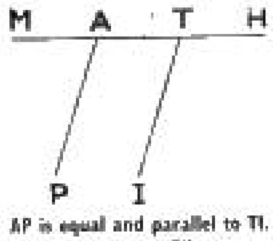
JUNIOR CROSS-FIGURE No. 16

ACROSS

1. The profit made when a car is sold for £918 at a gain of $13\frac{1}{3}$%.
4. Angle ITH, if angle PAM = 26° (see figure).
6. Square yards in a square chain.
7. xy if $2x - 5y = 3x - 8y = 7$.
9. The area, in sq. m., of a rectangle 7 Dm. 2m. by 8 cm.

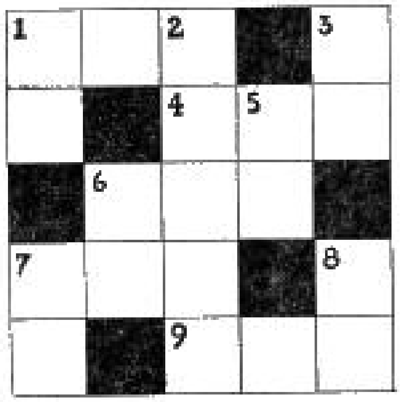


AP is equal and parallel to TI.
MA = AT = TH.

DOWN

1. A river flows at 6 m.p.h. Find the speed of the boat which can travel twice as fast downstream as it can upstream.
2. 16s. $4\frac{1}{2}$d. as a decimal of £1.
3. Length of MH, if AP = 16 cm., $\angle$PAM = 26° and $\angle$APH = 13° (see figure).
5. $5\ (10\frac{1}{10} + 2\frac{1}{2}) \div \frac{5}{8}$ of $17\frac{1}{2}$.
6. The circumference of a circular flower bed of area "4 across" sq. ft. ($\pi = 3\frac{1}{7}$).
7. $\frac{1}{2}(x + y)$ (see "7 across").
8. Four times the square root of "9 across."

I.H.



SOLUTIONS TO PROBLEMS IN ISSUE No. 17

HUNDREDFOLD

Seven men each received £10, forty-nine women 10/– and forty-four children 2/6d. This solution is unique.

SENIOR CROSSWORD

Across : (1) 1.69 ; (3) 201° ; (5) 1.15 miles ; (7) 98° ; (9) 72° ; (10) 63360 ; (11) 16 ; (12) 71° ; (14) 4.51 miles ; (16) .504 ft. ; (17) 33.5 sq. ins.
Down : (1) —1.19 ; (2) 91 apples ; (3) 2.5 cm. ; (4) 16.2 in. ; (6) £173.25 ; (8) 8.66 in. ; (9) 7.07 in. ; (11) .125 sq. in. ; (13) 175 gm. ; (14) 44 ; (15) 1.3.

FALLACY No. 17

The fallacy lies in the third equation in the working. If equal fractions have equal numerators, then either the denominators are also equal, or the numerators are each zero. In this case the equation is satisfied by $x = 10$, which shows that the original equation would hold whatever the denominator of the right-hand side.

WELL! WELL!

(a) Fill the 5-pint jar and, from this, fill the 3-pint jar.
(b) Empty the 3-pint jar and pour into it the remaining 2 pints from the 5-pint jar. The 3-pint jar then needs 1 pint to fill it.
(c) Fill the 5-pint jar, and from this fill up the 3-pint jar, leaving 4 pints in the 5-pint jar. All the woman needed was time and patience!

MATHEMATICAL LIMERICK No. 3

The ball weighed 2 ounces.

SERIOUS SERIES

(a) 14, 21 ; (b) 50, 65 ; (c) 39, 77 ; (d) 10, 7 ; (e) 47, 93 ; (f) 13, 20 ; (g) 9, 6.

GATE CRASHERS

There are more than six theorems.

The parallel lines give you (a) Alternate angles ; (b) Co-interior angles ; (c) Corresponding angles ; (d) Adjacent angles ; (e) Vertically opposite angles.

The tranversals with the parallel lines show the equal intercept theorem and give several sets of similar triangles.

If the cross stays of the gate meet at the centre of the top bar, the corner triangles are congruent and the centre triangle is isosceles. Pythagoras theorem can be applied to the right-angled triangles.

The gate also shows the angle sum of a triangle and a quadrilateral, and a quadrilateral as the sum of two triangles.

