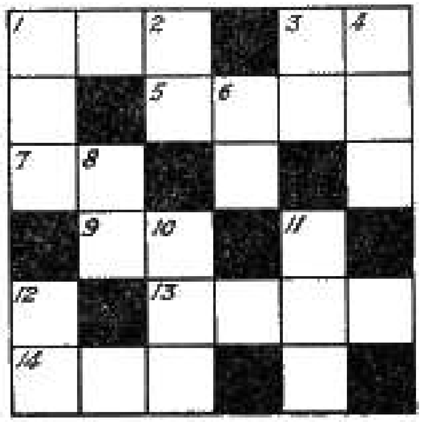
JUNIOR CROSS-FIGURE No. 20

Suggested by A. P. Milne, Form 4, King Edward's School, Bath.



ACROSS

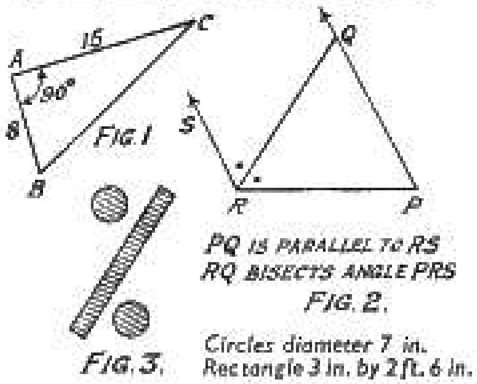
1. The L.C.M. of 91 and 117.
3. Length BC (see Fig. 1).
5. $\dfrac{1\cdot89\ (3\cdot1 - \cdot4)}{\cdot4\ (3\cdot1 + \cdot4)}$
7. Three and three-tenths of a score.
9. Three-quarters of a quarter in pounds.
13. Number of feet in a nautical mile.
14. The H.C.F. of 819, 702 and 1287.

DOWN

1. The product of "3 across" and the average of "6 down" and "12 down."
2. Thrice "12 down."
3. $\frac{1}{3}$ of $3\frac{9}{14} + 1\frac{2}{7} \div \frac{1}{8}$.
4. The cost price of a car which was sold for £795 at a profit of 6%.
6. Angle PQR if angle QPR = 50°. (See Fig. 2).
8. A multiple of "12 down."
10. Find, in sq. in., the shaded area taking $\pi = \frac{22}{7}$ (see Fig. 3).
11. xy when $3x - 5y = 4y - 2x = 4$.
12. A prime number.

Check clue: The sum of all the digits inserted is 92.

I.H.



Answer to "How Long?": 1 hour—why?

SOLUTIONS TO PROBLEMS IN ISSUE No. 21

MARBLE PIE

11 marbles in a large packet and 5 in a small packet (find a multiple of 19 and a smaller multiple of 3 so that thay add up to 224).

S(H)OCKS

By choosing 3 socks you are sure to have 2 alike.

HIGHWAY CODE

H452		H317	
42	H } can be	37	H } can be
34306	D } 1, 7, 9	23246	D } 5, 8, 9
604D		643D	
4H84D		3H03D	

Begin with the unit column where 2T + R = 10 or 20 and then find similar equations for each of the other columns.

SENIOR CROSS-FIGURE No. 21

Across: (1) 8·7 cm.; (2) 33 mi.; (4) 11°; (6) 30°; (7) ·1980; (9) 192; (11) ·704; (13) 27·38; (15) 16; (16) 54; (17) 1·4 in.; (18) 5·8 cm.

Down: (1) 81; (3) £3,300; (4) 1·11 sq. in.; (5) 59°; (7) 1·27; (8) ·878 sq. km.; (10) 9,261; (12) 42·4 sq. cm.; (14) 31°; (16) 5·8 cm.

Note: Clue "9 across" should have read $p^{\frac{1}{2}} \div q^{-3}$, not $p^{\frac{1}{2}} \div q^{\frac{1}{3}}$.

FALLACY No. 21

With the given information ∠DAC must be $22\frac{1}{2}$° in which case both methods make ∠DCB = $112\frac{1}{2}$°.

