MAGIC SQUARES FOR HALLOWE'EN

In issue No. 83, we described a variation on the well-known idea of a "magic square" – replacing each number by the corresponding letter of the alphabet (A instead of 1, B instead of 2, and so on).

The two complete squares below show a "four by four" magic square and its matching letter square. As you probably know, if you add together the numbers in any row, column, or diagonal you will get the same total. In a "four by four" magic square any "block" of four numbers also gives the same total.

Can you use these ideas to find which letters are missing from the incomplete squares, and in each case use the letters to make a "Hallowe'en" word? For the second one, you should find the answer more easily using a "key" word: LOZA.

E.G.

13	8	12	1
2	11	7	14
3	10	6	15
16	5	9	4

M	H	L	A
B	K	G	N
C	J	F	O
P	E	I	D

	C	B	
F		T	
	G	H	U
	Y	X	

B	U		V
X	G		D
	R	F	Y

100

The expression (aaa − aa) ÷ a has the value 100 for all values of a from 1 to 9. Can you make 100 using a repeated single digit in other ways which will be different for the different digits?

B.A.

THE LAST PAGE

One hundred issues we have done,
In thirty years and more,
Issue One was rather short,
With pages only four.

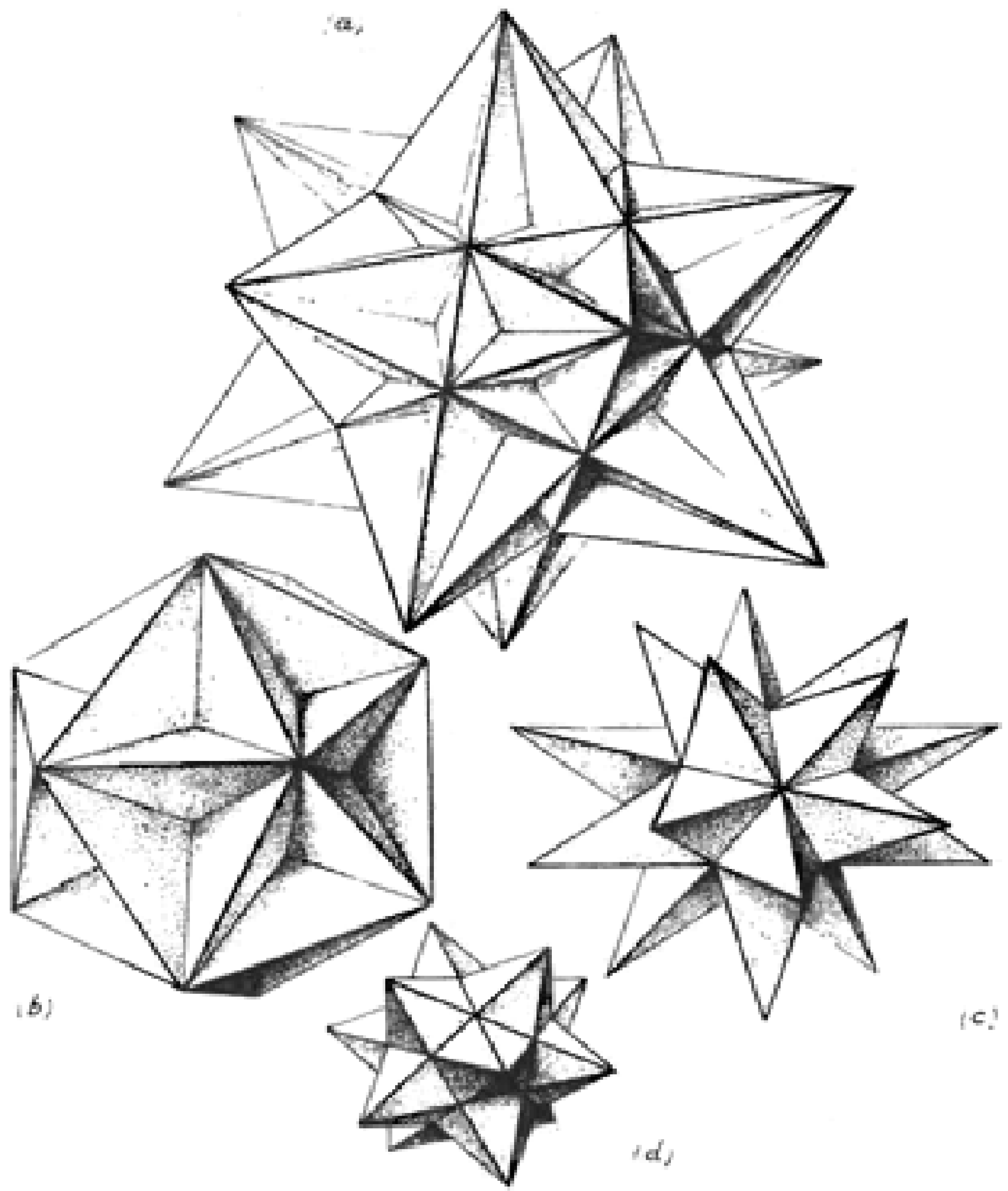
This explains the funny thing,
About the end of this,
The page we use to end this Pie
Is numbered seven nine six.