JUNIOR CROSSWORD No. 15

Across : (1) 1369 ; (4) 1529 ; (5) 165 ; (7) 96 ; (8) 67 ; (10) 54 ; (12) 43.
Down : (1) 1536 ; (2) 32 ; (3) .6916 ; (6) 675 ; (7) 931 ; (9) 84 ; (11) 42°.

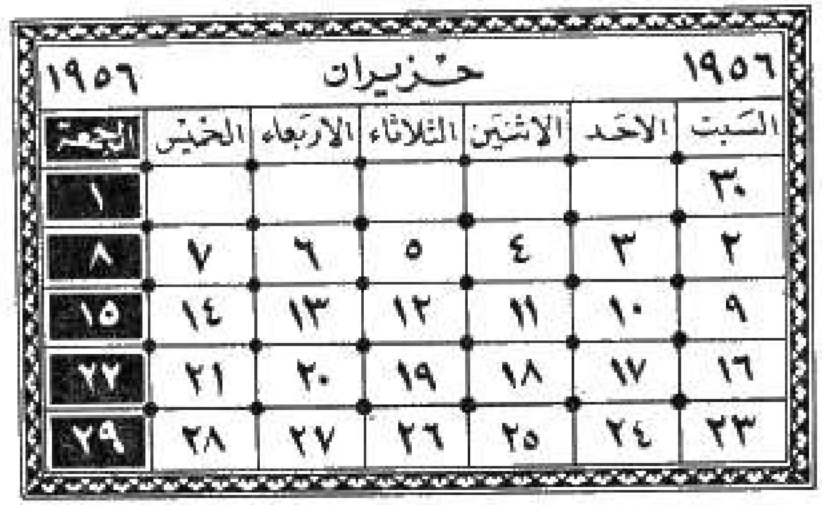
I.L.C.

13393 60726 02491 41273 72458 70066 06315 58817

MATHEMATICAL PIE

NUMBER EIGHTEEN — *Editorial Offices:* 97 Chequer Road, Doncaster — MAY 1956

ARABIAN (K)NIGHT'S HOMEWORK



١٩٥٦			حزيران			١٩٥٦
الجمعة	الخميس	الاربعاء	الثلاثاء	الاثنين	الاحد	السبت
١						٣٠
٨	٧	٦	٥	٤	٣	٢
١٥	١٤	١٣	١٢	١١	١٠	٩
٢٢	٢١	٢٠	١٩	١٨	١٧	١٦
٢٩	٢٨	٢٧	٢٦	٢٥	٢٤	٢٣

Those readers who are busy preparing for an examination must make frequent reference to the calendar to keep track of their progress. The form of the monthly tear-off leaf consequently becomes so familiar that it is hard to realise that other countries may have something very different such as, for example, the leaf which is reproduced above.

Our illustration, taken from a calendar which would be seen in countries where the Arabic language is used, represents the leaf for the month of June, 1956. We notice straight away that the figures are different from those which our arithmetic books refer to as "Arabic numerals" in distinguishing ordinary figures from the Roman system. In fact, from a historical point of view, our numbers could more correctly be termed "Hindu numerals" or "Indian numerals."

At a first glance, the year appears to be 1907, so, since we know it is really 1956, we deduce that the small "o" represents 5 and that the figure which looks like a seven is really 6. The next thing to notice is that the writing proceeds from right to left. The reason that the figures in the year seem to be in the usual order is that this would be read as "six and fifty and nine hundred and a thousand."

The days of the week, naturally, are also written in order from right to left commencing with Saturday on the extreme right and finishing with Friday, which is the religious day of rest, on the left.

With these hints, the reader should have no difficulty in identifying the Arabic numbers from 1 to 30 and would then be in a position to write down in the same notation the answers to the problems given below. Remember that the work proceeds from right to left and that the figures on the extreme right are the problem numbers.

= ٣ × ٩	٦	= ٧ + ٣	١
= ٧ ÷ ٢١	٧	= ٣ − ٨	٢
= ٦٠٠ + ١٠٠	٨	= ٥ × ٥	٣
= ١٢ × ١٣	٩	= ٣ − ٩	٤
= ٣٦ + ١٩ + ٨	١٠	= ٢٠ − ٣٠	٥

J.F.H.

$\pi = 3{\cdot}14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971$

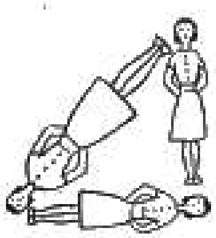
MATHEMATICAL LIMERICK No. 4

Contributed by R. D. Byers, Esq., B.Sc., Varndean School for Boys, Brighton.