SPOT THE SERIES

(i) Increase the differences by 4, 5, 6 etc. and the next 2 terms are 84, 120.
(ii) The terms are: $^5C_4$, $^6C_4$, $^7C_4$, etc. so the next 2 terms are 210, 330.
(iii) This series is made up of the coefficients of $(a+b)^6$ so the next 2 terms are 6, 1.
(iv) Reverse the figures of successive powers of 3; next 2 terms are 7812, 1656.
(v) Series is cyclic, next 2 terms are 12, 23.
(vi) Reverse the figures of successive perfect squares, next 2 terms are 94, 46.

SIMPLE ARITHMETIC

The numbers are 1, 2, 3.

MARBLE(OUS) MATHEMATICS

The boys started with 65, 35, 20 marbles. Working backwards, the boys each had 40 marbles at the end of the game; therefore, before the third miss they must have had 20, 20, 80 marbles. Repeat this process for each of the other two misses.

JUNIOR CROSS-FIGURE No. 19

Across: (1) 2704; (4) 8·2; (5) 7·5 cm.; (7) 6·375 sq. cm.; (10) ·725.

Down: (1) 28°; (2) —726; (3) 477; (6) 5517; (8) 385 sq. yd.; (9) 27°.

Note: Fig. 2 should have stated "side of square 40·0 mm." and "radius of circle 17·5 mm."

I.L.C.

54252 78625 51818 41757 46728 90977 77279 38000

MATHEMATICAL PIE

NO. 22 *Editorial Offices:* 97 Chequer Road, Doncaster. OCTOBER, 1957

MATHEMATICAL INSTRUMENTS No. 3

The Astrolabe—Part II

Fig. A.

Fig. A is based on a Persian astrolabe made in the early thirteenth century. This has a heavy metal disc, called the matrix, which is suspended from a ring, so that it hangs vertically. One face has a degree scale, with a sighting arm (alidade) pivoted at its centre. With this, the angle of elevation of the sun, or a star, can be measured.

On the other side, which is shown in the figure, the matrix is rather like a shallow pie-dish with a short spindle in its centre. The broad rim is marked off in degrees. Into the dish fit, first, one of a set of plates, each designed for use in a different latitude, then a cut-away disc, called the spider, and lastly, a pointer, called the label. These are all held in place by a cotter-pin, which fits into a spindle.

The plate is fixed by dowels to the matrix, so that it cannot move. On its upper part are engraved radiating lines, representing compass directions, and circles, which correspond to the different angles of elevation. Note that, as this represents a map of the sky, the north is the bottom of the plate. The spindle of the matrix represents the Pole Star, and cuts through the circle corresponding to the latitude for which the plate is designed.

The spider, which represents the celestial sphere, can rotate about the spindle. The sharp points represent stars, and the circle represents the ecliptic. This is marked with the signs of the Zodiac, and with 365 divisions representing the days of the year.

To set the spider by night, the angle of elevation of a star is found by using the alidade. The spider is then turned so that the point representing this star lies on the circle corresponding to this angle—on the left of the plate if the star is toward the West, and on the right if it is toward the East. The angles of elevation and compass bearings of all the stars can then be read off from the plate.

62080 46684 25906 94912 93313 67702 89891 52104

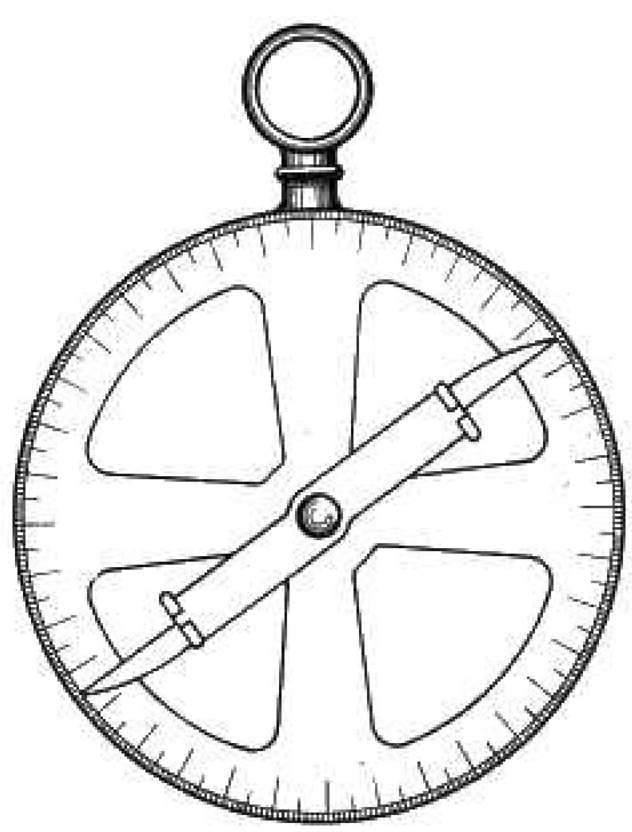


Fig. B.

