.

How did he know?

82796 79766 81454 10095 38837 86360 95068 00642

CURVED OR STRAIGHT?

Draw an isosceles triangle ABC with A the vertex; AB = AC = 5 in. and BC = 4 in.

Divide AB and AC into 10 equal parts and number the points of division on AB in order from 0 to 10, 0 being at A and 10 at B. Now number the points of division on AC from 0 to 10, but let 0 be at C and 10 at A.

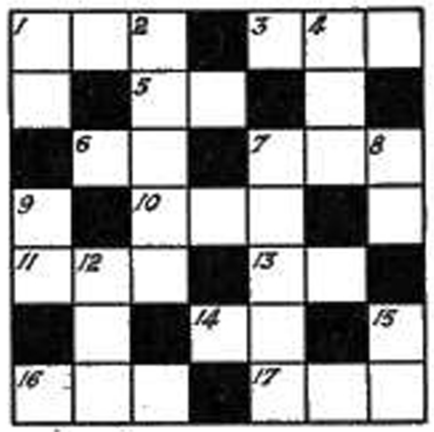
With a ruler join all the points on AB and AC which have equal numbers. You will see that the joining lines appear to enclose a curve. By sketching this curve in coloured pencil you will obtain a figure which may remind you of certain church windows, with an arch which is a parabola, and to which the lines are tangents. The curve is called the envelope of the lines.

J.G.

FROM AN ANCIENT BOOK

"Best of clocks, how much of the day is past?
There remains twice two-thirds of what is gone." J.F.H.

SENIOR CROSS-FIGURE No. 23



ACROSS

1. The square of a root of $7x^2 + 161x + 924 = 0$.
3. Area of the inscribed circle of ABCDE (see fig. 1).
5. The annual income, in pounds, from either investment (see 11 across).
6. Angle QSR (see fig. 2).
7. Diameter of the circumcircle of the pentagon (see fig. 1).
10. 35% of the product of "6 across" and "1 down".
11. The value, in pounds, of 20 $4\frac{3}{8}$% £5 shares if by selling 270 of them and reinvesting the proceeds in $3\frac{1}{2}$% stock at 87, there is no change of income.
13. Angle ACE (see fig. 1).
14. Diameter of circle, given PS = 8 cm. (see fig. 2).
16. Area in sq. cm., of ABCDE (see fig. 1).
17. The number of miles per gallon of petrol a car can travel if it goes 15 kilometres per litre. Take 1 cm. = ·394 in. and 1 litre = ·220 gal.

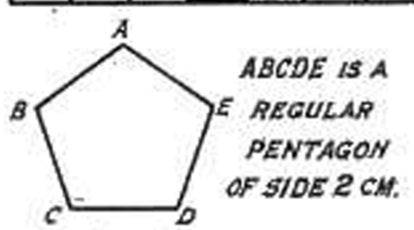


FIG. 1.

DOWN

1. The value of y if $\frac{1}{7(y+7)} - \frac{3(3y+4)}{9y+11} + 1 = 0$
2. The sum of the first 19 terms of an arithmetic progression whose 2nd term is 12 and 5th term is 324.
4. The larger root of $z^2 - 4z - 47 = 0$.
7. The sum of the first 9 terms of a geometric progression whose 2nd term is 12 and 5th term is 324.
8. $\frac{\cdot 259(\cdot 715 - \cdot 286)}{11\cdot 1(\cdot 715 + \cdot 286)}$.
9. The next number in the sequence 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, . . .
12. Angle ABC (see fig. 1).
15. The number of seconds it will take a rocket travelling at 2200 ft./sec. to reach an aeroplane flying at 500 m.p.h. in the same straight line away from it, the distance between them being 15 miles.

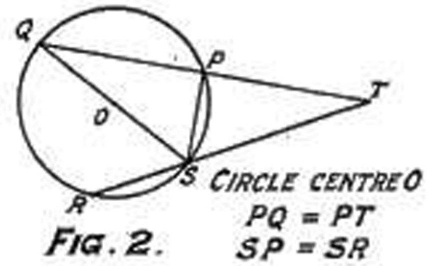


FIG. 2.

Check clue: The sum of all the digits inserted is a palindrome.

I.H.

43852 33239 07394 14333 45477 62416 86251 89835

JUST RIGID

At first sight one would think that a bridge could not possibly be over rigid and that one which was only just rigid might not be very safe. In fact excessive rigidity may lead to trouble. Engineers use the expressions over rigid and just rigid in ways which can be explained by the structures shown in figures 1 to 4.

Figure 1 represents two bars AP and PB, fastened together at P by a bolt which acts as a hinge, and screwed to a board at A and B. (An empty circle, O, is used to show that two parts are fastened together but not otherwise fixed; and a solid circle, •, is used to show that a point of the structure

FIG. 1

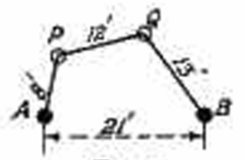


FIG. 2

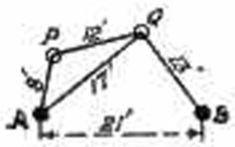


FIG. 3

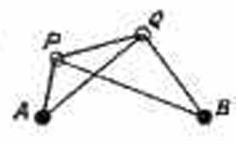


FIG. 4

is fixed.) Now if we let y represent the length of the perpendicular from P to the line AB and x the distance of the foot of this perpendicular from A, we can use Pythagoras' theorem to obtain two simultaneous equations $x^2 + y^2 = 20^2$, $(21 - x)^2 + y^2 = 13^2$. Two simultaneous equations in two unknowns can be solved. In this case we find that $x = 16$ and $y = \pm 12$. The choice of sign for y simply means that P may be either 12 feet above AB or 12 feet below AB. P, therefore, must be in one of two definite positions, and this simple structure is said to be delisminate or just rigid.