C.B.A.

MATHEMATICAL PIE

No. 100 *Editorial Address :* West View, Fiveways, Nr. Warwick AUTUMN, 1983

THE REGULAR POLYHEDRA



MAGIC TRICKS WITH DICE No. 1

A die with numbers or spots showing 1 to 6 has some fairly obvious number properties which can be used in simple number tricks. See if you can work out how these two tricks are done, by looking carefully at a normal die. If you can't see the trick, the answers are on page 795.

Trick number one: "What's the difference?"

Ask a friend to choose any three numbers on the die and add them together. They must remember (or write down — you should have your back turned to them anyway) this total. Now they must total the three numbers that are left out the first time. You ask them to work out the difference between the two totals and tell you the result. You then tell them what their two totals were!

Trick number two: "All but one"

Again, turn your back while your friend throws a die. This time they must add together the five numbers which can be seen (all but the one touching the table). Then they have to turn the die upside down and total up again. Finally they must add the two totals. They are probably expecting you to ask them what the total is — but you tell *them*. If you prefer, you can write down the total before they start!

Don't do "All but one" more than once in front of the same person; but "What's the difference?" can be done several times without "giving the game away". Don't forget, if you can't work out how to do these the answers are on page 795. In the next issue we will go on to tricks with two or three dice.

E.G.

FIND THE LIMIT

$$\sqrt{1} + 1 = 2$$

$$\sqrt{\sqrt{1} + 1} + 1 =$$

$$\sqrt{\sqrt{\sqrt{1} + 1} + 1} + 1 =$$

In the sequence on the left, each expression is the square root of the previous expression plus one. What is the limit of the sequence?

B.A.

CENTURIONS

The Romans used the letter X to represent ten and we use x for multiply. How many Roman numbers can make 100? Example: 62 is LXII which can be read 50 x 2 = 100.

Take the factors of 100 and multiply them together. How many 0's are there in the answer. You should not need a calculator to work it out.

C.B.A.

IT'S IN THE BAG

I have three bags, one contains only raisins, one contains only nuts and the third contains nuts and raisins. Without looking in the bag, what is the minimum number of draws from each bag so as to be sure of knowing what is in each bag?

R.H.C.

ROOM

Our architect friend decided to build himself a four-bedroomed bungalow and being lazy decided to make it so that from any of the rooms he could get into the other three without having to go through another room. What kind of layout did he need?

He would not have succeeded with a five-room bungalow.

R.H.C.

DICE TRICKS SECRETS

What's the difference? Add 21 to the difference, then divide by 2. This gives one of their totals. Take the result from 21 to get the other.

All but one. The overall total is always 35.

Now — what two important properties of the die are involved? Answer next time.

E.G.

SOLUTIONS TO PROBLEMS IN ISSUE No. 99

The Root of the Problem (a) (i) 1, (ii) 1.414, (iii) 1.554, (iv) 1.723; (b) (i) 1.414, (ii) 1.848, (iii) 1.962, (iv) 1.990.

Muddy Waters The plant was 24 ft high.

Wrong Labels Remove one item from the box labelled nuts and raisins and the rest follows simply.

Substitution A is 1, V is 5 and E can be 6, 7 or 8 with corresponding values for D and M.