By day, the angle of elevation of the sun is measured, and then the spider is set by turning it so that the division representing the day of the year lies on the corresponding circle of elevation.

To tell the time, the label is turned so that it passes through the division of the Zodiac circle representing the day of the year. From the intersection of the label with the scale on the rim of the matrix, the time can be calculated. (The lowest point of the rim represents midnight, and 15° are equivalent to one hour.)

The copy of Chaucer's works in your school library probably contains his Treatise on the Astrolabe, which explains a number of other uses of the instrument.

The introduction of the magnetic compass made the spider unnecessary for navigation. It went out of use in the sixteenth century, but simpler instruments without the spider were used until the eighteenth century for measuring angles of elevation at sea. Fig. B shows a nautical astrolabe, made in 1603, which was dredged from the St. Lawrence River about 85 years ago.

The description of the reflecting type is another story, as is the method of projection of the spider and plate (from the S pole to a tangent plane at the N. plane). C.V.G.

HOW LONG ?

Using the same scale on both axes take values of x from — 7 to + 7 and values of y from — 1 to + 11, then draw the following as accurately as possible.

(*a*) semicircle upwards from (— 3·5, 7·7) to (— ·5, 6·3) centre (— 2, 7) radius 1·7 units.
(*b*) semicircle upwards from (·7, 5) to (2·7, 6) centre (1·7, 5·5) radius 1·1 units.
(*c*) straight lines from (4, 6) to (5, 8) ; from (5, 6) to (7, 8) ; from (5, 5) to (7, 6) ; from (— 4, 8) to (— 5, 11) ; from (— 5, 7) to (— 7, 10).
(*d*) circle centre (— 3, 2·7) radius ·8 units.
(*e*) circle centre (2·2, 2·7) radius ·5 units.
(*f*) straight lines from (— 3·5, 7·7) to (— 5·6, 5) to (— 7, 4·5) ; from (2·7, 6) to (4, 4·3) to (5, 4).
(*g*) straight lines from (— 7, 0) to (— 5, 2) ; from (— 5, — 1) to (— 4, 1) ; from (4, 1) to (5, — 1) ; (7, — 1) to (5, 1) ; from (7, 1) to (5, 2).
(*h*) straight lines from (— 1, 1·5) to (— 1·4, 2·8) to (— ·5, 6·3) ; from (1, 2) to (1·3, 3) to (·7, 5) ; from (1·2, 6·5) to (0, 9) ; from (— 1·2, 8·5) to (0, 11).
(*i*) join with a smooth curve the points : (— 3·5, 4) ; (— 2·4, 3·3) ; (— 1·5, 2·5) ; (— 1, 1·5) ; (— 2, 1·2) ; (— 3·2, 1·5) ; (— 4·5, 2) ; (— 5·6, 2·7) ; (— 6·6, 3·7) ; (— 7, 4·5) ; (— 6, 4·7) ; (— 4·7, 4·5) ; (—3·5, 4).
(*j*) join with a smooth curve the points : (3·3, 2·4) ; (4, 2·8) ; (4·7, 3·4) ; (5, 4) ; (4·2, 4·1) ; (3·4, 3·9) ; (2·7, 3·6) ; (2, 3·1) ; (1·4, 2·6) ; (1, 2) ; (1·7, 1·9) ; (2·6, 2·1) ; (3·3, 2·4).

For solution to "How Long?" see p. 172.

I.H.

The equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ is used in Physical Optics to find the focal length of a lens. If AB (Fig. 1) represents the distance u (Fig. 5) and CD represents the distance v, then OX represents the focal length f.