In figure 2 we need four measurements to fix the positions of P and Q, but Pythagoras' theorem will give only three equations. We cannot find definite positions for P and Q and this structure is not rigid. It can be made rigid by adding another bar as in figure 3. Pythagoras' theorem will now give four equations from which four unknowns can be found.

If we try to improve the structure by joining B to P as well as A to Q, as in figure 4, we have what an engineer calls a redundant or over rigid

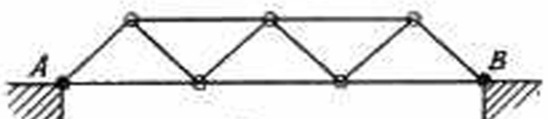


FIG. 5

FIG. 6

structure. We still need to find four measurements to fix the positions of P and Q, but we have, from Pythagoras, five equations which these four unknowns must satisfy. Now we can, given time enough, find the four unknowns from the first four equations and would be very lucky if we found that the solutions fitted the fifth equation. At first sight this might not seem very important, all we have to do is to make BQ exactly the right length and everything will fit together.

Unfortunately the parts of a structure do not remain of constant length. When loads are applied some parts are extended and some are compressed and all the parts expand or contract with changes of temperature. A Warren girder bridge built as in figure 5 would be over rigid (10 unknowns in 11 equations). The steel would expand or contract when the temperature changed but the distance between the supports would remain constant, so the bridge would either buckle or snap. To avoid such a

69485 56209 92192 22184 27255 02542 56887 67179

disaster one end only of the bridge is fixed and the other end left free to slide on its support, figure 6. This gives another unknown, the distance AB, so that the bridge is now only just rigid.

It is interesting to look for expansion joints and hinges on steel bridges. The two bridges shown in figures 8 and 9 look alike but structurally they are quite different. The bridge in figure 8 is a "three hinged arch". The curved ribs AP and PB behave very much in the same way as the stones of the masonry arch in figure 7 and are supported by oblique forces at A and B. In the masonry bridge the roadway is supported by the courses of stone which rest on the arch, while in the steel bridge the flexible roadway hangs on steel rods (suspenders).

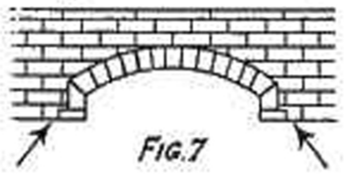


FIG. 7

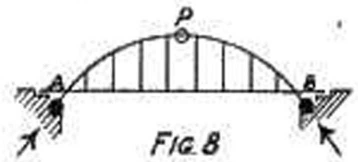


FIG. 8

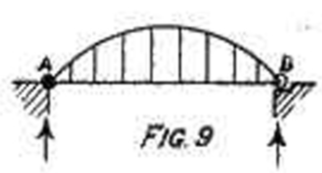


FIG. 9

The stone bridge is not much affected by changes of temperature. It will crack if the abutments settle, but small cracks can be mortared up again. To prevent temperature changes or settlement of the abutments destroying the steel arch a hinge is provided at P, and the ends of the roadway are free to slide on the abutments. This bridge is now very much like the simple structure of figure 1.

The bridge in figure 9 is called a "bow-string girder". It is a huge built up girder, deep at the centre where the bending stresses are greatest and tapering towards the ends. There is no hinge at the top of the "arch" and the roadway does not hang idly, like the roadway in the three hinged arch, but acts as a tie, holding the two ends together, so that there is no sideways thrust on the supports. To allow for temperature changes one end is fixed and the other slides on its support.

Some bridges are built as "two hinged arches". They are like the bridge in figure 8, but without the hinge at the apex. The arch rib in these bridges must have a certain amount of flexibility.

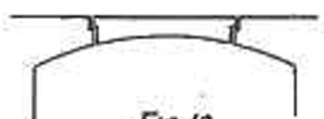


FIG. 10

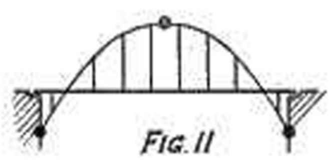


FIG. 11

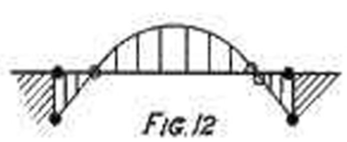


FIG. 12

You may find hinges or expansion joints in unexpected places, as on Waterloo Bridge in London. On some concrete bridges you will find none at all. Temperature changes do not affect concrete structures so much as they affect steel, and prestressed concrete is more elastic than rolled steel. Hinges are provided when a concrete bridge is being built, but they are very often filled in after the concrete has hardened and the bridge has bedded down.

Waterloo Bridge, figure 10, is an interesting deception. It looks like an arched bridge but structurally it is a cantilever bridge like the Forth Bridge. The centre of each arch is a reinforced concrete girder and the outer parts are cantilever brackets.

Figures 11 and 12 show two more bridges which look alike but are structurally quite different.

C.V.G.

04946 01653 46680 49886 27232 79178 60857 84383