A batsman, by name Clutterbuck,
Squared his total of runs, just for luck,
Then subtracted his score,
Took off 156 more,
And found the result was a duck.

Find the number of runs Clutterbuck made.

TRIPLETS



A triad is a group of three things or three numbers associated according to some rule or plan. A clover leaf; three notes forming a chord in music; three blind mice, are triads.

Pythagorean triads are groups of three lengths which may form the sides of a right-angled triangle. The simplest is the group 3, 4 and 5; but you may find as many as you like from the following simple rule:—

(1) Think of any two numbers.

(2) Write down (*a*) twice their product; (*b*) the difference of their squares; (*c*) the sum of their squares.

You will thus obtain three numbers which fit the sides of a right-angled triangle, the third being the hypotenuse. The results may be checked by the usual "converse of Pythagoras" method.

E.g., numbers chosen, 5 and 3.

Twice product = 30. Difference of squares, 16. Sum of squares, 34.
Check . . . $30^2 = 900$ . . $16^2 = 256$ . . $34^2 = 1156$ (= 900 + 256).

E.g. also. Numbers chosen, 365 and 273.

Twice product = 199290 . . . Difference of squares, 58596. Sum of squares, 207654.

These three lengths form the sides of a right-angled triangle. Should you doubt the truth of this statement, verify by squaring. I suggest that you test your accuracy by doing so.

Coda.

1. Can you prove that the rule always yields correct results?

2. With what two numbers would you commence to obtain the common triads 3, 4, 5, and 5, 12, 13?

3. Does the rule give *all* possible right-angled triangles?

"D'you know the one about the square on the hypotenuse?" (Reproduced by permission of the Proprietors of *Punch*).

J.G.

MATHEMATICAL REASONING?

Contributed by F. A., The Academy, Forfar, Angus.

"Dad, did you know that the light of a candle is brighter than the sun?"
"That's nonsense, my boy."
"But you agree that candle-light is brighter than nothing?"
"Of course it is."
"Well, nothing is brighter than the sun."

would not be found until we came to 496. After that we should reach our fourth at 8,128.

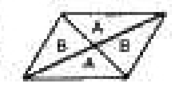
The first two perfect numbers were known to the early Hindu and Greek mathematicians. The third and fourth were discovered at about 100 A.D. by Nichomachus. Since his day several others have been found. It is a long stretch before we reach the fifth known member of this select company: it is 33,550,336. At the present time we know of only twelve perfect numbers altogether, the largest being a number of 77 digits!

Needless to say, the larger perfect numbers were not discovered by the impossibly laborious process of adding together their divisors. We are indebted to the genius of Euclid for the statement that the expression $2^{n-1}(2^n - 1)$ is perfect provided that $(2^n - 1)$ is a prime number. For instance, when $n = 3$, $2^n - 1$ is 7, which is prime; and on putting $n = 3$ in the whole expression we obtain 28, which is our second perfect number. By giving n the values 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, we obtain the twelve perfect numbers at present known. As $2^{127} - 1$ is, in this year of grace, the largest known prime, we cannot as yet find any more perfect numbers by this method.

JIGSAW No. 2

Contributed by Canon D. B. Eperson, Bishop Otter College, Chichester

Divide a parallelogram into 4 triangles by cutting along the diagonals. Into how many different parallelograms can 2 or more of the triangles be formed? (The pieces may *not* be turned over).



You are good if you can make at least ten.

LETTERS TO THE EDITOR No. 1

Dear Math. Pie,

My sister says that "the cross section of a bee is its sting." I am inclined to agree with her, having recently sat on a bee.

Yours sincerely,
JUDITH FOOT.

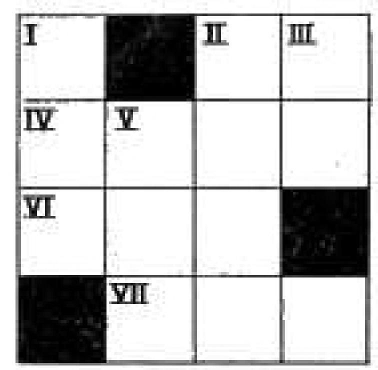
Cranborne Chase School,
Crichel, nr. Wimborne, Dorset.

ROMAN CROSS-FIGURE I.

(Place Roman Numerals in the squares)

Across: II. Add III down to the square of VII across.
IV. Square III down.
VI. The same both ways.
VII. Half of V down.

Down: I. VI across times the sum of II across and VII across.
II. The average of IV across and VI across.
III. Reverse the square of VI across.
V. Add III down to VI across.