Fig. 6 is an electric wiring diagram, showing two resistances r_1 and r_2 (ohms). It is known that the resistance R of the combination is given by $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$. R may therefore be found from the poles diagram by measuring OX.

If the hot water tap can fill a bath in 20 minutes, and the cold tap in 10 minutes, how long does it take to fill the bath if both taps are turned on?

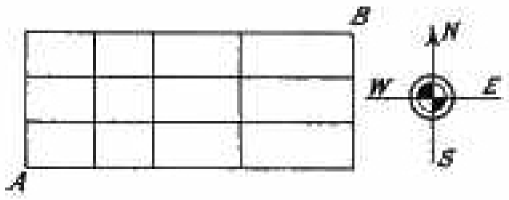
The answer is not 15 minutes. Let AB = 10 and CD = 20 ; then OX represents the answer. Can you explain why?

In Fig. 8, DEF is a triangle, base a, altitude h. It is required to inscribe a square inside the triangle as shown.

If x is the length of a side of the square, then it can be shown that $\frac{1}{x} = \frac{1}{a} + \frac{1}{h}$. (Can *you* prove it?). Thus, if AB in Fig. 1 is drawn equal to the base, and CD to the altitude, OX is the length of the side of the required square.

The problems above seem to be so different that they may be said to be "poles apart." Is it not strange that there is the same underlying unity, discernible only in mathematics? J.G.

COPS AND ROBBERS



The diagram represents the street plan of part of an American town. The police are out to capture a bank robber who is making his way from A to B, and they must catch him before he reaches B where he has an accomplice waiting in a car. The police can run faster than the robber who is hampered by his loot. How many policemen must be dispatched from A so that every possible route from A to B is covered to make sure that the robber will be caught? Only consider travel northwards and eastwards. J.G. ; J.F.H.

BONFIRE-NIGHT BARGAIN

Jock paid a shilling for a bundle of sparklers but found that some of them were damaged. He complained to the shopkeeper, who gave him two extra. Being careful with his money, Jock worked out that he had now got his sparklers for a penny less per dozen. How many sparklers were in the original bundle? J.F.H.

LETTER TO THE EDITOR

A correspondent recently inquired about the source of the figures at the bottom of each page of this journal, commenting that he suspected secretly that the Editor maintained a squad of boys in detention whose task it was to work out the figures for each coming issue.

Wishing to clear his name, the Editor states that the figures, comprising the value of π to 2035 decimal places, were calculated by the electronic machine ENIAC in 1949 and published by the National Research Council, Washington, D.C., in 1950.

FALLACY No. 22

Contributed by Barbara Blair, Girls' Collegiate School, Enniskillen.

Two men bought 60 oranges and divided them between themselves so that each got 30. One man sold his share at 3 for 1d. and the other sold his at 2 for 1d. Between them they collected 2/1.

They then bought 60 more oranges, but this time together sold them at 5 for 2d.—which is the same as 2 for 1d. and 3 for 1d.—This time they collected 2/–. Where did they lose the penny?

POLES APART

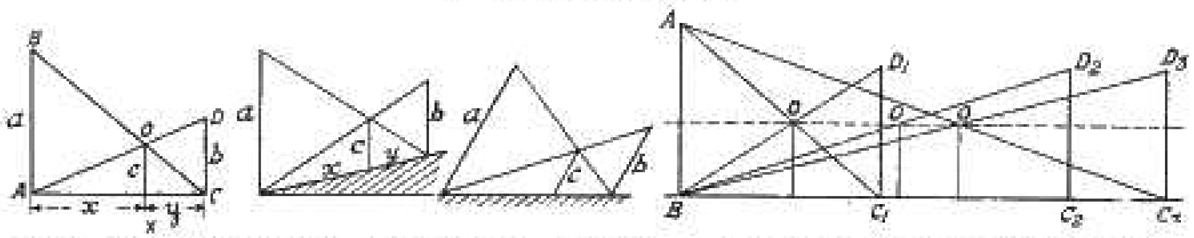


Fig. 1.—Ready for Algebra.

Fig. 2.—Poles on incline. $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$

Fig. 3.—Inclined, but parallel. $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$

Fig. 4.—The heights of the points marked O are all equal.