Odd One Out No. 3

1. 2 — because it is not a multiple of 5, 30 — because it is not a factor of 20, 5 — because its name does not begin with "t", or 10 — because its name is the only one to make sense backwards!
2. Circle — it is not a polygon.
3. Pair of compasses — the others are used to *measure* things.
4. Maths — none of the others are abbreviations of longer words.

Zero In On This 10^{10} needs ten zeros. $10^{10^{10}}$ needs one hundred zeros.

What Did I Buy? I bought a watch, a calculator and a slide rule.

Senior Cross Figure No. 71 Across: 1. 343; 3. 11; 5. 36; 6. 3525; 8. 156; 9. 12; 10. 25; 12. 675; 15. 2605; 17. 90; 18. 88; 19. 387.
Down: 1. 36; 2. 335; 3. 12; 4. 152; 5. 30; 7. 5625; 8. 1260; 11. 628; 13. 753; 14. 90; 16. 68; 17. 97.

Who's My Friend A, GEORGE; B, SARAH; C, TANYA; D, PADDY; E, SARAH.

Biblical Cross Figure Across: 1. 319; 4. 66; 6. 969; 7. 70; 8. 1310; 10. 1100; 12. 12; 13. 294; 16. 70; 17. 300.
Down: 1. 39; 2. 16; 3. 9910; 4. 671; 5. 600; 9. 3023; 10. 117; 11. 120; 14. 90; 15. 40.

B.A.

The Great Dodecahedron

Thirty straws 8 cm long and 60 straws 4.9 cm long are needed. There are 12 vertices with 10 arms and 20 vertices with 3 arms. With the 8 cm edges and the 10 arm vertices, an icosahedron is constructed. The 4.9 cm edges and the 3 arm vertices are then used to form a trihedral dimple in each face of the icosahedron (Fig. iv).

The Great Icosahedron

Thirty straws 8 cm long, 120 straws 5.2 cm long, and 120 straws 13.3 cm long are needed. There are 20 vertices with 12 arms, 12 vertices with 10 arms and 60 vertices with 3 arms. With the 8 cm edges and 12 arm vertices, a dodecahedron is constructed. 60 of the 13.3 cm edges and alternate arms of the 10 arm vertices are then used to form a pentagonal pyramid on each face of the dodecahedron (Fig. v). The model is completed by using the 5.2 cm edges, the remaining 13.3 cm edges and the 3 arm vertices to form a trihedral dimple in each face of the pentagonal pyramids (Fig. vi).

Mathematical Models by H. M. Cundy and A. P. Rollett (OUP) and *Polyhedron Models* by Magnus J. Wenninger (CUP) provide detailed information on these and other polyhedra. D.I.B.

WHO'S MY FRIEND? No. 3

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

R	S	T	U	V	W	X	Y	Z
18	19	20	21	22	23	24	25	26

Using the above substitution and the following clues, can you find the names of my friends?

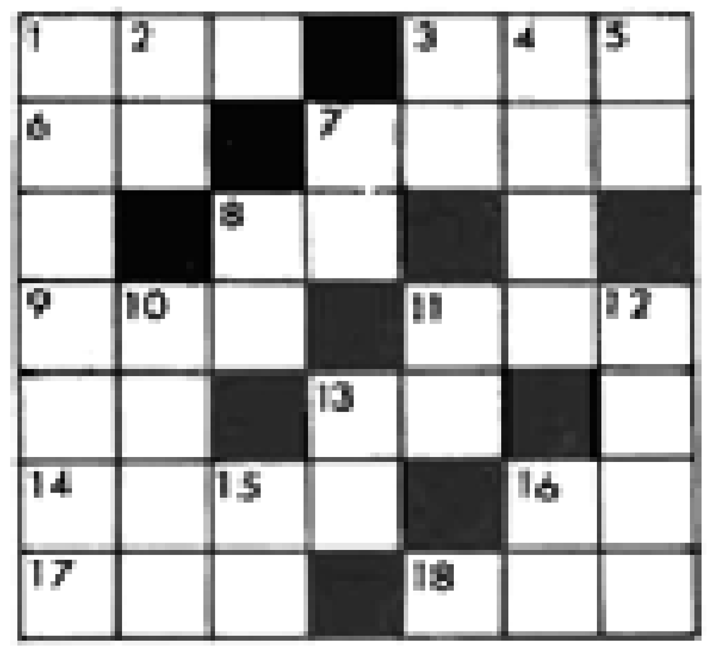
A. Two of the odd squares are here. The double letter is in the 2, 3, 6 and 9 times tables and the first letter is in the 2, 3, 4, 6 and 12 times tables. His twin brother is very similar, only his first letter is changed. He has both cubes as well as the squares.