I.H.

FUN WITH NUMBERS No. 4

$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$

Our old friend, Charlie Cook, found a new way of cancelling fractions—and, of course, he has found the right answer. Unfortunately, it only works like this with three other fractions in which both numerator and denominator are less than 100. Can you find these other fractions?

I saw a similar curve before a cup-tie. One entrance through a narrow turnstile was marked "Boys only." It was closed ; but a very large crowd of boys gathered ready for the opening. Instead of forming a queue, they bunched together. Fig. 8 shows an approximate sketch plan of the bunch. It appears to be another probability curve, because the further from the turnstile, the fewer boys were gathered there. The maximum depth was naturally opposite the turnstile.

Suppose now an allotment holder grows potatoes, and wishes to estimate accurately the results of his harvest. He grades his yield (disregarding the obviously unsuitable specimens) let us say according to their weight into 12 grades. Then, since counting is the basis of all statistics, he counts all the members of each grade, and, in order to see at a glance the results of his counting, puts the numbers into a graph, as shown in Fig. 9. The graph shows a remarkable similarity to the previous two. It is another probability graph ; but why the resemblance?

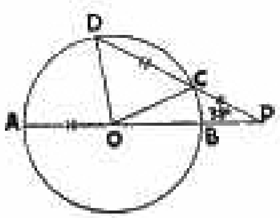
The answer to this and the previous question may be briefly stated : the properties of the things illustrated give similar algebraical equations for the curves ; and similar equations give similar graphs.

Another question may be asked : has every curve a formula or equation? Is there a formula for the outline of a herring, an oak leaf, a swallow's wing, or the profile of Napoleon? I have, unfortunately, no space to answer this most interesting question.

The centre picture is a sketch of a Scottish landscape. The main outlines have been drawn from the equations specified. The artistic merit may be doubtful ; but it is, I think, better than that of many examples of modern art.

J.G.

FALLACY No. 18



AB, diameter of a circle centre O, is produced to P such that the line PCD cutting the circle at C and D makes PC = CD = AO (radius).
To find $\angle COP$ when $\angle DPA = 35^\circ$.

Method 1.
OC = PC (given)
$\therefore \angle COP = 35^\circ$ (base angles of an isosceles $\triangle$)

Method 2.
$\triangle OCD$ is equilateral (given OC = CD = OD)
$\therefore \angle OCP = 120^\circ$ (exterior angle)
$\therefore \angle COP = 25^\circ$ (angle sum of $\triangle$ COP)
Which is correct?

I.H.

FEW ARE PERFECT

Contributed by D. J. Chittenden, M.A., Monks' Dyke School, Louth.

Perfect numbers appear to be almost as rare as perfect human beings. A number is said to be perfect if it possesses the simple property of being equal to the sum of its divisors. The smallest number which satisfies this condition is 6, which is equal to the sum of its divisors, $1 + 2 + 3$. If we proceed to examine the numbers one by one, we find that the next perfect number is 28 ($= 1 + 2 + 4 + 7 + 14$).

Encouraged by these two early discoveries, we might rashly suspect that perfect numbers were fairly common things. A continuation of the search would show how wrong this supposition is, because the next one

44288 10975 66593 34461 28474 64823 37867 83165

PROBLEM CORNER

Contributed by T. G. Moses, Esq., Pembroke Grammar School.

Judging by the tremendous response, the word "challenge" in the Square Numbers Problem in issue No. 16, was ill chosen. The task of reading through all the entries received indeed proved herculean.

Although the problem was set for pupils at school, I am grateful for the number of solutions received from adult enthusiasts, and, without presuming to pass judgment on their efforts, I would like to mention four—

Rev. W. M. Aitken, of Fettes College, Edinburgh, for brevity and wit ;
Mr. W. O. Storer, of University of Birmingham, for an exhaustive solution ;
Canon D. B. Eperson, of Bishop Otter College, Chichester, for rigour ; and
Mrs. L. Stafford, of Newport, I. of W., who, with the aid of a calculating machine, produced some dozen numbers, the last of which is of the order 3×10^{18}.

Of the solutions received from school pupils, some 40 only produced the correct solutions (rejecting 0, they are 48, 1680, 57120, 1940448, &c.) Of these pupils, the following have been awarded 5/- book tokens—

S. Fairthorne, Farnborough Grammar School.
S. D. Hawkins, St. Marylebone Grammar School.
G. Sargent, Scunthorpe Technical High School.
A. Day, Hele's School, Exeter.
C. P. Willans, Bradford, Yorks.
V. Goodfellow, Priory Secondary Girls School, Newport, I. of W.