AB and CD are two vertical poles of unequal lengths, supporting a wireless aerial BD. The top of each pole is connected by a straight wire to the base of the other, the two wires meeting at a point O. The problem is to find the height of the intersection O when the lengths of the poles are known.

This height can be found either by calculation or scale drawing. If the poles are 10 ft. and 20 ft., the height OX is $6\frac{2}{3}$ ft.

This problem is a geometrical curiosity, because part of the necessary data appears to be missing. The distance between the poles is not given. Nor is it required, for the height of the intersection is the same for given lengths of poles no matter how far apart they may be. Fig. 4 illustrates this fact. Three pole positions are shown.

The proof depends on similar triangles (Fig. 1). From the triangles OXA, DAC, $\frac{c}{b} = \frac{x}{x+y}$. From the triangles OXC, BAC, $\frac{c}{a} = \frac{y}{x+y}$.

On addition, $\frac{c}{a} + \frac{c}{b} = \frac{y+x}{x+y}$. Hence $\frac{c}{a} + \frac{c}{b} = 1$.

This result is usually written in two ways:—

(*a*) Dividing throughout by c, $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$.

(*b*) Multiplying by ab, $b + a = \frac{ab}{c}$ $\therefore c = \frac{ab}{a+b}$.

Thus, if $a = 20$ and $b = 10$, $c = \frac{20 \times 10}{20 + 10} = 6\frac{2}{3}$.

The diagrams shown above may illustrate and solve many other problems.

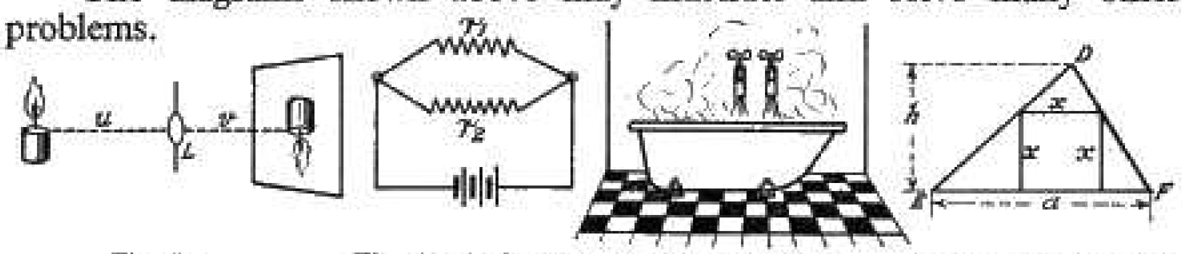


Fig. 5. Fig. 6.—Resistances in parallel. Fig. 7.—A Friday night problem? Fig. 8.—A geometric puzzle.

90927 21079 75093 02955 32116 53449 87202 75596

HANDY PROBLEM

The geometry master, feeling the need for some fresh air, glanced at the clock as he went out at some time between 4 and 5 p.m.

On his return about 3 hours later, he noticed that the hands of the clock had exactly changed places. What was the time of his departure?

J.F.H.

GREETINGS

$$\frac{(90^\circ - c_1)\,(90^\circ - c_2)\,(90^\circ - c_3)\;\ldots\;(90^\circ - c_n)}{2u}$$

The compliments of the season to you. (The complements of the *c's on* 2 u).

I.H.

STAMP COLLECTOR'S CORNER No. 3

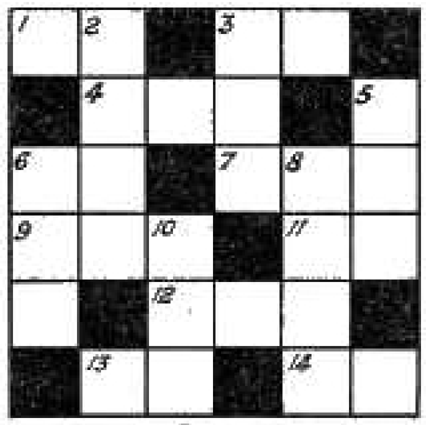
Tycho Brahe

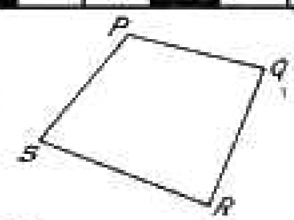
Denmark 1946. 20 ore dark red.