B. My friend from the Clyde has moved to Loch Lomond, can you find her name? Her three vowels do not include the largest nor the smallest. The double letter is seven times the one we do not know yet and both are even.

C. Issue No. 87 told you of my canine friend. The first letter is even and square; the second follows the first (in the alphabet). Together they add up to make the fifth letter. The double letter is five less than the last letter. The last letter is the highest prime in the teens. What is his name?

D. Can you name my South Seas Island friend? Her two digit even square number appears three times. Both cubic numbers occur once each. The remaining two unknown numbers both appear twice; one is odd, square and has a single digit; the other is found in the 2, 3, 4, 6 and 12 times tables but not in the 8's. A clue to get you started: find the sound 'F' even though there is no 6 to be found. C.B.A.

SENIOR CROSS FIGURE No. 72



Take π as $3^1/_7$ and ignore decimal points in solutions.

CLUES ACROSS

1. Surface area of a cube of volume 125 cm^3.
3. Vertex D of a kite ABCD in which A = (12, 0), B = 0, 5) and C = (3, 9).
6. Area of a semicircle which has a perimeter of 36 cm.
7. Surface area of a sphere which has a volume of 38,808 cm^3.
8. Perimeter (in units) of ABCD in 3 across.
9. 1000.01 (base-two) expressed in base-ten.
11. Gradient, when $x = 8.75$ in $y = 5x^2 - 17x + 14$.
13. Value of y when $x = 8$ on the straight line which passes through the points (2, 8) and (5, 17).
14. Area of a circle which has a diameter four times greater than that of another circle of area 64.5 cm^2.
16. Average speed (km h^{-1}) to travel 176 km in 2 hours 45 minutes.
17. Interior angle of a regular pentagon.
18. Value of x if $\log.x = \log.600 - \log.4$.

CLUES DOWN

1. $81^{\frac{3}{4}} \times 16^{1\frac{1}{2}}$.
2. Principal sum of money invested if simple interest = £51.30, rate = 18% per year and time = 5 years.
3. Gradient of CD in 3 across.
4. Roots (smaller first) of equation $5x^2 - 17x + 14 = 0$.
5. $\begin{pmatrix} 1 & 2 \\ 4 & 9 \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix}$.
7. Price of an item without VAT if its price, including VAT of 15%, is £64.40.
8. Larger value of x when $\dfrac{15}{2x+3} + \dfrac{7}{4x} = 2$.
10. 233rd term of the series: 28, 36.5, 45, 53.5, . . .
11. Angle ABC (to nearest degree) in 3 across.
12. Number of different arrangements that can be made by using all the letters of the word COMPUTE.
13. Area (units^2) bounded by $y = 0, x = 0, x = 2$ and the curve $y = 3x^2 + 2x + 5$.
14. $424 \div 34$ in base-five.
15. Value of x when $\dfrac{4}{5x+1} = \dfrac{1}{5}$.
16. New position $\begin{pmatrix} x \\ y \end{pmatrix}$ of D in 3 across after it has been rotated about C through 90° clockwise.

D.I.B.

Fig. (i)
Sections of pipe-cleaner twisted together form arms of the "vertices" on which drinking-straw "edges" are pushed.