Here is another problem to which our readers are invited to send in solutions. On your entry give details of your age and school ; book tokens will be awarded for the best answers received by the end of July. Here is another competition problem.

SEASON TICKETS

My season ticket is carried in a small leather case. After producing it for inspection one morning, I noticed that it had been creased down the centre. The ticket number was 3025 ; the crease had split this in half, so that the 30 appeared on one side, and the 25 on the other. Unconsciously I began to make the following calculation : 30 added to 25 gives 55, and 55 squared (3025) is the original ticket number.

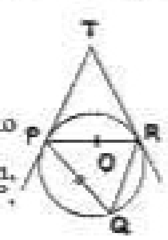
Ordinary level pupils are asked to find *one* other number, of four figures, *all different*, which will give the same result. (2025 is rejected since the digits are *not* all different).

Advanced level pupils are asked to investigate the same problem with 6 digit numbers. (Four figure tables of squares will again be found useful).

SENIOR CROSS-FIGURE No. 18

1	■	2	3		■
4	5			■	6
■		■	7	8	
9		10	■		■
	■	11	12		13
■	14			■	

TP, TR are tangents to circle centre O. PR = PQ = 13 cm. and angle PQR = 61°.



CLUES

ACROSS

2. The product, y, of the roots of $24z^2 - 17z + 3 = 0$.
4. The number of tins a dealer can buy, after selling 3,000 tins at a profit of $6\frac{1}{4}\%$ for £100.
7. The larger root of $24z^2 - 17z + 3 = 0$.
9. Twice the length of RQ (see figure).
11. The total rateable value of a row of "9 down" houses, if each householder pays £39 19s. 4d., the rate being 18/2d. in the £1.
14. Find x if $\dfrac{2(x+1)}{x-9} = \dfrac{4x+21}{2(x-5)}$

DOWN

1. "5 down" in feet per second.
2. One ninth of angle OPT (see figure).
3. One quarter of the product of "9 down" and "12 down."
5. The average speed of a lorry that travelled at 27 m.p.h. for 2 hr. 40 min. and then at half this speed for half the time.
6. The cost price, in pence, of a tin (see "4 across").
8. Radius of circle PQR (see figure).
9. The number of sides of a regular polygon whose interior angle is one half of "13 down" times its exterior angle.
10. The product of the two numbers, whose sum is twice their quotient, and also the difference between "12 down" and "13 down."
12. Find angle OTP (see figure).
13. $x^{1/3}y^{-1/3}$ (see "2 across" and "14 across").

I.H.