At the expense of Frederick II of Denmark, Tycho Brahe (1546-1601) built an observatory which he called Uraniborg, or the Castle of the Heavens. After the death of Frederick he was established by the Emperor Rudolph II in an observatory near Prague where Kepler became his assistant. He designed astronomical instruments capable of measuring angles to small fractions of a degree. The observations which he made of the motions of the planets over many years were the basis of the laws of planetary motion later published by Kepler.

C.V.G.

SENIOR CROSS-FIGURE No. 22





PQRS IS A QUADRILATERAL
FIG. I

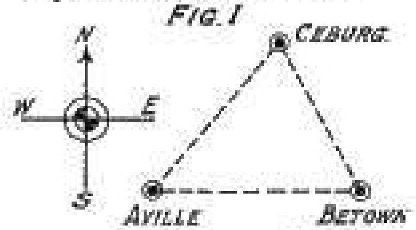


FIG. 2 PLAN—Betown is 7 miles due E. of Aville and Ceburg 4 miles N. 30° W. of Betown.

ACROSS

1. Square root of "10 down."
3. The angle, to the nearest degree, subtended at Aville by the road joining Betown to Ceburg. (See Fig. 2).
4. The weight in Kg. of a solid metal cylinder with diameter 7 cm. and length 3 m. 1 c.c. of metal weighs $6\frac{4}{11}$ gm. Take $\pi = \frac{22}{7}$.
6. Angle S when PQRS is cyclic, $\angle P = \frac{1}{2}\angle Q$ and $\angle R = \frac{4}{5}\angle Q$. (See Fig. 1).
7. A root of $x^3 + 6x - 1 = 0$.
9. The average speed, in m.p.h., of a car which takes $14\frac{4}{5}$ minutes to complete the road circuit Aville to Aville. (See Fig. 2).
11. $\left\{\frac{6}{7} - \left(1\frac{5}{7} \times \frac{1}{5}\right)\right\}^2$.
12. Length PR, when PQ is parallel to SR, QR is parallel to PS, PQ = QR, $\angle Q = 90^\circ$ and the perimeter is 62 cm. (See Fig. 1).
13. The distance, in miles, saved by travelling direct from Aville to Ceburg instead of going via Betown. (See Fig. 2).
14. Angle SPR when PQ = QR = RS = SP and $\angle R : \angle Q = 4 : 1$. (See Fig. 1).

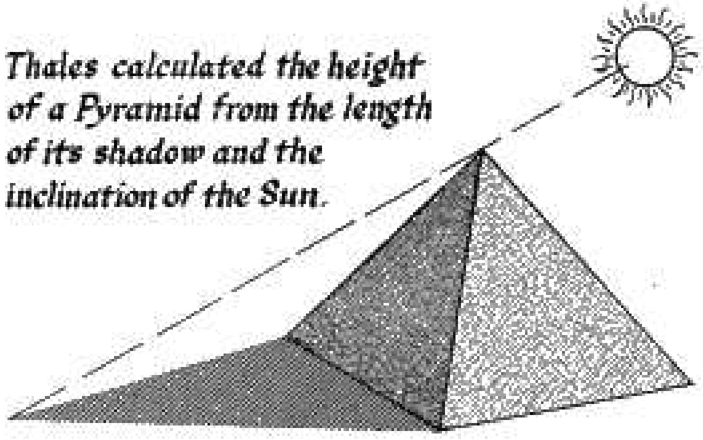
DOWN

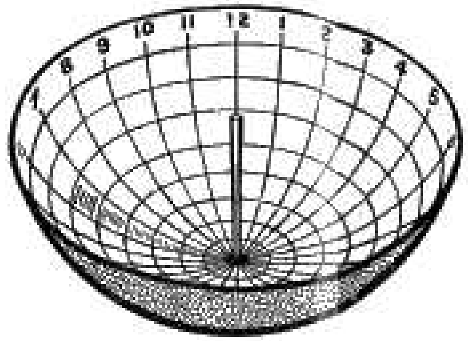
2. Area, in acres, of the ground enclosed by the three roads connecting Aville, Betown and Ceburg. (See Fig. 2).
3. $a^2 b$ if $9a - 2b = b - 4a = 1$.
5. Area, in sq. cm., when PQ is parallel to SR, PQ = 6 cm., SR = 28 cm., and PS = QR = 22 cm. (See Fig. 1).
6. Find c if $\frac{3}{5(3 + c)} - \frac{1 - 3c}{3c} = 1$.
8. The shortest distance, in yards, from Ceburg to the road joining Aville and Betown. (See Fig. 2).
10. A perfect cube.