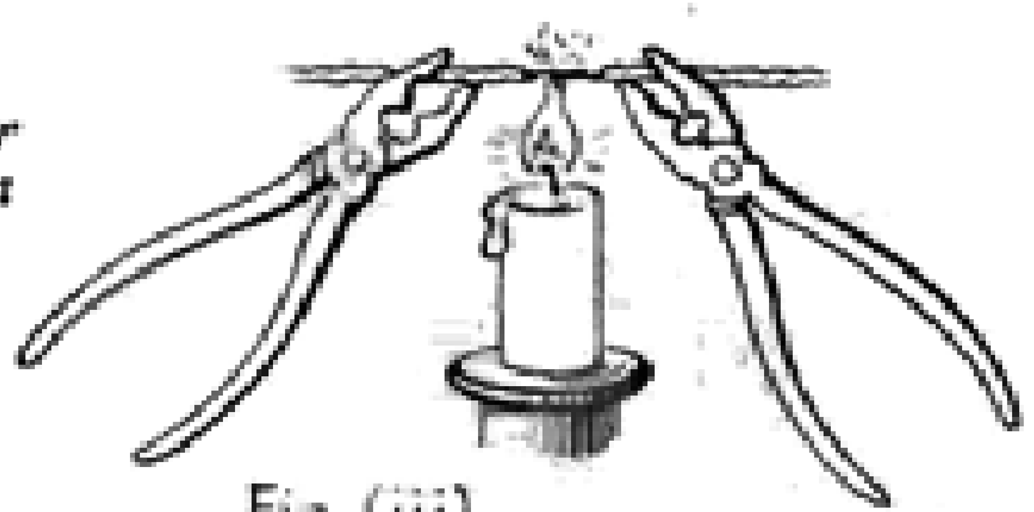


Fig. (iii)

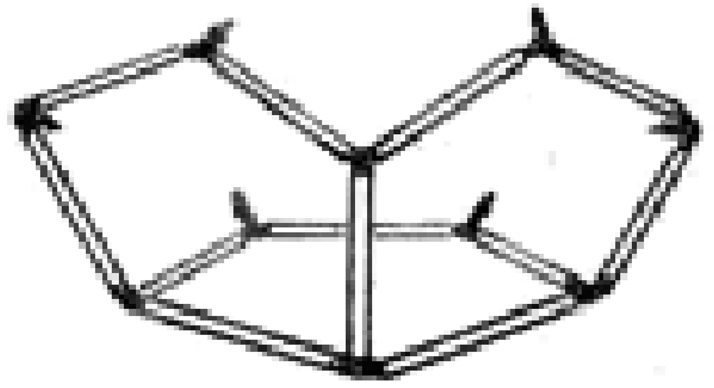


Fig. (ii)
Example of construction method showing how the dodecahedron is built up from a pentagon base.

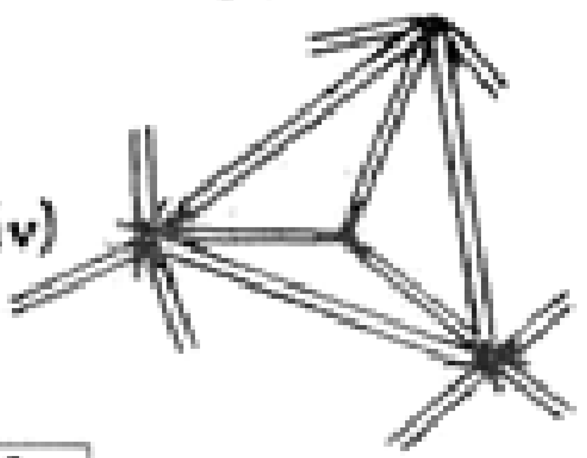


Fig. (iv)

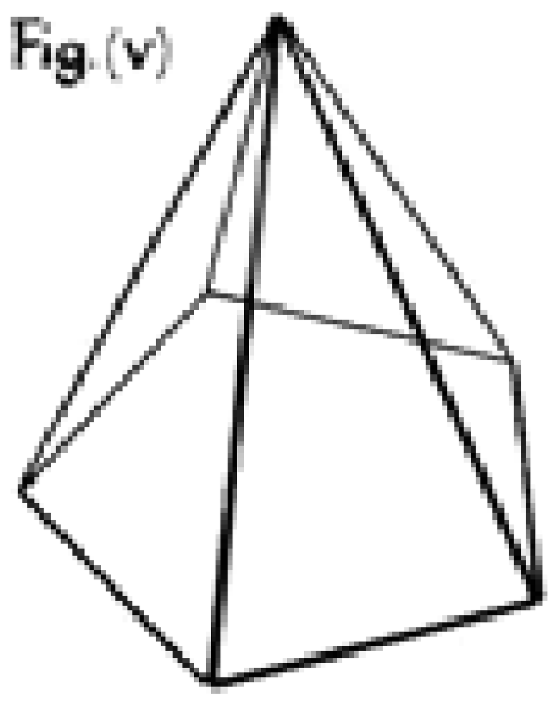
Polyhedron	No. of equal-length (e.g. 8cm) straw edges.	No. of arms per pipe-cleaner vertex	No. of vertices	No. of faces
Tetrahedron	6	3	4	4 equilat. △'s
Cube	12	3	8	6 squares
Octahedron	12	4	6	8 equilat. △'s
Dodecahedron	30	3	20	12 reg. pentagons
Icosahedron	30	5	12	20 equilat. △'s