86280 34825 34211 70679 82148 08651 32823 06647

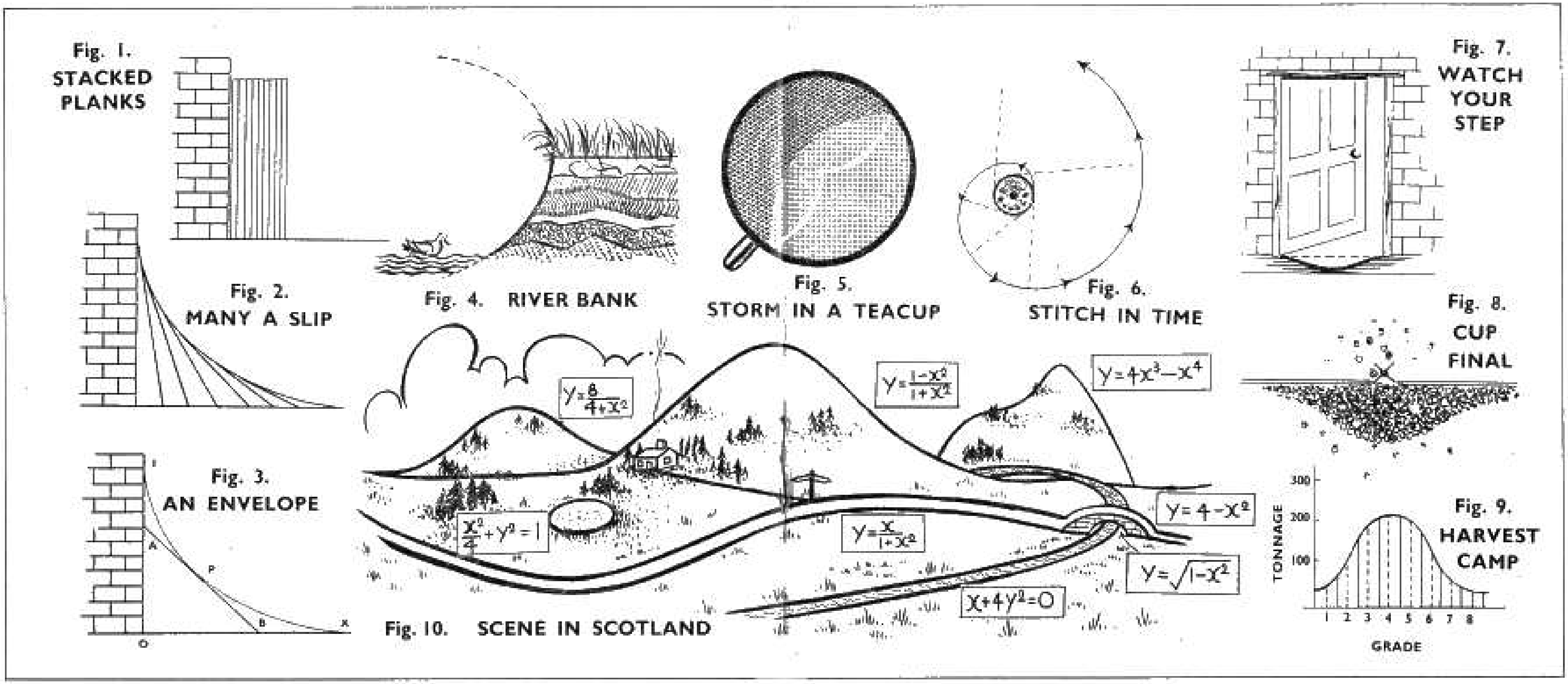


Fig. 1. STACKED PLANKS

Fig. 2. MANY A SLIP

Fig. 3. AN ENVELOPE

Fig. 4. RIVER BANK

Fig. 5. STORM IN A TEACUP

Fig. 6. STITCH IN TIME

Fig. 7. WATCH YOUR STEP

Fig. 8. CUP FINAL

Fig. 9. HARVEST CAMP

Fig. 10. SCENE IN SCOTLAND

Science grows from what we see, and one of its chief purposes is to explain what is observed. You may have seen some of the things described below ; each of them can be explained mathematically.

Have you ever noticed an overhanging river bank which looks like Fig. 4? The outline is a definite curve ; but it is not formed unless the composition of the bank is more or less the same throughout. The curve is part of what is called a parabola, the broken line showing more of the curve. It is seen at Lord's or Headingley, when a batsman is caught on the boundary ; but it may also be the curve followed by an electron under the action of electric and magnetic forces. Is it not curious that the overhanging bank, the flight of a cricket ball, and the path of an electron in a vacuum tube, should resemble each other? Why should they? . . . .

A timber merchant stacks his long thin floorboards against a wall. (Fig. 1). They slip, and form an apparently awkward heap (Fig. 2) ; but observation will show that the slipped boards may now enclose a quite definite curve. (Fig. 3). Each board is a tangent to the curve, which is called the *envelope* of the collection of boards. Envelopes (not the correspondence kind) are important things in mathematical theory and technology.

Take a bobbin of thread ; hold it still on a piece of paper on a table and, keeping the thread tight, unwind it from the bobbin. Trace the spiral path of the end of the thread in pencil. (Fig. 6). No matter where the thread is, it is a tangent to the bobbin. The bobbin (a circle) is the envelope of all the positions of the thread.

If you look at the surface of a cup of tea, or milk, in a good light, you may see a remarkable curve of light plainly visible on the top of the liquid. (Fig. 5). It appears almost cherry shaped, and is formed by the extra illumination given to the surface when the rays of light reflected from the circular cup cross and reinforce one another. This curve is the envelope of the reflected rays, and is part of a curve known as an epicycloid. Such a curve may be used in the design of gear wheels, because it is the path traced out by a tooth of a cogwheel which rolls upon another.

Fig. 7 is a sketch of a very old, well-used doorstep, worn down by the treading of countless feet to a quite definite shape. Though the behaviour of mankind is often unpredictable, most people would instinctively tread on the step near the middle ; the further from the middle, the less the wear on the step. We might say that it is probable that the depth worn down from the original horizontal level at any point of the step is a measure of the number of people who have entered at that particular point, that is to say, of the *frequency* with which that part of the step has been used. That is, the outline of the step may be regarded as a *probability* or *frequency* curve.