Check clue:
The sum of all the digits inserted is 110. I.H.

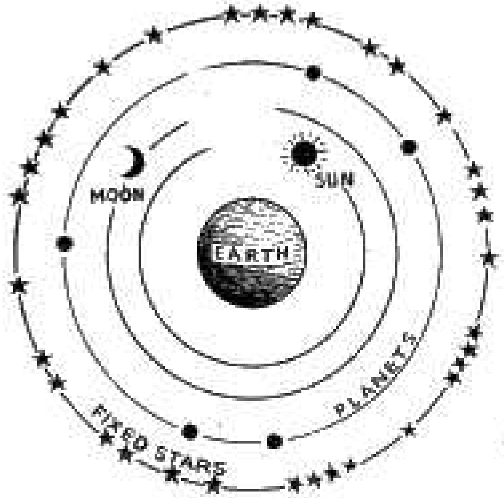
35587 64024 74964 73263 91419 92726 04269 92279

Thales calculated the height of a Pyramid from the length of its shadow and the inclination of the Sun.





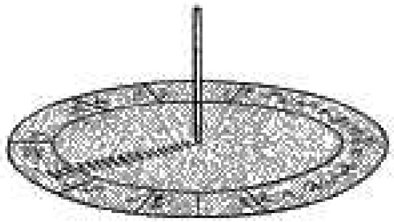
A Bowl Sundial or Hemispherical Gnomon. The bowl contains a vertical peg and markings to show the time of day. The full circles enable the height of the shadow to be measured.



The earliest Greek ideas made the stars fixed and the sun, moon and planets revolve on different circular orbits about the earth.

The Constellation Orion. The early Greeks noticed groups of prominent stars and associated with them fancied resemblances to mythical characters.

In Greek Mythology Atlas held the Universe on his shoulders.



The Vertical Gnomon, a form of Sundial was first used by the Babylonians and then by the Greeks.

Most of our information regarding the history of early Greek astronomy has been obtained from manuscripts written centuries after the original workers lived. Because of this it is difficult for us to know how the Greek mathematical astronomy of the 6th and 5th centuries B.C. arose out of the primitive knowledge existing at the time of Thales (640-546 B.C.).

The Babylonians and Egyptians studied the heavens from purely utilitarian motives and recorded their observations carefully and accurately, but the Greeks were rapidly developing into a nation of philosophers. Many of the early philosophers, of whom Pythagoras was one, had attended the school of Thales, in Miletus, and subsequently travelled in Egypt and, Mesopotamia to learn from the priests and scribes of these countries. In this way they learned to predict solar and lunar eclipses and compared records of the movements of the planets.

In the hands of the Greeks, Astronomy became a pure science. No longer was it simply the observation of phenomena but a study of the nature of the Universe. The main problems were not so much how the sun, moon and planets moved among the stars, but why they followed their particular paths.

67823 54781 63600 93417 21641 21992 45863 15030

The firm belief that the earth was the centre of the Universe made this work very difficult. The Pythagoreans aimed at finding a perfect system involving circles and spheres, but fell far short of explaining the true motion of the planets. It is believed that Plato (429-347 B.C.) set his pupils the task of determining a perfect system for the Universe but realised, in his old age, that only by assuming the earth to be moving could such a system be evolved.

Aristotle (384-322 B.C.) developed the ideas of his predecessors, Eudoxus and Callipus, and evolved a complicated system of spheres on which the sun, moon and planets moved. These spheres revolved about the earth and originally were 27 in number—one for the stars, three each for the sun and moon and four each for the five planets. The final result, due to Aristotle, was a system of 56 spheres, since more had been added to counteract the effect of the spheres on each other. This was, of course, a very confusing conception, since Aristotle regarded them as material entities. Unfortunately for future progress, Aristotle was held in such high esteem that his system was regarded as a correct representation of celestial motion for many centuries.

I.L.C.

28618 29745 55706 74983 85054 94588 58692 69956