The five Platonic solids were described in Issue No. 3. Each solid is convex and has regular congruent faces with identical vertices. It is easy to prove that no other regular solids of this type can exist.

If the faces of the dodecahedron and the icosahedron are extended, four non-convex regular polyhedra are formed. These are illustrated on the front page. The small stellated dodecahedron, Fig. (d), and the great stellated dodecahedron, Fig. (c), were discovered by the German astronomer, Johann Kepler (1571–1630) and the great dodecahedron, Fig. (b), and the great icosahedron, Fig. (a), by the French mathematician Louis Poinsot (1777–1859). These four polyhedra are often called the Kepler–Poinsot solids.

Plastic drinking straws and pipe cleaners provide an attractive medium for constructing models of polyhedra. For the Platonic solids use equal straw lengths for the edges – 8 cm is a convenient size. The Kepler–Poinsot solids are more complicated and the method of construction is shown in Figs i–iv. As the Kepler–Poinsot polyhedra have up to twelve arms per vertex, "bunching" would occur when the pipe cleaners are twisted together. To avoid this, the fluffy outer layer of the pipe cleaner is burned away from the middle section exposing the thin wire for twisting (Fig. iii). Gripped by a pair of pliers in each hand, the required length of pipe cleaner is held over a candle flame for a few seconds. Proper safety precautions should always be taken when a naked flame is used and children should ensure that they are supervised by an adult.

Fig. (v)



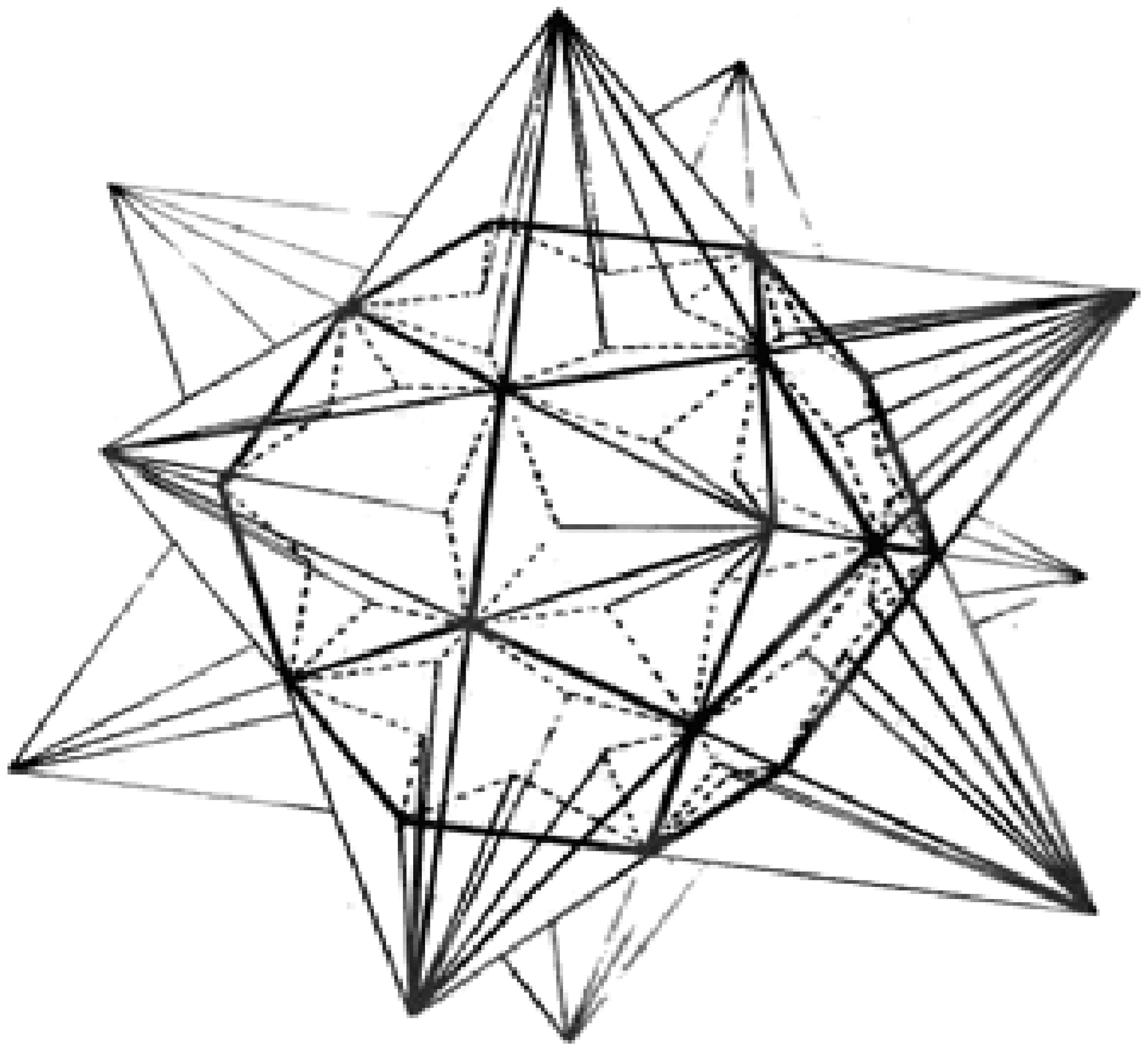
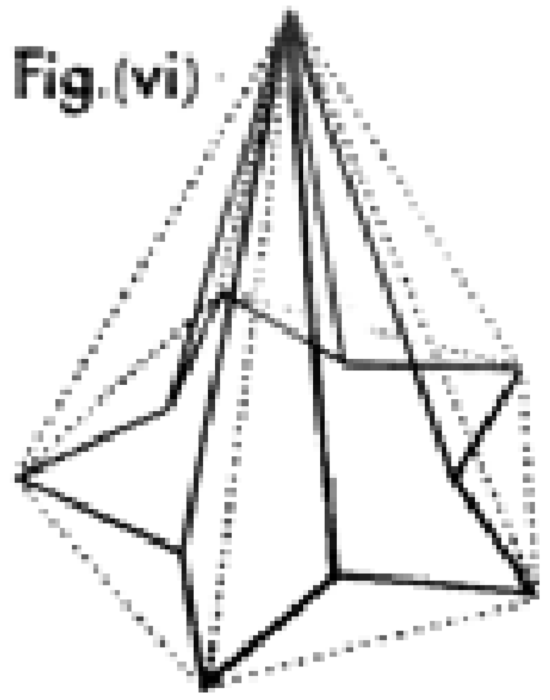


Fig. (vi)



The suggested dimensions give relatively inexpensive models of a convenient size but the dimensions can be increased or decreased proportionately if desired. The pipe cleaners cost £5 for a box of 1,000 from Messrs E. J. Arnold, educational suppliers, and a box of 500 drinking straws costs 40p from most dairies. A little plastic modelling glue applied to the vertices is recommended – though not essential – for the construction of the rather complex 'great icosahedron'.

The Great Stellated Dodecahedron

Thirty straws 8 cm long and 60 straws 12.9 cm long are needed. There are 12 vertices with 10 arms – i.e. 5 pipe cleaner sections – and 20 vertices with 3 arms. With the 8 cm edges and 10 arm vertices, an icosahedron is constructed. The model is completed by using the 12.9 cm edges and 3 arm vertices to form a pyramid on each face of the icosahedron.

The Small Stellated Dodecahedron

Thirty straws 8 cm long and 60 straws 12.9 cm long are needed. There are 20 vertices with 6 arms and 12 vertices with 5 arms. With the 8 cm edges and 6 arm vertices, a dodecahedron is constructed. The model is completed by using the 12.9 cm edges and 5 arm vertices to form a pyramid on each face of the dodecahedron.